

Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/193-
7.3.3-d+e-x-^m-a+b-arctanh-c-xⁿ-^p

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [49]. This is test number [193].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (49)	0.00 (0)
Mathematica	97.96 (48)	2.04 (1)
Maple	97.96 (48)	2.04 (1)
Maxima	59.18 (29)	40.82 (20)
Fricas	34.69 (17)	65.31 (32)
Mupad	34.69 (17)	65.31 (32)
Giac	34.69 (17)	65.31 (32)
Sympy	20.41 (10)	79.59 (39)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

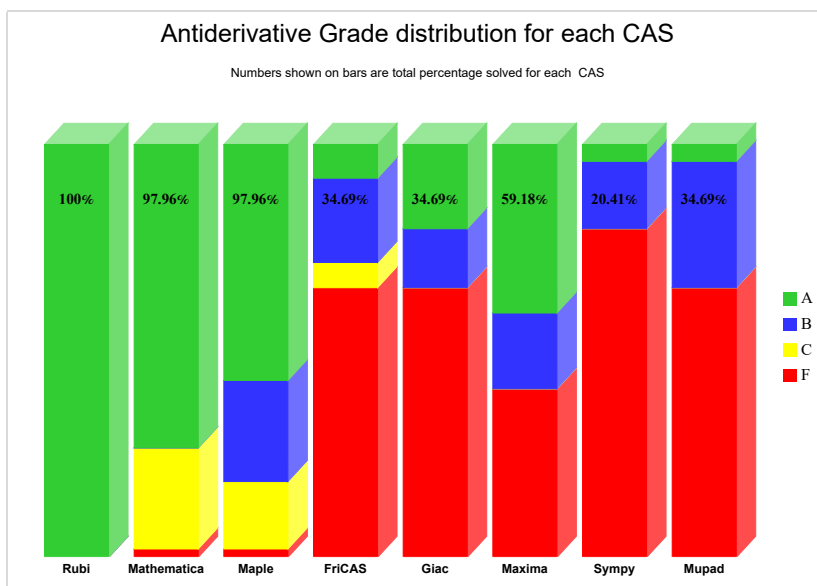
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

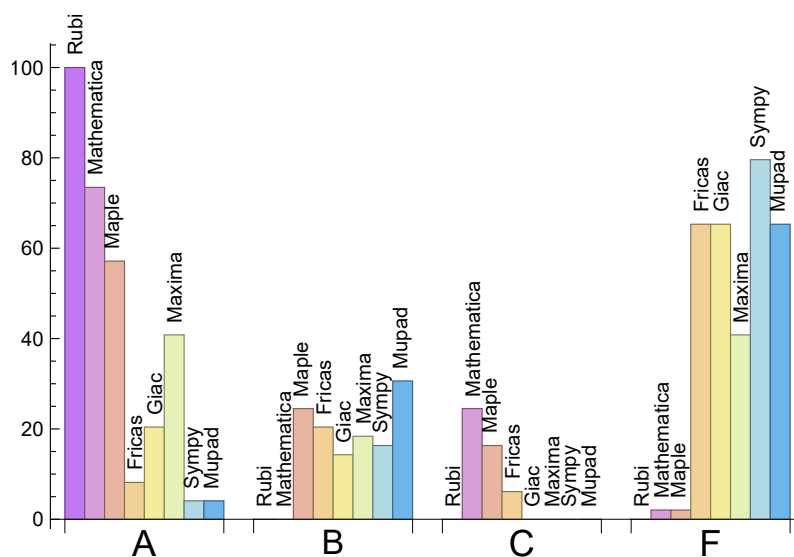
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	73.47	0.00	24.49	2.04
Maple	57.14	24.49	16.33	2.04
Maxima	40.82	18.37	0.00	40.82
Giac	20.41	14.29	0.00	65.31
Fricas	8.16	20.41	6.12	65.31
Mupad	N/A	30.61	0.00	65.31
Sympy	4.08	16.33	0.00	79.59

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Fricas	32	100.00 %	0.00 %	0.00 %
Giac	32	100.00 %	0.00 %	0.00 %
Maxima	20	100.00 %	0.00 %	0.00 %
Sympy	39	66.67 %	33.33 %	0.00 %
Mupad	32	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

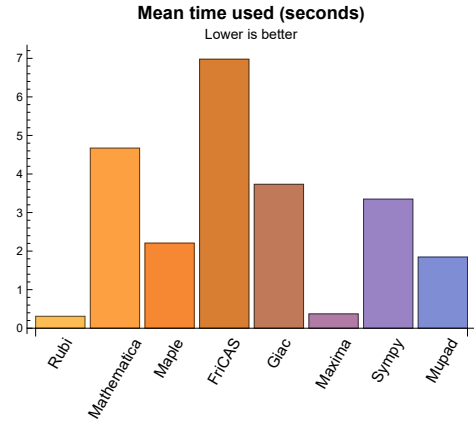
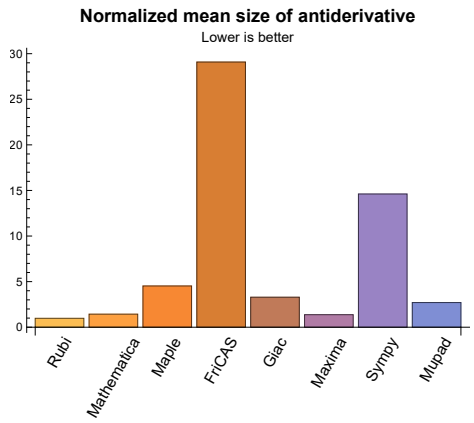
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.31	270.80	0.98	192.00	1.00
Mathematica	4.67	348.40	1.43	273.00	1.08
Maple	2.21	2051.25	4.52	327.50	1.58
Maxima	0.37	234.34	1.37	208.00	1.33
Fricas	6.98	8906.29	29.08	975.00	6.25
Sympy	3.35	2278.60	14.61	721.50	6.93
Giac	3.73	500.59	3.29	279.00	1.60
Mupad	1.85	592.82	2.70	309.00	2.07

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



1.4 list of integrals that has no closed form antiderivative

{30, 31}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {18, 21, 22}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

The following fields are present only in Rubi Table file

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 15, 16, 17, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49 }

B grade: { }

C grade: { 5, 12, 13, 14, 18, 19, 21, 22, 26, 34, 44, 46 }

F grade: { 20 }

2.1.3 Maple

A grade: { 4, 5, 6, 7, 8, 14, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49 }

B grade: { 1, 2, 3, 9, 10, 11, 13, 39, 40, 41, 42, 43 }

C grade: { 12, 15, 16, 17, 18, 19, 20, 34 }

F grade: { 29 }

2.1.4 Maxima

A grade: { 2, 3, 4, 6, 7, 8, 23, 24, 25, 27, 28, 30, 31, 32, 33, 35, 36, 37, 38, 39 }

B grade: { 1, 9, 10, 11, 22, 40, 41, 42, 43 }

C grade: { }

F grade: { 5, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 29, 34, 44, 45, 46, 47, 48, 49 }

2.1.5 FriCAS

A grade: { 4, 25, 30, 31 }

B grade: { 1, 2, 3, 6, 7, 8, 23, 24, 27, 28 }

C grade: { 32, 33, 35 }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 29, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

2.1.6 Sympy

A grade: { 4, 25 }

B grade: { 1, 2, 3, 6, 7, 8, 23, 24 }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

2.1.7 Giac

A grade: { 23, 24, 25, 27, 28, 30, 31, 32, 33, 35 }

B grade: { 1, 2, 3, 4, 6, 7, 8 }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 29, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

2.1.8 Mupad

A grade: { 30, 31 }

B grade: { 1, 2, 3, 4, 6, 7, 8, 23, 24, 25, 27, 28, 32, 33, 35 }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 29, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbrev-

	Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
viated to MMA.	grade	A	A	A	B	B	B	B	B	B
	verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
	size	149	149	274	353	269	975	381	1576	272
	N.S.	1	1.00	1.84	2.37	1.81	6.54	2.56	10.58	1.83
	time (sec)	N/A	0.107	0.085	0.148	0.258	0.387	0.531	0.434	1.696

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	205	280	207	594	279	970	197
N.S.	1	1.00	1.64	2.24	1.66	4.75	2.23	7.76	1.58
time (sec)	N/A	0.105	0.063	0.142	0.262	0.451	0.402	0.420	1.150

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	129	204	137	316	178	532	127
N.S.	1	1.00	1.34	2.12	1.43	3.29	1.85	5.54	1.32
time (sec)	N/A	0.087	0.049	0.141	0.261	0.388	0.306	0.440	0.941

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	96	99	85	137	92	245	67
N.S.	1	1.00	1.14	1.18	1.01	1.63	1.10	2.92	0.80
time (sec)	N/A	0.057	0.011	0.089	0.267	0.382	0.220	0.418	0.827

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	257	158	0	0	0	0	-1
N.S.	1	1.00	2.25	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.239	0.753	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	102	128	99	393	690	243	115
N.S.	1	1.00	1.10	1.38	1.06	4.23	7.42	2.61	1.24
time (sec)	N/A	0.048	0.099	0.151	0.335	0.467	1.271	0.415	3.596

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	133	158	184	1555	3216	809	427
N.S.	1	1.00	1.02	1.22	1.42	11.96	24.74	6.22	3.28
time (sec)	N/A	0.100	0.101	0.188	0.268	0.468	3.257	0.423	3.998

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	173	227	322	4030	10946	1792	418
N.S.	1	1.00	0.99	1.30	1.84	23.03	62.55	10.24	2.39
time (sec)	N/A	0.140	0.186	0.215	0.269	0.635	5.318	0.435	2.295

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	506	1373	808	0	0	0	-1
N.S.	1	1.00	1.41	3.82	2.25	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.384	0.637	0.376	0.429	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	319	1011	548	0	0	0	-1
N.S.	1	1.00	1.24	3.93	2.13	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	0.431	0.348	0.420	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	174	442	329	0	0	0	-1
N.S.	1	1.00	1.09	2.76	2.06	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.257	0.281	0.407	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	938	1197	0	0	0	0	-1
N.S.	1	1.00	4.99	6.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	10.586	5.763	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	317	649	0	0	0	0	-1
N.S.	1	1.00	0.99	2.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	3.571	1.234	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	470	828	0	0	0	0	-1
N.S.	1	1.00	0.98	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	6.002	1.355	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	830	6004	0	0	0	0	-1
N.S.	1	1.00	1.35	9.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.965	1.399	27.531	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	591	4520	0	0	0	0	-1
N.S.	1	1.00	1.53	11.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.552	1.285	16.236	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	331	12134	0	0	0	0	-1
N.S.	1	1.00	1.36	49.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.425	0.437	6.658	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	2179	2415	0	0	0	0	-1
N.S.	1	1.00	8.01	8.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.042	31.923	11.467	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	990	3591	0	0	0	0	-1
N.S.	1	1.00	1.91	6.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	11.584	8.039	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	953	953	0	53542	0	0	0	0	-1
N.S.	1	1.00	0.00	56.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.756	62.534	15.394	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	109	240	101	0	0	0	0	-1
N.S.	1	1.63	3.58	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.180	0.264	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	108	272	127	144	0	0	0	-1
N.S.	1	1.23	3.09	1.44	1.64	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.066	0.184	0.459	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	254	276	238	1137	2786	227	823
N.S.	1	1.00	1.40	1.52	1.31	6.25	15.31	1.25	4.52
time (sec)	N/A	0.200	0.340	0.348	0.471	0.404	8.669	7.829	2.045

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	170	205	171	677	3465	171	309
N.S.	1	1.00	1.08	1.30	1.08	4.28	21.93	1.08	1.96
time (sec)	N/A	0.133	0.097	0.283	0.465	0.461	7.494	1.309	1.385

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	104	92	97	315	753	135	242
N.S.	1	1.00	0.89	0.79	0.83	2.69	6.44	1.15	2.07
time (sec)	N/A	0.087	0.040	0.153	0.472	0.361	6.011	0.553	1.310

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	285	362	0	0	0	0	-1
N.S.	1	1.00	0.88	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.414	13.633	0.142	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	261	181	171	1762	0	269	727
N.S.	1	1.00	1.57	1.09	1.03	10.61	0.00	1.62	4.38
time (sec)	N/A	0.199	0.239	0.334	0.461	2.568	0.000	0.447	2.598

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	379	310	300	8795	0	362	2016
N.S.	1	1.00	1.68	1.37	1.33	38.92	0.00	1.60	8.92
time (sec)	N/A	0.261	0.456	0.342	0.480	19.412	0.000	22.470	5.280

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1085	1085	684	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.340	2.129	0.056	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	30.389	0.098	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	25.991	0.096	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	299	482	295	79566	0	346	1081
N.S.	1	1.00	0.89	1.43	0.88	236.80	0.00	1.03	3.22
time (sec)	N/A	0.530	0.171	0.249	0.465	6.555	0.000	11.887	1.819

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	333	363	250	14519	0	279	621
N.S.	1	1.00	1.17	1.27	0.88	50.94	0.00	0.98	2.18
time (sec)	N/A	0.310	0.069	0.145	0.471	1.294	0.000	1.086	1.033

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	515	182	0	0	0	0	-1
N.S.	1	1.00	0.98	0.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.711	69.941	0.177	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	534	591	393	36636	0	554	2638
N.S.	1	1.00	1.29	1.43	0.95	88.49	0.00	1.34	6.37
time (sec)	N/A	0.522	0.378	0.299	0.468	84.382	0.000	14.891	1.405

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	160	277	246	0	0	0	-1
N.S.	1	1.00	0.82	1.42	1.26	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.418	0.479	0.456	0.411	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	130	244	208	0	0	0	-1
N.S.	1	1.00	0.81	1.52	1.30	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.301	0.291	0.424	0.394	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	96	211	166	0	0	0	-1
N.S.	1	1.00	0.80	1.76	1.38	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.141	0.362	0.401	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	75	162	101	0	0	0	-1
N.S.	1	1.00	0.96	2.08	1.29	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.060	0.223	0.571	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	72	217	159	0	0	0	-1
N.S.	1	1.00	1.04	3.14	2.30	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.079	0.252	0.373	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	118	278	248	0	0	0	-1
N.S.	1	1.00	1.01	2.38	2.12	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.270	0.222	0.260	0.409	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	158	311	291	0	0	0	-1
N.S.	1	1.00	1.01	1.98	1.85	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.342	0.387	0.271	0.415	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	187	344	330	0	0	0	-1
N.S.	1	1.00	0.97	1.79	1.72	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.405	0.558	0.277	0.412	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	558	706	0	0	0	0	-1
N.S.	1	1.00	1.21	1.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.576	2.659	0.648	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	337	589	0	0	0	0	-1
N.S.	1	1.00	0.90	1.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	1.027	0.638	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	432	505	0	0	0	0	-1
N.S.	1	1.00	1.36	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.230	1.481	0.559	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	302	540	0	0	0	0	-1
N.S.	1	1.00	0.84	1.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	0.826	0.645	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	360	683	0	0	0	0	-1
N.S.	1	1.00	0.87	1.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.534	1.129	0.671	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	394	808	0	0	0	0	-1
N.S.	1	1.00	0.78	1.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.638	1.829	0.649	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [36] had the largest ratio of [26]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	16	0.250
2	A	6	4	1.00	16	0.250
3	A	6	4	1.00	16	0.250
4	A	6	4	1.00	14	0.286
5	A	4	4	1.00	16	0.250
6	A	6	4	1.00	16	0.250
7	A	4	3	1.00	16	0.188
8	A	4	3	1.00	16	0.188
9	A	19	14	1.00	18	0.778
10	A	15	12	1.00	18	0.667
11	A	12	9	1.00	16	0.562
12	A	1	1	1.00	18	0.056
13	A	12	6	1.00	18	0.333
14	A	18	10	1.00	18	0.556
15	A	29	15	1.00	18	0.833
16	A	20	13	1.00	18	0.722
17	A	14	10	1.00	16	0.625
18	A	1	1	1.00	18	0.056
19	A	9	7	1.00	18	0.389
20	A	21	11	1.00	18	0.611
21	A	4	4	1.63	17	0.235
22	A	4	4	1.23	15	0.267
23	A	13	10	1.00	18	0.556
24	A	11	9	1.00	18	0.500
25	A	10	8	1.00	16	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	19	8	1.00	18	0.444
27	A	9	6	1.00	18	0.333
28	A	9	6	1.00	18	0.333
29	A	77	24	1.00	18	1.333
30	A	0	0	0.00	0	0.000
31	A	0	0	0.00	0	0.000
32	A	24	14	1.00	18	0.778
33	A	22	12	1.00	16	0.750
34	A	25	8	1.00	18	0.444
35	A	19	11	1.00	18	0.611
36	A	19	10	1.00	26	0.385
37	A	14	10	1.00	26	0.385
38	A	9	9	1.00	24	0.375
39	A	5	5	1.00	23	0.217
40	A	5	7	1.00	26	0.269
41	A	9	9	1.00	26	0.346
42	A	14	9	1.00	26	0.346
43	A	20	9	1.00	26	0.346
44	A	20	11	1.00	23	0.478
45	A	15	10	1.00	21	0.476
46	A	11	6	1.00	20	0.300
47	A	15	11	1.00	23	0.478
48	A	19	13	1.00	23	0.565
49	A	24	13	1.00	23	0.565

Chapter 3

Listing of integrals

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3.1 $\int (d + ex)^4 (a + b \tanh^{-1}(cx)) dx$

Optimal. Leaf size=149

$$\frac{bde(2c^2d^2 + e^2)x}{c^3} + \frac{be^2(10c^2d^2 + e^2)x^2}{10c^3} + \frac{bde^3x^3}{3c} + \frac{be^4x^4}{20c} + \frac{(d + ex)^5 (a + b \tanh^{-1}(cx))}{5e} + \frac{b(cd + e)^5 \log(1 - cx)}{10c^5e}$$

[Out] b*d*e*(2*c^2*d^2+e^2)*x/c^3+1/10*b*e^2*(10*c^2*d^2+e^2)*x^2/c^3+1/3*b*d*e^3*x^3/c+1/20*b*e^4*x^4/c+1/5*(e*x+d)^5*(a+b*arctanh(c*x))/e+1/10*b*(c*d+e)^5*ln(-c*x+1)/c^5/e-1/10*b*(c*d-e)^5*ln(c*x+1)/c^5/e

Rubi [A]

time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6063, 716, 647, 31}

$$\frac{(d + ex)^5 (a + b \tanh^{-1}(cx))}{5e} - \frac{b(cd - e)^5 \log(cx + 1)}{10c^5e} + \frac{b(cd + e)^5 \log(1 - cx)}{10c^5e} + \frac{be^2x^2(10c^2d^2 + e^2)}{10c^3} + \frac{bdex(2c^2d^2 + e^2)}{c^3} + \frac{bde^3x^3}{3c} + \frac{be^4x^4}{20c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^4*(a + b*ArcTanh[c*x]),x]

[Out] (b*d*e*(2*c^2*d^2 + e^2)*x)/c^3 + (b*e^2*(10*c^2*d^2 + e^2)*x^2)/(10*c^3) + (b*d*e^3*x^3)/(3*c) + (b*e^4*x^4)/(20*c) + ((d + e*x)^5*(a + b*ArcTanh[c*x]))/(5*e) + (b*(c*d + e)^5*Log[1 - c*x])/(10*c^5*e) - (b*(c*d - e)^5*Log[1 + c*x])/(10*c^5*e)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 647

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[(-a)*c]

Rule 716

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol
] := Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned} \int (d + ex)^4 (a + b \tanh^{-1}(cx)) dx &= \frac{(d + ex)^5 (a + b \tanh^{-1}(cx))}{5e} - \frac{(bc) \int \frac{(d+ex)^5}{1-c^2x^2} dx}{5e} \\ &= \frac{(d + ex)^5 (a + b \tanh^{-1}(cx))}{5e} - \frac{(bc) \int \left(-\frac{5de^2(2c^2d^2+e^2)}{c^4} - \frac{e^3(10c^2d^2+e^2)x}{c^4} \right) dx}{5e} \\ &= \frac{bde(2c^2d^2 + e^2)x}{c^3} + \frac{be^2(10c^2d^2 + e^2)x^2}{10c^3} + \frac{bde^3x^3}{3c} + \frac{be^4x^4}{20c} + \frac{(d + ex)^5}{5e} \\ &= \frac{bde(2c^2d^2 + e^2)x}{c^3} + \frac{be^2(10c^2d^2 + e^2)x^2}{10c^3} + \frac{bde^3x^3}{3c} + \frac{be^4x^4}{20c} + \frac{(d + ex)^5}{5e} \\ &= \frac{bde(2c^2d^2 + e^2)x}{c^3} + \frac{be^2(10c^2d^2 + e^2)x^2}{10c^3} + \frac{bde^3x^3}{3c} + \frac{be^4x^4}{20c} + \frac{(d + ex)^5}{5e} \end{aligned}$$

Mathematica [A]

time = 0.09, size = 274, normalized size = 1.84

$\frac{60c^2d(ac^2d^2 + be(2c^2d^2 + e^2))x + 6c^2(20ac^3d^2 + be(10c^2d^2 + e^2))x^2 + 20c^2de(6acd + be)x^3 + 3c^2c(20acd + be)x^4 + 12ac^2e^2x^5 + 12bc^2e^2(5d^4 + 10d^3ex + 5de^2x^2 + e^2x^4) \operatorname{arctanh}(cx) + 6b(5c^4d^4 + 10c^2d^2e + 10c^2d^2e^2 + 5de^3 + e^4) \log(1 - cx) + 6b(5c^4d^4 - 10c^2d^2e + 10c^2d^2e^2 - 5de^3 + e^4) \log(1 + cx)}{60c^5}$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (60*c^2*d*(a*c^3*d^3 + b*e*(2*c^2*d^2 + e^2))*x + 6*c^2*e*(20*a*c^3*d^3 + b
*e*(10*c^2*d^2 + e^2))*x^2 + 20*c^4*d*e^2*(6*a*c*d + b*e))*x^3 + 3*c^4*e^3*(
20*a*c*d + b*e)*x^4 + 12*a*c^5*e^4*x^5 + 12*b*c^5*x*(5*d^4 + 10*d^3*e*x + 1
0*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4)*ArcTanh[c*x] + 6*b*(5*c^4*d^4 + 10*c
^3*d^3*e + 10*c^2*d^2*e^2 + 5*c*d*e^3 + e^4)*Log[1 - c*x] + 6*b*(5*c^4*d^4
- 10*c^3*d^3*e + 10*c^2*d^2*e^2 - 5*c*d*e^3 + e^4)*Log[1 + c*x])/(60*c^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(137) = 274.

time = 0.15, size = 353, normalized size = 2.37

method	result
derivativdivides	$\frac{b \ln(cx+1)d^4}{2} + \frac{b \ln(cx-1)d^4}{2} + bc e^3 \operatorname{arctanh}(cx) d x^4 + 2be d^3 x + \frac{b e^3 dx}{e^2} + b e^2 d^2 x^2 + \frac{b e^3 d x^3}{3} + \frac{bc e^4 \operatorname{arctanh}(cx) x^5}{5} + \frac{b e^3 \ln(cx-1)}{2c^3}$

default	$\frac{b \ln(cx+1)d^4}{2} + \frac{b \ln(cx-1)d^4}{2} + bc e^3 \operatorname{arctanh}(cx) d x^4 + 2be d^3 x + \frac{b e^3 d x}{c^2} + b e^2 d^2 x^2 + \frac{b e^3 d x^3}{3} + \frac{bc e^4 \operatorname{arctanh}(cx) x^5}{5} + \frac{b e^3 \ln(cx-1)}{2c^3}$
risch	$\frac{b e^4 x^4}{20c} + e^3 a d x^4 + 2e^2 a d^2 x^3 + 2e a d^3 x^2 + a d^4 x + \frac{e^2 b d^2 x^2}{c} + \frac{2eb d^3 x}{c} + \frac{e^3 b d x}{c^3} - \frac{e \ln(cx+1) b d^3}{c^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^4*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)`

[Out] $1/c*(1/2*b*\ln(c*x+1)*d^4+1/2*b*\ln(c*x-1)*d^4+2*b*e*d^3*x+b/c^2*e^3*d*x+b*e^2*d^2*x^2+1/3*b*e^3*d*x^3+1/5*b*c*e^4*\operatorname{arctanh}(c*x)*x^5+2*b*c*e*\operatorname{arctanh}(c*x)*d^3*x^2+2*b*c*e^2*\operatorname{arctanh}(c*x)*d^2*x^3+b*c*e^3*\operatorname{arctanh}(c*x)*d*x^4+b*\operatorname{arctanh}(c*x)*d^4*c*x+1/2*b/c^3*e^3*\ln(c*x-1)*d+1/10*b/c^2*e^4*x^2+1/20*b*e^4*x^4-1/10*b*c/e*\ln(c*x+1)*d^5-b/c*e*\ln(c*x+1)*d^3+b/c^2*e^2*\ln(c*x+1)*d^2-1/2*b/c^3*e^3*\ln(c*x+1)*d+1/10*b*c/e*\ln(c*x-1)*d^5+b/c*e*\ln(c*x-1)*d^3+b/c^2*e^2*\ln(c*x-1)*d^2+1/5*b*c/e*\operatorname{arctanh}(c*x)*d^5+1/10*b/c^4*e^4*\ln(c*x+1)+1/10*b/c^4*e^4*\ln(c*x-1)+1/5*(c*e*x+c*d)^5*a/c^4/e)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(133) = 266.

time = 0.26, size = 269, normalized size = 1.81

$$\frac{1}{5} a^2 x^4 + a d x^4 + 2 a d^2 x^3 + 2 a d^3 x^2 + a d^4 x + \left(2 x \operatorname{arctanh}(c x) + c \left(\frac{2 x}{c^2} - \frac{\log(c x+1)}{c^2} + \frac{\log(c x-1)}{c^2} \right) \right) b d^4 + \left(2 c x \operatorname{arctanh}(c x) + \frac{\log(-c^2 x^2+1)}{2 c} \right) b d^3 + \left(2 x^2 \operatorname{arctanh}(c x) + c \left(\frac{x^2}{c^2} - \frac{\log(c^2 x^2-1)}{c^2} \right) \right) b d^2 + \frac{1}{5} \left(6 x^4 \operatorname{arctanh}(c x) + c \left(\frac{2(c^2 x^2+3 x)}{c^4} - \frac{3 \log(c x+1)}{c^4} + \frac{3 \log(c x-1)}{c^4} \right) \right) b d x + \frac{1}{20} \left(4 x^5 \operatorname{arctanh}(c x) + c \left(\frac{c^2 x^4+2 x^2}{c^4} + \frac{2 \log(c^2 x^2-1)}{c^4} \right) \right) b e^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")`

[Out] $1/5*a*x^5*e^4 + a*d*x^4*e^3 + 2*a*d^2*x^3*e^2 + 2*a*d^3*x^2*e + a*d^4*x + (2*x^2*\operatorname{arctanh}(c*x) + c*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3))*b*d^3*e + 1/2*(2*c*x*\operatorname{arctanh}(c*x) + \log(-c^2*x^2 + 1))*b*d^4/c + (2*x^3*\operatorname{arctanh}(c*x) + c*(x^2/c^2 + \log(c^2*x^2 - 1)/c^4))*b*d^2*e^2 + 1/6*(6*x^4*\operatorname{arctanh}(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*\log(c*x + 1)/c^5 + 3*\log(c*x - 1)/c^5))*b*d*e^3 + 1/20*(4*x^5*\operatorname{arctanh}(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*\log(c^2*x^2 - 1)/c^6))*b*e^4$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 975 vs. 2(133) = 266.

time = 0.39, size = 975, normalized size = 6.54

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")`

[Out] $1/60*(60*a*c^5*d^4*x + 3*(4*a*c^5*x^5 + b*c^4*x^4 + 2*b*c^2*x^2)*\cosh(1)^4 + 3*(4*a*c^5*x^5 + b*c^4*x^4 + 2*b*c^2*x^2)*\sinh(1)^4 + 20*(3*a*c^5*d*x^4 +$

$$\begin{aligned}
& b*c^4*d*x^3 + 3*b*c^2*d*x)*\cosh(1)^3 + 4*(15*a*c^5*d*x^4 + 5*b*c^4*d*x^3 + \\
& 15*b*c^2*d*x + 3*(4*a*c^5*x^5 + b*c^4*x^4 + 2*b*c^2*x^2)*\cosh(1))*\sinh(1)^3 \\
& + 60*(2*a*c^5*d^2*x^3 + b*c^4*d^2*x^2)*\cosh(1)^2 + 6*(20*a*c^5*d^2*x^3 + \\
& 10*b*c^4*d^2*x^2 + 3*(4*a*c^5*x^5 + b*c^4*x^4 + 2*b*c^2*x^2)*\cosh(1))^2 + 10 \\
& *(3*a*c^5*d*x^4 + b*c^4*d*x^3 + 3*b*c^2*d*x)*\cosh(1))*\sinh(1)^2 + 120*(a*c^5 \\
& *d^3*x^2 + b*c^4*d^3*x)*\cosh(1) + 6*(5*b*c^4*d^4 - 10*b*c^3*d^3*\cosh(1) + \\
& 10*b*c^2*d^2*\cosh(1)^2 - 5*b*c*d*\cosh(1)^3 + b*\cosh(1)^4 + b*\sinh(1)^4 - (5 \\
& *b*c*d - 4*b*\cosh(1))*\sinh(1)^3 + (10*b*c^2*d^2 - 15*b*c*d*\cosh(1) + 6*b*\co \\
& sh(1)^2)*\sinh(1)^2 - (10*b*c^3*d^3 - 20*b*c^2*d^2*\cosh(1) + 15*b*c*d*\cosh(1) \\
&)^2 - 4*b*\cosh(1)^3)*\sinh(1))*\log(c*x + 1) + 6*(5*b*c^4*d^4 + 10*b*c^3*d^3* \\
& \cosh(1) + 10*b*c^2*d^2*\cosh(1)^2 + 5*b*c*d*\cosh(1)^3 + b*\cosh(1)^4 + b*\sinh \\
& (1)^4 + (5*b*c*d + 4*b*\cosh(1))*\sinh(1)^3 + (10*b*c^2*d^2 + 15*b*c*d*\cosh(1) \\
&) + 6*b*\cosh(1)^2)*\sinh(1)^2 + (10*b*c^3*d^3 + 20*b*c^2*d^2*\cosh(1) + 15*b* \\
& c*d*\cosh(1)^2 + 4*b*\cosh(1)^3)*\sinh(1))*\log(c*x - 1) + 6*(b*c^5*x^5*\cosh(1) \\
& ^4 + b*c^5*x^5*\sinh(1)^4 + 5*b*c^5*d*x^4*\cosh(1)^3 + 10*b*c^5*d^2*x^3*\cosh(\\
& 1)^2 + 10*b*c^5*d^3*x^2*\cosh(1) + 5*b*c^5*d^4*x + (4*b*c^5*x^5*\cosh(1) + 5* \\
& b*c^5*d*x^4)*\sinh(1)^3 + (6*b*c^5*x^5*\cosh(1)^2 + 15*b*c^5*d*x^4*\cosh(1) + \\
& 10*b*c^5*d^2*x^3)*\sinh(1)^2 + (4*b*c^5*x^5*\cosh(1)^3 + 15*b*c^5*d*x^4*\cosh(\\
& 1)^2 + 20*b*c^5*d^2*x^3*\cosh(1) + 10*b*c^5*d^3*x^2)*\sinh(1))*\log(-(c*x + 1) \\
& / (c*x - 1)) + 12*(10*a*c^5*d^3*x^2 + 10*b*c^4*d^3*x + (4*a*c^5*x^5 + b*c^4*x \\
& x^4 + 2*b*c^2*x^2)*\cosh(1))^3 + 5*(3*a*c^5*d*x^4 + b*c^4*d*x^3 + 3*b*c^2*d*x) \\
&)*\cosh(1)^2 + 10*(2*a*c^5*d^2*x^3 + b*c^4*d^2*x^2)*\cosh(1))*\sinh(1))/c^5
\end{aligned}$$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(134) = 268.

time = 0.53, size = 381, normalized size = 2.56

$$\left\{ \begin{array}{l} ad^3x + 2ad^2cx^2 + 2ad^2c^2x^3 + ad^2c^3 + \frac{ad^2c^4}{c} + bd^2x \operatorname{atanh}(cx) + 2bd^2cx^2 \operatorname{atanh}(cx) + 2bd^2c^2x^3 \operatorname{atanh}(cx) + bd^2c^3 \operatorname{atanh}(cx) + \frac{bd^2c^4 \operatorname{atanh}(cx)}{c} + \frac{bd^2 \log(-c-1)}{c} + \frac{bd^2 \operatorname{atanh}(cx)}{c} + 2bd^2c + \frac{bd^2c^2}{c} + \frac{bd^2c^3}{c} + \frac{bd^2c^4}{c} - \frac{2bd^2c \operatorname{atanh}(cx)}{c} - \frac{2bd^2c^2 \log(-c-1)}{c} + \frac{2bd^2c^3 \operatorname{atanh}(cx)}{c} + \frac{bd^2c^4}{c} + \frac{bd^2c^5}{c} - \frac{bd^2 \operatorname{atanh}(cx)}{c} + \frac{bd^2 \log(-c-1)}{c} + \frac{bd^2 \operatorname{atanh}(cx)}{c} \\ a(d^3x + 2d^2cx^2 + 2d^2c^2x^3 + d^2c^3 + \frac{d^2c^4}{c}) \end{array} \right. \text{ for } c \neq 0 \text{ otherwise}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**4*(a+b*atanh(c*x)),x)

[Out] Piecewise((a*d**4*x + 2*a*d**3*e*x**2 + 2*a*d**2*e**2*x**3 + a*d*e**3*x**4 + a*e**4*x**5/5 + b*d**4*x*atanh(c*x) + 2*b*d**3*e*x**2*atanh(c*x) + 2*b*d**2*e**2*x**3*atanh(c*x) + b*d*e**3*x**4*atanh(c*x) + b*e**4*x**5*atanh(c*x)/5 + b*d**4*log(x - 1/c)/c + b*d**4*atanh(c*x)/c + 2*b*d**3*e*x/c + b*d**2*e**2*x**2/c + b*d*e**3*x**3/(3*c) + b*e**4*x**4/(20*c) - 2*b*d**3*e*atanh(c*x)/c**2 + 2*b*d**2*e**2*log(x - 1/c)/c**3 + 2*b*d**2*e**2*atanh(c*x)/c**3 + b*d*e**3*x/c**3 + b*e**4*x**2/(10*c**3) - b*d*e**3*atanh(c*x)/c**4 + b*e**4*log(x - 1/c)/(5*c**5) + b*e**4*atanh(c*x)/(5*c**5), Ne(c, 0)), (a*(d**4*x + 2*d**3*e*x**2 + 2*d**2*e**2*x**3 + d*e**3*x**4 + e**4*x**5/5), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1576 vs. 2(137) = 274.

time = 0.43, size = 1576, normalized size = 10.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")

[Out]
$$\frac{1}{15}c^3(5(c^2x+1)^4b^4d^4/(c^2x-1)^4 - 20(c^2x+1)^3b^4d^4/(c^2x-1)^3 + 30(c^2x+1)^2b^4d^4/(c^2x-1)^2 - 20(c^2x+1)b^4d^4/(c^2x-1) + 5b^4d^4 + 20(c^2x+1)^4b^3d^3e/(c^2x-1)^4 - 60(c^2x+1)^3b^3d^3e/(c^2x-1)^3 + 60(c^2x+1)^2b^3d^3e/(c^2x-1)^2 - 20(c^2x+1)b^3d^3e/(c^2x-1) + 30(c^2x+1)^4b^2d^2e^2/(c^2x-1)^4 - 60(c^2x+1)^3b^2d^2e^2/(c^2x-1)^3 + 40(c^2x+1)^2b^2d^2e^2/(c^2x-1)^2 - 20(c^2x+1)b^2d^2e^2/(c^2x-1) + 10b^2d^2e^2 + 20(c^2x+1)^4b^2d^2e^2/(c^2x-1)^4 - 20(c^2x+1)^3b^2d^2e^2/(c^2x-1)^3 + 20(c^2x+1)^2b^2d^2e^2/(c^2x-1)^2 - 20(c^2x+1)b^2d^2e^2/(c^2x-1) + 5(c^2x+1)^4b^2d^2e^2/(c^2x-1)^4 + 10(c^2x+1)^2b^2d^2e^2/(c^2x-1)^2 + b^2d^2e^2) \cdot \log(-(c^2x+1)/(c^2x-1)) / ((c^2x+1)^5c^6/(c^2x-1)^5 - 5(c^2x+1)^4c^6/(c^2x-1)^4 + 10(c^2x+1)^3c^6/(c^2x-1)^3 - 10(c^2x+1)^2c^6/(c^2x-1)^2 + 5(c^2x+1)c^6/(c^2x-1) - c^6) + 2(15(c^2x+1)^4a^4d^4/(c^2x-1)^4 - 60(c^2x+1)^3a^4d^4/(c^2x-1)^3 + 90(c^2x+1)^2a^4d^4/(c^2x-1)^2 - 60(c^2x+1)a^4d^4/(c^2x-1) + 15a^4d^4 + 60(c^2x+1)^4a^3d^3e/(c^2x-1)^4 - 180(c^2x+1)^3a^3d^3e/(c^2x-1)^3 + 180(c^2x+1)^2a^3d^3e/(c^2x-1)^2 - 60(c^2x+1)a^3d^3e/(c^2x-1) + 30(c^2x+1)^4b^3d^3e/(c^2x-1)^4 - 120(c^2x+1)^3b^3d^3e/(c^2x-1)^3 + 180(c^2x+1)^2b^3d^3e/(c^2x-1)^2 - 120(c^2x+1)b^3d^3e/(c^2x-1) + 30b^3d^3e + 90(c^2x+1)^4a^2d^2e^2/(c^2x-1)^4 - 180(c^2x+1)^3a^2d^2e^2/(c^2x-1)^3 + 120(c^2x+1)^2a^2d^2e^2/(c^2x-1)^2 - 60(c^2x+1)a^2d^2e^2/(c^2x-1) + 30a^2d^2e^2 + 30(c^2x+1)^4b^2d^2e^2/(c^2x-1)^4 - 90(c^2x+1)^3b^2d^2e^2/(c^2x-1)^3 + 90(c^2x+1)^2b^2d^2e^2/(c^2x-1)^2 - 30(c^2x+1)b^2d^2e^2/(c^2x-1) + 60(c^2x+1)^4a^2d^2e^2/(c^2x-1)^4 - 60(c^2x+1)^3a^2d^2e^2/(c^2x-1)^3 + 60(c^2x+1)^2a^2d^2e^2/(c^2x-1)^2 - 60(c^2x+1)a^2d^2e^2/(c^2x-1) + 30(c^2x+1)^4b^2d^2e^2/(c^2x-1)^4 - 90(c^2x+1)^3b^2d^2e^2/(c^2x-1)^3 + 110(c^2x+1)^2b^2d^2e^2/(c^2x-1)^2 - 70(c^2x+1)b^2d^2e^2/(c^2x-1) + 20b^2d^2e^2 + 15(c^2x+1)^4a^2e^4/(c^2x-1)^4 + 30(c^2x+1)^2a^2e^4/(c^2x-1)^2 + 3a^2e^4 + 6(c^2x+1)^4b^2e^4/(c^2x-1)^4 - 12(c^2x+1)^3b^2e^4/(c^2x-1)^3 + 12(c^2x+1)^2b^2e^4/(c^2x-1)^2 - 6(c^2x+1)b^2e^4/(c^2x-1)) / ((c^2x+1)^5c^6/(c^2x-1)^5 - 5(c^2x+1)^4c^6/(c^2x-1)^4 + 10(c^2x+1)^3c^6/(c^2x-1)^3 - 10(c^2x+1)^2c^6/(c^2x-1)^2 + 5(c^2x+1)c^6/(c^2x-1) - c^6) - 3(5b^4c^4d^4 + 10b^4c^2d^2e^2 + b^4e^4) \cdot \log(-(c^2x+1)/(c^2x-1) + 1)/c^6 + 3(5b^4c^4d^4 + 10b^4c^2d^2e^2 + b^4e^4) \cdot \log(-(c^2x+1)/(c^2x-1))/c^6)$$

Mupad [B]

time = 1.70, size = 272, normalized size = 1.83

$$\frac{a^4d^4}{3} + a^4d^4x + \frac{b^4d^4 \ln(c^2x^2-1)}{2c} + \frac{b^4d^4 \ln(c^2x^2-1)}{10c^3} + 2ad^4e^2x^2 + \frac{b^4d^4}{20c} + \frac{b^4d^4}{10c^3} + bd^4x \operatorname{atanh}(cx) + 2ad^4e^2x + ad^4e^2x^2 + \frac{b^4d^4 \operatorname{atanh}(cx)}{5} + \frac{2bd^4e^2x}{c} + \frac{bd^4e^2x}{c^3} - \frac{2bd^4e^2 \operatorname{atanh}(cx)}{c^2} - \frac{bd^4e^2 \operatorname{atanh}(cx)}{c^4} + 2bd^4e^2x^2 \operatorname{atanh}(cx) + bd^4e^2x \operatorname{atanh}(cx) + \frac{bd^4e^2x^2}{3c} + 2bd^4e^2x^2 \operatorname{atanh}(cx) + \frac{bd^4e^2 \ln(c^2x^2-1)}{c^2} + \frac{bd^4e^2x^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c*x))*(d + e*x)^4,x)`

[Out] $(a*e^4*x^5)/5 + a*d^4*x + (b*d^4*\log(c^2*x^2 - 1))/(2*c) + (b*e^4*\log(c^2*x^2 - 1))/(10*c^5) + 2*a*d^2*e^2*x^3 + (b*e^4*x^4)/(20*c) + (b*e^4*x^2)/(10*c^3) + b*d^4*x*atanh(c*x) + 2*a*d^3*e*x^2 + a*d*e^3*x^4 + (b*e^4*x^5*atanh(c*x))/5 + (2*b*d^3*e*x)/c + (b*d*e^3*x)/c^3 - (2*b*d^3*e*atanh(c*x))/c^2 - (b*d*e^3*atanh(c*x))/c^4 + 2*b*d^3*e*x^2*atanh(c*x) + b*d*e^3*x^4*atanh(c*x) + (b*d*e^3*x^3)/(3*c) + 2*b*d^2*e^2*x^3*atanh(c*x) + (b*d^2*e^2*\log(c^2*x^2 - 1))/c^3 + (b*d^2*e^2*x^2)/c$

3.2 $\int (d + ex)^3 (a + b \tanh^{-1}(cx)) dx$

Optimal. Leaf size=125

$$\frac{be(6c^2d^2 + e^2)x}{4c^3} + \frac{bde^2x^2}{2c} + \frac{be^3x^3}{12c} + \frac{(d + ex)^4 (a + b \tanh^{-1}(cx))}{4e} + \frac{b(cd + e)^4 \log(1 - cx)}{8c^4e} - \frac{b(cd - e)^4 \log(1 + cx)}{8c^4e}$$

[Out] 1/4*b*e*(6*c^2*d^2+e^2)*x/c^3+1/2*b*d*e^2*x^2/c+1/12*b*e^3*x^3/c+1/4*(e*x+d)^4*(a+b*arctanh(c*x))/e+1/8*b*(c*d+e)^4*ln(-c*x+1)/c^4/e-1/8*b*(c*d-e)^4*ln(c*x+1)/c^4/e

Rubi [A]

time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6063, 716, 647, 31}

$$\frac{(d + ex)^4 (a + b \tanh^{-1}(cx))}{4e} - \frac{b(cd - e)^4 \log(cx + 1)}{8c^4e} + \frac{b(cd + e)^4 \log(1 - cx)}{8c^4e} + \frac{be(6c^2d^2 + e^2)x}{4c^3} + \frac{bde^2x^2}{2c} + \frac{be^3x^3}{12c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3*(a + b*ArcTanh[c*x]),x]

[Out] (b*e*(6*c^2*d^2 + e^2)*x)/(4*c^3) + (b*d*e^2*x^2)/(2*c) + (b*e^3*x^3)/(12*c) + ((d + e*x)^4*(a + b*ArcTanh[c*x]))/(4*e) + (b*(c*d + e)^4*Log[1 - c*x])/(8*c^4*e) - (b*(c*d - e)^4*Log[1 + c*x])/(8*c^4*e)

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 647

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[(-a)*c]

Rule 716

Int[((d_) + (e_)*(x_))^(m_)/((a_) + (c_)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 6063

Int[((a_) + ArcTanh[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_))^(q_), x_Symbol] := Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b

`*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]`

Rubi steps

$$\begin{aligned} \int (d + ex)^3 (a + b \tanh^{-1}(cx)) dx &= \frac{(d + ex)^4 (a + b \tanh^{-1}(cx))}{4e} - \frac{(bc) \int \frac{(d+ex)^4}{1-c^2x^2} dx}{4e} \\ &= \frac{(d + ex)^4 (a + b \tanh^{-1}(cx))}{4e} - \frac{(bc) \int \left(-\frac{e^2(6c^2d^2+e^2)}{c^4} - \frac{4de^3x}{c^2} - \frac{e^4x^2}{c^2} + \dots \right) dx}{4e} \\ &= \frac{be(6c^2d^2 + e^2)x}{4c^3} + \frac{bde^2x^2}{2c} + \frac{be^3x^3}{12c} + \frac{(d + ex)^4 (a + b \tanh^{-1}(cx))}{4e} - \frac{bc}{4e} \int \frac{(d+ex)^4}{1-c^2x^2} dx \\ &= \frac{be(6c^2d^2 + e^2)x}{4c^3} + \frac{bde^2x^2}{2c} + \frac{be^3x^3}{12c} + \frac{(d + ex)^4 (a + b \tanh^{-1}(cx))}{4e} + \frac{bc}{4e} \int \frac{(d+ex)^4}{1-c^2x^2} dx \\ &= \frac{be(6c^2d^2 + e^2)x}{4c^3} + \frac{bde^2x^2}{2c} + \frac{be^3x^3}{12c} + \frac{(d + ex)^4 (a + b \tanh^{-1}(cx))}{4e} + \frac{bc}{4e} \int \frac{(d+ex)^4}{1-c^2x^2} dx \end{aligned}$$

Mathematica [A]

time = 0.06, size = 205, normalized size = 1.64

$$\frac{6c(4ac^3d^3 + be(6c^2d^2 + e^2))x + 12c^3de(3acd + be)x^2 + 2c^3e^2(12acd + be)x^3 + 6ac^4e^3x^4 + 6bc^4x(4d^3 + 6d^2cx + 4de^2x^2 + e^3x^3) \tanh^{-1}(cx) + 3b(4c^3d^3 + 6c^2d^2e + 4cde^2 + e^3) \log(1 - cx) + 3b(4c^3d^3 - 6c^2d^2e + 4cde^2 - e^3) \log(1 + cx)}{24c^4}$$

Antiderivative was successfully verified.

`[In] Integrate[(d + e*x)^3*(a + b*ArcTanh[c*x]),x]`

`[Out] (6*c*(4*a*c^3*d^3 + b*e*(6*c^2*d^2 + e^2))*x + 12*c^3*d*e*(3*a*c*d + b*e)*x^2 + 2*c^3*e^2*(12*a*c*d + b*e)*x^3 + 6*a*c^4*e^3*x^4 + 6*b*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcTanh[c*x] + 3*b*(4*c^3*d^3 + 6*c^2*d^2*e + 4*c*d*e^2 + e^3)*Log[1 - c*x] + 3*b*(4*c^3*d^3 - 6*c^2*d^2*e + 4*c*d*e^2 - e^3)*Log[1 + c*x])/(24*c^4)`

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(113) = 226.

time = 0.14, size = 280, normalized size = 2.24

method	result
derivativedivides	$\frac{\frac{(ce+dc)^4 a}{4c^3 e} + \frac{bc \operatorname{arctanh}(cx) d^4}{4e} + b \operatorname{arctanh}(cx) d^3 cx + \frac{3bce \operatorname{arctanh}(cx) d^2 x^2}{2} + bce^2 \operatorname{arctanh}(cx) d x^3 + \frac{bc e^3 \operatorname{arctanh}(cx) x^4}{4} + \frac{3be d^2}{2}}$
default	$\frac{(ce+dc)^4 a}{4c^3 e} + \frac{bc \operatorname{arctanh}(cx) d^4}{4e} + b \operatorname{arctanh}(cx) d^3 cx + \frac{3bce \operatorname{arctanh}(cx) d^2 x^2}{2} + bce^2 \operatorname{arctanh}(cx) d x^3 + \frac{bc e^3 \operatorname{arctanh}(cx) x^4}{4} + \frac{3be d^2}{2}$

risch	$\frac{(ex+d)^4 b \ln(cx+1)}{8e} - \frac{e^3 b x^4 \ln(-cx+1)}{8} - \frac{e^2 b d x^3 \ln(-cx+1)}{2} + \frac{e^3 a x^4}{4} - \frac{3eb d^2 x^2 \ln(-cx+1)}{4} + e^2 a d x^3 -$
-------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{c} \left(\frac{1}{4} (c e^x + c d)^4 \frac{a}{c^3} e^{1/4} \frac{b}{c} e^{\arctanh(c x)} d^4 + b \arctanh(c x) d^3 e^x + \frac{3}{2} b c e^{\arctanh(c x)} d^2 x^2 + b c e^{2 \arctanh(c x)} d x^3 + \frac{1}{4} b c e^3 \arctanh(c x) x^4 + \frac{3}{2} b e d^2 x + \frac{1}{2} b e^2 d x^2 + \frac{1}{12} b e^3 x^3 + \frac{1}{4} \frac{b}{c^2} e^3 x + \frac{1}{8} \frac{b}{c} e \ln(c x - 1) d^4 + \frac{1}{2} \frac{b}{c} \ln(c x - 1) d^3 + \frac{3}{4} \frac{b}{c} e \ln(c x - 1) d^2 + \frac{1}{2} \frac{b}{c^2} e^2 \ln(c x - 1) d + \frac{1}{8} \frac{b}{c^3} e^3 \ln(c x - 1) - \frac{1}{8} \frac{b}{c} e \ln(c x + 1) d^4 + \frac{1}{2} \frac{b}{c} \ln(c x + 1) d^3 - \frac{3}{4} \frac{b}{c} e \ln(c x + 1) d^2 + \frac{1}{2} \frac{b}{c^2} e^2 \ln(c x + 1) d - \frac{1}{8} \frac{b}{c^3} e^3 \ln(c x + 1) \right)$$

Maxima [A]

time = 0.26, size = 207, normalized size = 1.66

$$\frac{1}{4} a x^4 e^3 + a d x^3 e^2 + \frac{3}{2} a d^2 x^2 e + a d^3 x + \frac{3}{4} (2 x^2 \arctanh(c x) + c \left(\frac{2 x}{c^2} - \frac{\log(c x + 1)}{c^2} + \frac{\log(c x - 1)}{c^2} \right)) b d^2 e + \frac{(2 c x \arctanh(c x) + \log(-c^2 x^2 + 1)) b d^2}{2 c} + \frac{1}{2} (2 x^3 \arctanh(c x) + c \left(\frac{x^2}{c^2} + \frac{\log(c^2 x^2 - 1)}{c^2} \right)) b d e^2 + \frac{1}{24} (6 x^4 \arctanh(c x) + c \left(\frac{2(c^2 x^3 + 3 x)}{c^4} - \frac{3 \log(c x + 1)}{c^4} + \frac{3 \log(c x - 1)}{c^4} \right)) b e^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")`

[Out]
$$\frac{1}{4} a x^4 e^3 + a d x^3 e^2 + \frac{3}{2} a d^2 x^2 e + a d^3 x + \frac{3}{4} (2 x^2 \arctanh(c x) + c \left(\frac{2 x}{c^2} - \frac{\log(c x + 1)}{c^2} + \frac{\log(c x - 1)}{c^2} \right)) b d^2 e + \frac{1}{2} (2 x^3 \arctanh(c x) + \log(-c^2 x^2 + 1)) b d^3 / c + \frac{1}{2} (2 x^3 \arctanh(c x) + c \left(\frac{x^2}{c^2} + \frac{\log(c^2 x^2 - 1)}{c^2} \right)) b d e^2 + \frac{1}{24} (6 x^4 \arctanh(c x) + c \left(\frac{2(c^2 x^3 + 3 x)}{c^4} - \frac{3 \log(c x + 1)}{c^4} + \frac{3 \log(c x - 1)}{c^4} \right)) b e^3$$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(111) = 222.

time = 0.45, size = 594, normalized size = 4.75

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")`

[Out]
$$\frac{1}{24} (24 a c^4 d^3 x + 2 (3 a c^4 x^4 + b c^3 x^3 + 3 b c x) \cosh(1)^3 + 2 (3 a c^4 x^4 + b c^3 x^3 + 3 b c x) \sinh(1)^3 + 12 (2 a c^4 d x^3 + b c^3 d x^2) \cosh(1)^2 + 6 (4 a c^4 d x^3 + 2 b c^3 d x^2 + (3 a c^4 x^4 + b c^3 x^3 + 3 b c x) \cosh(1)) \sinh(1)^2 + 36 (a c^4 d^2 x^2 + b c^3 d^2 x) \cosh(1) + 3 (4 b c^3 d^3 - 6 b c^2 d^2 \cosh(1) + 4 b c d \cosh(1)^2 - b \cosh(1)^3 - b \sinh(1)^3 + (4 b c d - 3 b c \cosh(1)) \sinh(1)^2 - (6 b c^2 d^2 - 8 b c d \cosh(1) + 3 b \cosh(1)^2) \sinh(1)) \log(c x + 1) + 3 (4 b c^3 d^3 + 6 b c^2 d^2 \cosh(1) + 4 b c d \cosh(1)^2 + b \cosh(1)^3 + b \sinh(1)^3 + (4 b c d + 3 b$$

$\cosh(1))\sinh(1)^2 + (6*b*c^2*d^2 + 8*b*c*d*\cosh(1) + 3*b*\cosh(1)^2)*\sinh(1))*\log(c*x - 1) + 3*(b*c^4*x^4*\cosh(1)^3 + b*c^4*x^4*\sinh(1)^3 + 4*b*c^4*d*x^3*\cosh(1)^2 + 6*b*c^4*d^2*x^2*\cosh(1) + 4*b*c^4*d^3*x + (3*b*c^4*x^4*\cosh(1) + 4*b*c^4*d*x^3)*\sinh(1)^2 + (3*b*c^4*x^4*\cosh(1)^2 + 8*b*c^4*d*x^3*\cosh(1) + 6*b*c^4*d^2*x^2)*\sinh(1))*\log(-(c*x + 1)/(c*x - 1)) + 6*(6*a*c^4*d^2*x^2 + 6*b*c^3*d^2*x + (3*a*c^4*x^4 + b*c^3*x^3 + 3*b*c*x)*\cosh(1)^2 + 4*(2*a*c^4*d*x^3 + b*c^3*d*x^2)*\cosh(1))*\sinh(1))/c^4$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(109) = 218$.

time = 0.40, size = 279, normalized size = 2.23

$$\begin{cases} ad^3x + \frac{3ad^2cx^2}{2} + ad^2x^3 + \frac{a^2x^4}{4} + bd^3x \operatorname{atanh}(cx) + \frac{3bd^2cx^2 \operatorname{atanh}(cx)}{2} + bde^2x^3 \operatorname{atanh}(cx) + \frac{be^3x^4 \operatorname{atanh}(cx)}{4} + \frac{bd^3 \log(x-\frac{1}{c})}{c} + \frac{bd^2 \operatorname{atanh}(cx)}{c} + \frac{3bd^2cx}{2c} + \frac{bd^2x^2}{12c} - \frac{3bd^2 \operatorname{atanh}(cx)}{2c^2} + \frac{bd^2 \log(x-\frac{1}{c})}{c^3} + \frac{bd^2 \operatorname{atanh}(cx)}{c^3} + \frac{bd^2}{4c^3} - \frac{bd^2 \operatorname{atanh}(cx)}{4c^3} & \text{for } c \neq 0 \\ a(d^3x + \frac{3d^2cx^2}{2} + d^2x^3 + \frac{a^2x^4}{4}) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**3*(a+b*atanh(c*x)),x)

[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b*d**3*x*atanh(c*x) + 3*b*d**2*e*x**2*atanh(c*x)/2 + b*d*e**2*x**3*atanh(c*x) + b*e**3*x**4*atanh(c*x)/4 + b*d**3*log(x - 1/c)/c + b*d**3*atanh(c*x)/c + 3*b*d**2*e*x/(2*c) + b*d*e**2*x**2/(2*c) + b*e**3*x**3/(12*c) - 3*b*d**2*e*atanh(c*x)/(2*c**2) + b*d*e**2*log(x - 1/c)/c**3 + b*d*e**2*atanh(c*x)/c**3 + b*e**3*x/(4*c**3) - b*e**3*atanh(c*x)/(4*c**4), Ne(c, 0)), (a*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(113) = 226$.

time = 0.42, size = 970, normalized size = 7.76

([...])

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")

[Out] $1/3*c*(3*((c*x + 1)^3*b*c^3*d^3/(c*x - 1)^3 - 3*(c*x + 1)^2*b*c^3*d^3/(c*x - 1)^2 + 3*(c*x + 1)*b*c^3*d^3/(c*x - 1) - b*c^3*d^3 + 3*(c*x + 1)^3*b*c^2*d^2*e/(c*x - 1)^3 - 6*(c*x + 1)^2*b*c^2*d^2*e/(c*x - 1)^2 + 3*(c*x + 1)*b*c^2*d^2*e/(c*x - 1) + 3*(c*x + 1)^3*b*c*d*e^2/(c*x - 1)^3 - 3*(c*x + 1)^2*b*c*d*e^2/(c*x - 1)^2 + (c*x + 1)*b*c*d*e^2/(c*x - 1) - b*c*d*e^2 + (c*x + 1)^3*b*e^3/(c*x - 1)^3 + (c*x + 1)*b*e^3/(c*x - 1))*\log(-(c*x + 1)/(c*x - 1)) / ((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1)^3 + 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5) + (6*(c*x + 1)^3*a*c^3*d^3/(c*x - 1)^3 - 18*(c*x + 1)^2*a*c^3*d^3/(c*x - 1)^2 + 18*(c*x + 1)*a*c^3*d^3/(c*x - 1) - 6*a*c^3*d^3 + 18*(c*x + 1)^3*a*c^2*d^2*e/(c*x - 1)^3 - 3*6*(c*x + 1)^2*a*c^2*d^2*e/(c*x - 1)^2 + 18*(c*x + 1)*a*c^2*d^2*e/(c*x - 1)$

$$\begin{aligned}
& + 9*(c*x + 1)^3*b*c^2*d^2*e/(c*x - 1)^3 - 27*(c*x + 1)^2*b*c^2*d^2*e/(c*x - 1)^2 + 27*(c*x + 1)*b*c^2*d^2*e/(c*x - 1) - 9*b*c^2*d^2*e + 18*(c*x + 1)^3 \\
& *a*c*d*e^2/(c*x - 1)^3 - 18*(c*x + 1)^2*a*c*d*e^2/(c*x - 1)^2 + 6*(c*x + 1) \\
& *a*c*d*e^2/(c*x - 1) - 6*a*c*d*e^2 + 6*(c*x + 1)^3*b*c*d*e^2/(c*x - 1)^3 - \\
& 12*(c*x + 1)^2*b*c*d*e^2/(c*x - 1)^2 + 6*(c*x + 1)*b*c*d*e^2/(c*x - 1) + 6* \\
& (c*x + 1)^3*a*e^3/(c*x - 1)^3 + 6*(c*x + 1)*a*e^3/(c*x - 1) + 3*(c*x + 1)^3 \\
& *b*e^3/(c*x - 1)^3 - 6*(c*x + 1)^2*b*e^3/(c*x - 1)^2 + 5*(c*x + 1)*b*e^3/(c \\
& *x - 1) - 2*b*e^3)/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - \\
& 1)^3 + 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5) - 3 \\
& *(b*c^2*d^3 + b*d*e^2)*\log(-(c*x + 1)/(c*x - 1) + 1)/c^4 + 3*(b*c^2*d^3 + b \\
& *d*e^2)*\log(-(c*x + 1)/(c*x - 1))/c^4)
\end{aligned}$$

Mupad [B]

time = 1.15, size = 197, normalized size = 1.58

$$\frac{a^2 x^4}{4} + a d^3 x + \frac{b d^3 \ln(c^2 x^2 - 1)}{2c} + \frac{b e^3 x^3}{12c} + b d^3 x \operatorname{atanh}(c x) + \frac{3 a d^2 e x^2}{2} + a d e^2 x^3 + \frac{b e^3 x}{4c^3} - \frac{b e^3 \operatorname{atanh}(c x)}{4c^3} + \frac{b e^3 x^4 \operatorname{atanh}(c x)}{4} + \frac{3 b d^2 e x}{2c} - \frac{3 b d^2 e \operatorname{atanh}(c x)}{2c^2} + \frac{3 b d^2 e x^2 \operatorname{atanh}(c x)}{2} + b d e^2 x^3 \operatorname{atanh}(c x) + \frac{b d e^2 \ln(c^2 x^2 - 1)}{2c^3} + \frac{b d e^2 x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c*x))*(d + e*x)^3,x)`

[Out] $(a*e^3*x^4)/4 + a*d^3*x + (b*d^3*\log(c^2*x^2 - 1))/(2*c) + (b*e^3*x^3)/(12*c) + b*d^3*x*\operatorname{atanh}(c*x) + (3*a*d^2*e*x^2)/2 + a*d*e^2*x^3 + (b*e^3*x)/(4*c^3) - (b*e^3*\operatorname{atanh}(c*x))/(4*c^4) + (b*e^3*x^4*\operatorname{atanh}(c*x))/4 + (3*b*d^2*e*x)/(2*c) - (3*b*d^2*e*\operatorname{atanh}(c*x))/(2*c^2) + (3*b*d^2*e*x^2*\operatorname{atanh}(c*x))/2 + b*d*e^2*x^3*\operatorname{atanh}(c*x) + (b*d*e^2*\log(c^2*x^2 - 1))/(2*c^3) + (b*d*e^2*x^2)/(2*c)$

3.3 $\int (d + ex)^2 (a + b \tanh^{-1}(cx)) dx$

Optimal. Leaf size=96

$$\frac{bdex}{c} + \frac{be^2x^2}{6c} + \frac{(d+ex)^3(a+b\tanh^{-1}(cx))}{3e} + \frac{b(cd+e)^3\log(1-cx)}{6c^3e} - \frac{b(cd-e)^3\log(1+cx)}{6c^3e}$$

[Out] $b*d*e*x/c+1/6*b*e^2*x^2/c+1/3*(e*x+d)^3*(a+b*\arctanh(c*x))/e+1/6*b*(c*d+e)^3*\ln(-c*x+1)/c^3/e-1/6*b*(c*d-e)^3*\ln(c*x+1)/c^3/e$

Rubi [A]

time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6063, 716, 647, 31}

$$\frac{(d+ex)^3(a+b\tanh^{-1}(cx))}{3e} - \frac{b(cd-e)^3\log(cx+1)}{6c^3e} + \frac{b(cd+e)^3\log(1-cx)}{6c^3e} + \frac{bdex}{c} + \frac{be^2x^2}{6c}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d + e*x)^2*(a + b*\text{ArcTanh}[c*x]), x]$

[Out] $(b*d*e*x)/c + (b*e^2*x^2)/(6*c) + ((d + e*x)^3*(a + b*\text{ArcTanh}[c*x]))/(3*e) + (b*(c*d + e)^3*\text{Log}[1 - c*x])/(6*c^3*e) - (b*(c*d - e)^3*\text{Log}[1 + c*x])/(6*c^3*e)$

Rule 31

$\text{Int}[(a + (b*x)^(-1)), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ /; FreeQ}\{a, b\}, x]$

Rule 647

$\text{Int}[(d + (e*x))/(a + (c*x)^2), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(a*c, 2)], \text{Dist}[e/2 + c*(d/(2*q)), \text{Int}[1/(-q + c*x), x], x] + \text{Dist}[e/2 - c*(d/(2*q)), \text{Int}[1/(q + c*x), x], x]\} \text{ /; FreeQ}\{a, c, d, e\}, x \ \&\& \ \text{NiceSqrtQ}[(a)*c]$

Rule 716

$\text{Int}[(d + (e*x))^m/(a + (c*x)^2), x_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[(d + e*x)^m, a + c*x^2, x], x] \text{ /; FreeQ}\{a, c, d, e\}, x \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{IGtQ}[m, 1] \ \&\& \ (\text{NeQ}[d, 0] \ || \ \text{GtQ}[m, 2])]$

Rule 6063

$\text{Int}[(a + \text{ArcTanh}[c*x])*(b*x)^q/(d + e*x), x_Symbol] \rightarrow \text{Simp}[(d + e*x)^(q + 1)*((a + b*\text{ArcTanh}[c*x])/(e*(q + 1))), x] - \text{Dist}[b$

`*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]`

Rubi steps

$$\begin{aligned}
 \int (d + ex)^2 (a + b \tanh^{-1}(cx)) dx &= \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))}{3e} - \frac{(bc) \int \frac{(d+ex)^3}{1-c^2x^2} dx}{3e} \\
 &= \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))}{3e} - \frac{(bc) \int \left(-\frac{3de^2}{c^2} - \frac{e^3x}{c^2} + \frac{c^2d^3+3de^2+e(3c^2d^2+e^3)}{c^2(1-c^2x^2)} \right) dx}{3e} \\
 &= \frac{bdex}{c} + \frac{be^2x^2}{6c} + \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))}{3e} - \frac{b \int \frac{c^2d^3+3de^2+e(3c^2d^2+e^3)}{1-c^2x^2} dx}{3ce} \\
 &= \frac{bdex}{c} + \frac{be^2x^2}{6c} + \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))}{3e} + \frac{(b(cd - e)^3) \int \frac{1}{-c-c^2x} dx}{6ce} \\
 &= \frac{bdex}{c} + \frac{be^2x^2}{6c} + \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))}{3e} + \frac{b(cd + e)^3 \log(1 - cx)}{6c^3e}
 \end{aligned}$$

Mathematica [A]

time = 0.05, size = 129, normalized size = 1.34

$$\frac{1}{6} \left(\frac{6d(acd + be)x}{c} + \frac{e(6acd + be)x^2}{c} + 2ae^2x^3 + 2bx(3d^2 + 3dex + e^2x^2) \tanh^{-1}(cx) + \frac{b(3c^2d^2 + 3cde + e^2) \log(1 - cx)}{c^3} + \frac{b(3c^2d^2 - 3cde + e^2) \log(1 + cx)}{c^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2*(a + b*ArcTanh[c*x]), x]

[Out] ((6*d*(a*c*d + b*e)*x)/c + (e*(6*a*c*d + b*e)*x^2)/c + 2*a*e^2*x^3 + 2*b*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcTanh[c*x] + (b*(3*c^2*d^2 + 3*c*d*e + e^2)*Log[1 - c*x])/c^3 + (b*(3*c^2*d^2 - 3*c*d*e + e^2)*Log[1 + c*x])/c^3)/6

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(88) = 176.

time = 0.14, size = 204, normalized size = 2.12

method	result
derivativedivides	$\frac{\frac{(cex+dc)^3 a}{3c^2 e} + \frac{bc \operatorname{arctanh}(cx) d^3}{3e} + b \operatorname{arctanh}(cx) d^2 cx + bce \operatorname{arctanh}(cx) dx^2 + \frac{bc e^2 \operatorname{arctanh}(cx) x^3}{3} + bedx + \frac{b e^2 x^2}{6} + \frac{bc \ln(cx-1) d^3}{6e}}{c}$
default	$\frac{\frac{(cex+dc)^3 a}{3c^2 e} + \frac{bc \operatorname{arctanh}(cx) d^3}{3e} + b \operatorname{arctanh}(cx) d^2 cx + bce \operatorname{arctanh}(cx) dx^2 + \frac{bc e^2 \operatorname{arctanh}(cx) x^3}{3} + bedx + \frac{b e^2 x^2}{6} + \frac{bc \ln(cx-1) d^3}{6e}}{c}$
risch	$\frac{(ex+d)^3 b \ln(cx+1)}{6e} - \frac{e^2 b x^3 \ln(-cx+1)}{6} - \frac{ebd x^2 \ln(-cx+1)}{2} + \frac{e^2 a x^3}{3} - \frac{b d^2 x \ln(-cx+1)}{2} + ead x^2 - \frac{\ln(cx-1) d^3}{6e}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)

[Out] $\frac{1}{c} * \left(\frac{1}{3} * (c * e * x + c * d)^3 * a / c^2 / e + \frac{1}{3} * b * c / e * \text{arctanh}(c * x) * d^3 + b * \text{arctanh}(c * x) * d^2 * c * x + b * c * e * \text{arctanh}(c * x) * d * x^2 + \frac{1}{3} * b * c * e^2 * \text{arctanh}(c * x) * x^3 + b * e * d * x + \frac{1}{6} * b * e^2 * x^2 + \frac{1}{6} * b * c / e * \ln(c * x - 1) * d^3 + \frac{1}{2} * b * \ln(c * x - 1) * d^2 + \frac{1}{2} * b / c * e * \ln(c * x - 1) * d + \frac{1}{6} * b / c^2 * e^2 * \ln(c * x - 1) - \frac{1}{6} * b * c / e * \ln(c * x + 1) * d^3 + \frac{1}{2} * b * \ln(c * x + 1) * d^2 - \frac{1}{2} * b / c * e * \ln(c * x + 1) * d + \frac{1}{6} * b / c^2 * e^2 * \ln(c * x + 1) \right)$

Maxima [A]

time = 0.26, size = 137, normalized size = 1.43

$$\frac{1}{3} a x^3 e^3 + a d x^2 e + a d^2 x + \frac{1}{2} \left(2 x^2 \operatorname{artanh}(c x) + c \left(\frac{2 x}{c^2} - \frac{\log(c x + 1)}{c^3} + \frac{\log(c x - 1)}{c^3} \right) \right) b d e + \frac{(2 c x \operatorname{artanh}(c x) + \log(-c^2 x^2 + 1)) b d^2}{2 c} + \frac{1}{6} \left(2 x^3 \operatorname{artanh}(c x) + c \left(\frac{x^2}{c^2} + \frac{\log(c^2 x^2 - 1)}{c^4} \right) \right) b e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")

[Out] $\frac{1}{3} * a * x^3 * e^2 + a * d * x^2 * e + a * d^2 * x + \frac{1}{2} * (2 * x^2 * \text{arctanh}(c * x) + c * (2 * x / c^2 - \log(c * x + 1) / c^3 + \log(c * x - 1) / c^3)) * b * d * e + \frac{1}{2} * (2 * c * x * \text{arctanh}(c * x) + \log(-c^2 * x^2 + 1)) * b * d^2 / c + \frac{1}{6} * (2 * x^3 * \text{arctanh}(c * x) + c * (x^2 / c^2 + \log(c^2 * x^2 - 1) / c^4)) * b * e^2$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(88) = 176.

time = 0.39, size = 316, normalized size = 3.29

$$\frac{a^2 d^3 x^3 + 3 a^2 d^2 x^2 + 3 a^2 d x + a^2}{3} + \frac{b d^2 x^3 \operatorname{atanh}(c x) + b d^2 x^2 \operatorname{atanh}(c x) + \frac{b d^2 \log(x - \frac{1}{c})}{c} + \frac{b d^2 \operatorname{atanh}(c x)}{c} + \frac{b d e x}{c} + \frac{b e^2 x^2}{6 c} - \frac{b d e \operatorname{atanh}(c x)}{c^2} + \frac{b e^2 \log(x - \frac{1}{c})}{3 c^3} + \frac{b e^2 \operatorname{atanh}(c x)}{3 c^3}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")

[Out] $\frac{1}{6} * (6 * a * c^3 * d^2 * x + (2 * a * c^3 * x^3 + b * c^2 * x^2) * \cosh(1)^2 + (2 * a * c^3 * x^3 + b * c^2 * x^2) * \sinh(1)^2 + 6 * (a * c^3 * d * x^2 + b * c^2 * d * x) * \cosh(1) + (3 * b * c^2 * d^2 - 3 * b * c * d * \cosh(1) + b * \cosh(1)^2 + b * \sinh(1)^2 - (3 * b * c * d - 2 * b * \cosh(1)) * \sinh(1)) * \log(c * x + 1) + (3 * b * c^2 * d^2 + 3 * b * c * d * \cosh(1) + b * \cosh(1)^2 + b * \sinh(1)^2 + (3 * b * c * d + 2 * b * \cosh(1)) * \sinh(1)) * \log(c * x - 1) + (b * c^3 * x^3 * \cosh(1)^2 + b * c^3 * x^3 * \sinh(1)^2 + 3 * b * c^3 * d * x^2 * \cosh(1) + 3 * b * c^3 * d^2 * x + (2 * b * c^3 * x^3 * \cosh(1) + 3 * b * c^3 * d * x^2) * \sinh(1)) * \log(-(c * x + 1) / (c * x - 1)) + 2 * (3 * a * c^3 * d * x^2 + 3 * b * c^2 * d * x + (2 * a * c^3 * x^3 + b * c^2 * x^2) * \cosh(1)) * \sinh(1)) / c^3$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(82) = 164.

time = 0.31, size = 178, normalized size = 1.85

$$\begin{cases} a d^2 x + a d e x^2 + \frac{a e^2 x^3}{3} + b d^2 x \operatorname{atanh}(c x) + b d e x^2 \operatorname{atanh}(c x) + \frac{b e^2 x^3 \operatorname{atanh}(c x)}{3} + \frac{b d^2 \log(x - \frac{1}{c})}{c} + \frac{b d^2 \operatorname{atanh}(c x)}{c} + \frac{b d e x}{c} + \frac{b e^2 x^2}{6 c} - \frac{b d e \operatorname{atanh}(c x)}{c^2} + \frac{b e^2 \log(x - \frac{1}{c})}{3 c^3} + \frac{b e^2 \operatorname{atanh}(c x)}{3 c^3} & \text{for } c \neq 0 \\ a(d^2 x + d e x^2 + \frac{e^2 x^3}{3}) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*(a+b*atanh(c*x)),x)

[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*atanh(c*x) + b*d*e*x**2*atanh(c*x) + b*e**2*x**3*atanh(c*x)/3 + b*d**2*log(x - 1/c)/c + b*d**2*atanh(c*x)/c + b*d*e*x/c + b*e**2*x**2/(6*c) - b*d*e*atanh(c*x)/c**2 + b*e**2*log(x - 1/c)/(3*c**3) + b*e**2*atanh(c*x)/(3*c**3), Ne(c, 0)), (a*(d**2*x + d*e*x**2 + e**2*x**3/3), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(88) = 176.

time = 0.44, size = 532, normalized size = 5.54

$$\frac{1}{3^4} \left(\frac{\frac{3(ac+1)^2bd^2}{(a-1)^2} - \frac{6(ac+1)bd^2}{a-1} + 3bd^2d^2 + \frac{6(ac+1)bd^2}{(a-1)^2} - \frac{6(ac+1)bd^2}{a-1} + \frac{3(ac+1)^2bd^2 + bd^2}{(a-1)^2} \log\left(-\frac{cax}{a-1}\right)}{\frac{3(ac+1)^2bd^2}{(a-1)^2} - \frac{6(ac+1)bd^2}{a-1} + \frac{3bd^2d^2}{c^4}} + \frac{2 \left(\frac{3(ac+1)^2bd^2}{(a-1)^2} - \frac{6(ac+1)bd^2}{a-1} + 3ac^2d^2 + \frac{6(ac+1)bd^2}{(a-1)^2} - \frac{6(ac+1)bd^2}{a-1} + \frac{3(ac+1)^2bd^2}{(a-1)^2} - \frac{6(ac+1)bd^2}{a-1} + 3bd^2d^2 + \frac{3(ac+1)^2bd^2}{(a-1)^2} + ac^2 + \frac{6(ac+1)bd^2}{(a-1)^2} - \frac{6(ac+1)bd^2}{a-1} \right)}{\frac{3(ac+1)^2bd^2}{(a-1)^2} - \frac{6(ac+1)bd^2}{a-1} + \frac{3bd^2d^2}{c^4}} \frac{(3bd^2d^2 + bd^2) \log\left(-\frac{cax}{a-1} + 1\right)}{c^4} + \frac{(3bd^2d^2 + bd^2) \log\left(-\frac{cax}{a-1}\right)}{c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")

[Out] $\frac{1}{3}c \left(\frac{3(c^2x^2 + 1)^2 b^2 c^2 d^2}{(cx - 1)^2} - \frac{6(c^2x^2 + 1) b^2 c^2 d^2}{(cx - 1)} + 3b^2 c^2 d^2 + \frac{6(c^2x^2 + 1)^2 b^2 c d e}{(cx - 1)^2} - \frac{6(c^2x^2 + 1) b^2 c d e}{(cx - 1)} + \frac{3(c^2x^2 + 1)^2 b^2 e^2}{(cx - 1)^2} + \frac{b^2 e^2}{(cx - 1)} \right) \log\left(-\frac{cx + 1}{cx - 1}\right) / \left(\frac{3c^4}{(cx - 1)^3} - \frac{3c^4}{(cx - 1)^2} + \frac{3(c^2x^2 + 1)c^4}{(cx - 1)} - c^4 \right) + \frac{2 \left(\frac{3(c^2x^2 + 1)^2 a^2 c^2 d^2}{(cx - 1)^2} - \frac{6(c^2x^2 + 1) a^2 c^2 d^2}{(cx - 1)} + 3a^2 c^2 d^2 + \frac{6(c^2x^2 + 1)^2 a^2 c d e}{(cx - 1)^2} - \frac{6(c^2x^2 + 1) a^2 c d e}{(cx - 1)} + \frac{3(c^2x^2 + 1)^2 b^2 c d e}{(cx - 1)^2} - \frac{6(c^2x^2 + 1) b^2 c d e}{(cx - 1)} + 3b^2 c d e + \frac{3(c^2x^2 + 1)^2 a^2 e^2}{(cx - 1)^2} + \frac{a^2 e^2}{(cx - 1)} + \frac{(c^2x^2 + 1)^2 b^2 e^2}{(cx - 1)^2} - \frac{(c^2x^2 + 1) b^2 e^2}{(cx - 1)} \right) / \left(\frac{3c^4}{(cx - 1)^3} - \frac{3c^4}{(cx - 1)^2} + \frac{3(c^2x^2 + 1)c^4}{(cx - 1)} - c^4 \right) - \frac{(3b^2 c^2 d^2 + b^2 e^2) \log\left(-\frac{cx + 1}{cx - 1} + 1\right)}{c^4} + \frac{(3b^2 c^2 d^2 + b^2 e^2) \log\left(-\frac{cx + 1}{cx - 1}\right)}{c^4}$

Mupad [B]

time = 0.94, size = 127, normalized size = 1.32

$$\frac{ae^2x^3}{3} + ad^2x + \frac{bd^2 \ln(c^2x^2 - 1)}{2c} + \frac{be^2 \ln(c^2x^2 - 1)}{6c^3} + \frac{be^2x^2}{6c} + adex^2 + bd^2x \operatorname{atanh}(cx) + \frac{be^2x^3 \operatorname{atanh}(cx)}{3} + \frac{bdex}{c} - \frac{bde \operatorname{atanh}(cx)}{c^2} + bde x^2 \operatorname{atanh}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))*(d + e*x)^2,x)

[Out] $\frac{a e^2 x^3}{3} + a d^2 x + \frac{b d^2 \log(c^2 x^2 - 1)}{2 c} + \frac{b e^2 \log(c^2 x^2 - 1)}{(6 c^3)} + \frac{b e^2 x^2}{(6 c)} + a d e x^2 + b d^2 x \operatorname{atanh}(c x) + \frac{b e^2 x^3 \operatorname{atanh}(c x)}{3} + \frac{b d e x}{c} - \frac{b d e \operatorname{atanh}(c x)}{c^2} + b d e x^2 \operatorname{atanh}(c x)$

3.4 $\int (d + ex) (a + b \tanh^{-1}(cx)) dx$

Optimal. Leaf size=84

$$\frac{bex}{2c} + \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))}{2e} + \frac{b(cd + e)^2 \log(1 - cx)}{4c^2e} - \frac{b(cd - e)^2 \log(1 + cx)}{4c^2e}$$

[Out] $1/2*b*e*x/c+1/2*(e*x+d)^2*(a+b*\arctanh(c*x))/e+1/4*b*(c*d+e)^2*\ln(-c*x+1)/c^2/e-1/4*b*(c*d-e)^2*\ln(c*x+1)/c^2/e$

Rubi [A]

time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6063, 716, 647, 31}

$$\frac{(d + ex)^2 (a + b \tanh^{-1}(cx))}{2e} - \frac{b(cd - e)^2 \log(cx + 1)}{4c^2e} + \frac{b(cd + e)^2 \log(1 - cx)}{4c^2e} + \frac{bex}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)*(a + b*ArcTanh[c*x]),x]

[Out] $(b*e*x)/(2*c) + ((d + e*x)^2*(a + b*ArcTanh[c*x]))/(2*e) + (b*(c*d + e)^2*\text{Log}[1 - c*x])/(4*c^2*e) - (b*(c*d - e)^2*\text{Log}[1 + c*x])/(4*c^2*e)$

Rule 31

Int[((a_) + (b_)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 647

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[(-a)*c]

Rule 716

Int[((d_) + (e_)*(x_))^(m)/((a_) + (c_)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 6063

Int[((a_) + ArcTanh[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_))^(q_), x_Symbol] := Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,

b, c, d, e, q}, x] && NeQ[q, -1]

Rubi steps

$$\begin{aligned}
 \int (d + ex) (a + b \tanh^{-1}(cx)) dx &= \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))}{2e} - \frac{(bc) \int \frac{(d+ex)^2}{1-c^2x^2} dx}{2e} \\
 &= \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))}{2e} - \frac{(bc) \int \left(-\frac{e^2}{c^2} + \frac{c^2 d^2 + e^2 + 2c^2 dex}{c^2(1-c^2x^2)} \right) dx}{2e} \\
 &= \frac{bex}{2c} + \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))}{2e} - \frac{b \int \frac{c^2 d^2 + e^2 + 2c^2 dex}{1-c^2x^2} dx}{2ce} \\
 &= \frac{bex}{2c} + \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))}{2e} + \frac{(b(cd - e)^2) \int \frac{1}{-c-c^2x} dx}{4e} - \frac{b(cd - e)^2 \int \frac{1}{c+c^2x} dx}{4e} \\
 &= \frac{bex}{2c} + \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))}{2e} + \frac{b(cd + e)^2 \log(1 - cx)}{4c^2e} - \frac{b(cd - e)^2 \log(1 + cx)}{4c^2e}
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 96, normalized size = 1.14

$$adx + \frac{bex}{2c} + \frac{1}{2}aex^2 + bdx \tanh^{-1}(cx) + \frac{1}{2}bex^2 \tanh^{-1}(cx) + \frac{be \log(1 - cx)}{4c^2} - \frac{be \log(1 + cx)}{4c^2} + \frac{bd \log(1 - c^2x^2)}{2c}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x]), x]

[Out] a*d*x + (b*e*x)/(2*c) + (a*e*x^2)/2 + b*d*x*ArcTanh[c*x] + (b*e*x^2*ArcTanh[c*x])/2 + (b*e*Log[1 - c*x])/(4*c^2) - (b*e*Log[1 + c*x])/(4*c^2) + (b*d*Log[1 - c^2*x^2])/(2*c)

Maple [A]

time = 0.09, size = 99, normalized size = 1.18

method	result
derivativedivides	$\frac{a(d^2cx + \frac{1}{2}e^2cx^2)}{e} + b \operatorname{arctanh}(cx)dcx + \frac{bc \operatorname{arctanh}(cx)e^2x^2}{2} + \frac{bex}{2} + \frac{b \ln(cx-1)d}{2} + \frac{b \ln(cx-1)e}{4c} + \frac{b \ln(cx+1)d}{2} - \frac{b \ln(cx+1)e}{4c}$
default	$\frac{a(d^2cx + \frac{1}{2}e^2cx^2)}{e} + b \operatorname{arctanh}(cx)dcx + \frac{bc \operatorname{arctanh}(cx)e^2x^2}{2} + \frac{bex}{2} + \frac{b \ln(cx-1)d}{2} + \frac{b \ln(cx-1)e}{4c} + \frac{b \ln(cx+1)d}{2} - \frac{b \ln(cx+1)e}{4c}$
risch	$\frac{bx(ex+2d) \ln(cx+1)}{4} - \frac{be x^2 \ln(-cx+1)}{4} - \frac{bdx \ln(-cx+1)}{2} + \frac{ae x^2}{2} + adx + \frac{\ln(cx+1)bd}{2c} + \frac{\ln(-cx+1)bd}{2c} + \dots$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)*(a+b*arctanh(c*x)), x, method=_RETURNVERBOSE)

[Out] $1/c*(a/c*(d*c^2*x+1/2*e*c^2*x^2)+b*\operatorname{arctanh}(c*x)*d*c*x+1/2*b*c*\operatorname{arctanh}(c*x)*e*x^2+1/2*b*e*x+1/2*b*\ln(c*x-1)*d+1/4*b/c*\ln(c*x-1)*e+1/2*b*\ln(c*x+1)*d-1/4*b/c*\ln(c*x+1)*e)$

Maxima [A]

time = 0.27, size = 85, normalized size = 1.01

$$\frac{1}{2}ax^2e + adx + \frac{1}{4}\left(2x^2 \operatorname{artanh}(cx) + c\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)be + \frac{(2cx \operatorname{artanh}(cx) + \log(-c^2x^2+1))bd}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")`

[Out] $1/2*a*x^2*e + a*d*x + 1/4*(2*x^2*\operatorname{arctanh}(c*x) + c*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3))*b*e + 1/2*(2*c*x*\operatorname{arctanh}(c*x) + \log(-c^2*x^2 + 1))*b*d/c$

Fricas [A]

time = 0.38, size = 137, normalized size = 1.63

$$\frac{4ac^2dx + 2(ac^2x^2 + bcx) \cosh(1) + (2bcd - b \cosh(1) - b \sinh(1)) \log(cx+1) + (2bcd + b \cosh(1) + b \sinh(1)) \log(cx-1) + (bc^2x^2 \cosh(1) + bc^2x^2 \sinh(1) + 2bc^2dx) \log\left(-\frac{cx+1}{cx-1}\right) + 2(ac^2x^2 + bcx) \sinh(1)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")`

[Out] $1/4*(4*a*c^2*d*x + 2*(a*c^2*x^2 + b*c*x)*\cosh(1) + (2*b*c*d - b*\cosh(1) - b*\sinh(1))*\log(c*x + 1) + (2*b*c*d + b*\cosh(1) + b*\sinh(1))*\log(c*x - 1) + (b*c^2*x^2*\cosh(1) + b*c^2*x^2*\sinh(1) + 2*b*c^2*d*x)*\log(-(c*x + 1)/(c*x - 1)) + 2*(a*c^2*x^2 + b*c*x)*\sinh(1))/c^2$

Sympy [A]

time = 0.22, size = 92, normalized size = 1.10

$$\begin{cases} adx + \frac{aex^2}{2} + bdx \operatorname{atanh}(cx) + \frac{bex^2 \operatorname{atanh}(cx)}{2} + \frac{bd \log\left(\frac{x-\frac{1}{c}}{c}\right)}{c} + \frac{bd \operatorname{atanh}(cx)}{c} + \frac{bex}{2c} - \frac{be \operatorname{atanh}(cx)}{2c^2} & \text{for } c \neq 0 \\ a\left(dx + \frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*atanh(c*x)),x)`

[Out] `Piecewise((a*d*x + a*e*x**2/2 + b*d*x*atanh(c*x) + b*e*x**2*atanh(c*x))/2 + b*d*log(x - 1/c)/c + b*d*atanh(c*x)/c + b*e*x/(2*c) - b*e*atanh(c*x)/(2*c**2), Ne(c, 0)), (a*(d*x + e*x**2/2), True))`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(76) = 152.

time = 0.42, size = 245, normalized size = 2.92

$$-c\left(\frac{bd \log\left(-\frac{cx+1}{cx-1} + 1\right)}{c^2} - \frac{\left(\frac{cx+1}{cx-1}bcd - bcd + \frac{cx+1}{cx-1}be\right) \log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2(cx+1)c^3}{cx-1} + c^3} - \frac{bd \log\left(-\frac{cx+1}{cx-1}\right)}{c^2} - \frac{\frac{2(cx+1)acd}{cx-1} - 2acd + \frac{2(cx+1)ae}{cx-1} + \frac{(cx+1)be}{cx-1} - be}{\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2(cx+1)c^3}{cx-1} + c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")

[Out] $-c*(b*d*\log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - ((c*x + 1)*b*c*d/(c*x - 1) - b*c*d + (c*x + 1)*b*e/(c*x - 1))*\log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2*c^3/(c*x - 1)^2 - 2*(c*x + 1)*c^3/(c*x - 1) + c^3) - b*d*\log(-(c*x + 1)/(c*x - 1))/c^2 - (2*(c*x + 1)*a*c*d/(c*x - 1) - 2*a*c*d + 2*(c*x + 1)*a*e/(c*x - 1) + (c*x + 1)*b*e/(c*x - 1) - b*e)/((c*x + 1)^2*c^3/(c*x - 1)^2 - 2*(c*x + 1)*c^3/(c*x - 1) + c^3)$

Mupad [B]

time = 0.83, size = 67, normalized size = 0.80

$$a dx + \frac{a e x^2}{2} + b dx \operatorname{atanh}(c x) + \frac{b e x}{2 c} - \frac{b e \operatorname{atanh}(c x)}{2 c^2} + \frac{b e x^2 \operatorname{atanh}(c x)}{2} + \frac{b d \ln(c^2 x^2 - 1)}{2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))*(d + e*x),x)

[Out] $a*d*x + (a*e*x^2)/2 + b*d*x*\operatorname{atanh}(c*x) + (b*e*x)/(2*c) - (b*e*\operatorname{atanh}(c*x))/(2*c^2) + (b*e*x^2*\operatorname{atanh}(c*x))/2 + (b*d*\log(c^2*x^2 - 1))/(2*c)$

3.5 $\int \frac{a+b \tanh^{-1}(cx)}{d+ex} dx$

Optimal. Leaf size=114

$$-\frac{(a+b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{e} + \frac{(a+b \tanh^{-1}(cx)) \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{b \text{PolyLog}\left(2, 1 - \frac{2}{1+cx}\right)}{2e} - \frac{b \text{PolyLog}\left(2, 1 - \frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2e}$$

[Out] $-(a+b*\text{arctanh}(c*x))*\ln(2/(c*x+1))/e+(a+b*\text{arctanh}(c*x))*\ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+1/2*b*\text{polylog}(2,1-2/(c*x+1))/e-1/2*b*\text{polylog}(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e$

Rubi [A]

time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6057, 2449, 2352, 2497}

$$\frac{(a+b \tanh^{-1}(cx)) \log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e} - \frac{\log\left(\frac{2}{cx+1}\right) (a+b \tanh^{-1}(cx))}{e} - \frac{b \text{Li}_2\left(1 - \frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e} + \frac{b \text{Li}_2\left(1 - \frac{2}{cx+1}\right)}{2e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{ArcTanh}[c*x])/(d + e*x), x]$

[Out] $-(((a + b*\text{ArcTanh}[c*x])*Log[2/(1 + c*x)]))/e + ((a + b*\text{ArcTanh}[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*\text{PolyLog}[2, 1 - 2/(1 + c*x)])/(2*e) - (b*\text{PolyLog}[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)$

Rule 2352

$\text{Int}[\text{Log}[(c_*)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] \rightarrow \text{Simp}[(-e^{(-1)})*\text{PolyLog}[2, 1 - c*x], x] /; \text{FreeQ}\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c*d, 0]$

Rule 2449

$\text{Int}[\text{Log}[(c_*)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[-e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x] \ \&\& \ \text{EqQ}[c, 2*d] \ \&\& \ \text{EqQ}[e^2*f + d^2*g, 0]$

Rule 2497

$\text{Int}[\text{Log}[u_]*(Pq_)^{(m_)}, x_Symbol] \rightarrow \text{With}\{\{C = \text{FullSimplify}[Pq^m*((1 - u)/D[u, x]]\}\}, \text{Simp}[C*\text{PolyLog}[2, 1 - u], x] /; \text{FreeQ}[C, x] /; \text{IntegerQ}[m] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{RationalFunctionQ}[u, x] \ \&\& \ \text{LeQ}[\text{RationalFunctionExponents}[u, x][[2]], \text{Expon}[Pq, x]]\}$

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := S
imp[(- (a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])*(Log[2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{a + b \tanh^{-1}(cx)}{d + ex} dx &= -\frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{e} + \frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{(bc)}{e} \int \frac{1}{1+cx} dx \\ &= -\frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{e} + \frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} - \frac{b \operatorname{Li}_2\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} \\ &= -\frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{e} + \frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{b \operatorname{Li}_2\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.24, size = 257, normalized size = 2.25

$$\frac{a \log(d+ex) + b \operatorname{arctanh}\left(\frac{cx}{d+ex}\right) \log\left(\frac{2}{1+cx}\right) - \frac{1}{2} \left[(a - 2b \operatorname{arctanh}\left(\frac{cx}{d+ex}\right))^2 + (b \operatorname{arctanh}\left(\frac{cx}{d+ex}\right))^2 + (c^2 d^2 - e^2) \operatorname{arctanh}\left(\frac{cx}{d+ex}\right) \log\left(\frac{2}{1+cx}\right) + 2 \operatorname{arctanh}\left(\frac{cx}{d+ex}\right) \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right) - (c^2 d^2 - e^2) \operatorname{arctanh}\left(\frac{cx}{d+ex}\right) \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right) - 2 \operatorname{arctanh}\left(\frac{cx}{d+ex}\right) \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right) \operatorname{arctanh}\left(\frac{cx}{d+ex}\right) - 2 \operatorname{arctanh}\left(\frac{cx}{d+ex}\right) \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right) \operatorname{arctanh}\left(\frac{cx}{d+ex}\right) \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)]}{2e} \right]}{e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x])/(d + e*x), x]

[Out] (a*Log[d + e*x] + b*ArcTanh[c*x]*(Log[1 - c^2*x^2]/2 + Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]]) - (I/2)*b*((-1/4*I)*(Pi - (2*I)*ArcTanh[c*x])^2 + I*(ArcTanh[(c*d)/e] + ArcTanh[c*x])^2 + (Pi - (2*I)*ArcTanh[c*x])*Log[1 + E^(2*ArcTanh[c*x])] + (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c*x])*Log[2/Sqrt[1 - c^2*x^2]] - (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[(2*I)*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]) - I*PolyLog[2, -E^(2*ArcTanh[c*x])] - I*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))])/e

Maple [A]

time = 0.75, size = 158, normalized size = 1.39

method	result
derivativedivides	$\frac{ac \ln(cx+dc)}{e} + \frac{bc \ln(cx+dc) \operatorname{arctanh}(cx)}{e} + \frac{bc \ln(cx+dc) \ln\left(\frac{cx-e}{-dc-e}\right)}{2e} + \frac{bc \operatorname{dilog}\left(\frac{cx-e}{-dc-e}\right)}{2e} - \frac{bc \ln(cx+dc) \ln\left(\frac{cx+e}{-dc+e}\right)}{2e} - \frac{bc \operatorname{dilog}\left(\frac{cx+e}{-dc+e}\right)}{2e}$

default	$\frac{\frac{ac \ln(cex+dc)}{e} + \frac{bc \ln(cex+dc) \operatorname{arctanh}(cx)}{e} + \frac{bc \ln(cex+dc) \ln\left(\frac{cex-e}{-dc-e}\right)}{2e} + \frac{bc \operatorname{dilog}\left(\frac{cex-e}{-dc-e}\right)}{2e} - \frac{bc \ln(cex+dc) \ln\left(\frac{cex+e}{-dc+e}\right)}{2e} - \frac{bc \operatorname{dilog}\left(\frac{cex+e}{-dc+e}\right)}{2e}}{c}$
risch	$-\frac{b \operatorname{dilog}\left(\frac{(-cx+1)e-dc-e}{-dc-e}\right)}{2e} - \frac{b \ln(-cx+1) \ln\left(\frac{(-cx+1)e-dc-e}{-dc-e}\right)}{2e} + \frac{a \ln((-cx+1)e-dc-e)}{e} + \frac{b \operatorname{dilog}\left(\frac{(cx+1)e+dc-e}{dc-e}\right)}{2e}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{c} \left(\frac{a \ln(cex+dc)}{e} + \frac{b \ln(cex+dc) \operatorname{arctanh}(cx)}{e} + \frac{1}{2} \frac{b \ln(cex+dc) \ln\left(\frac{cex-e}{-dc-e}\right)}{e} - \frac{1}{2} \frac{b \ln(cex+dc) \ln\left(\frac{cex+e}{-dc+e}\right)}{e} \right) + \frac{1}{2} \frac{b \operatorname{dilog}\left(\frac{cex-e}{-dc-e}\right)}{e} - \frac{1}{2} \frac{b \operatorname{dilog}\left(\frac{cex+e}{-dc+e}\right)}{e}$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")`

[Out] $a e^{-1} \log(xe + d) + \frac{1}{2} b \int (\log(cx + 1) - \log(-cx + 1)) / (xe + d) dx$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x))/(e*x+d),x, algorithm="fricas")`

[Out] $\int (b \operatorname{atanh}(cx) + a) / (xe + d) dx$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}(cx)}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x))/(e*x+d),x)`

[Out] $\int (a + b \operatorname{atanh}(cx)) / (d + ex) dx$

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/(e*x + d), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{atanh}(cx)}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x))/(d + e*x), x)
```

3.6 $\int \frac{a+b \tanh^{-1}(cx)}{(d+ex)^2} dx$

Optimal. Leaf size=93

$$-\frac{a+b \tanh^{-1}(cx)}{e(d+ex)} - \frac{bc \log(1-cx)}{2e(cd+e)} + \frac{bc \log(1+cx)}{2(cd-e)e} - \frac{bc \log(d+ex)}{c^2d^2 - e^2}$$

[Out] $(-a-b*\operatorname{arctanh}(c*x))/e/(e*x+d)-1/2*b*c*\ln(-c*x+1)/e/(c*d+e)+1/2*b*c*\ln(c*x+1)/(c*d-e)/e-b*c*\ln(e*x+d)/(c^2*d^2-e^2)$

Rubi [A]

time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6063, 720, 31, 647}

$$-\frac{a+b \tanh^{-1}(cx)}{e(d+ex)} - \frac{bc \log(d+ex)}{c^2d^2 - e^2} - \frac{bc \log(1-cx)}{2e(cd+e)} + \frac{bc \log(cx+1)}{2e(cd-e)}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcTanh[c*x])/(d + e*x)^2, x]`

[Out] $-((a + b*\operatorname{ArcTanh}[c*x])/(e*(d + e*x))) - (b*c*\operatorname{Log}[1 - c*x])/(2*e*(c*d + e)) + (b*c*\operatorname{Log}[1 + c*x])/(2*(c*d - e)*e) - (b*c*\operatorname{Log}[d + e*x])/(c^2*d^2 - e^2)$

Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 647

`Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[(-a)*c]`

Rule 720

`Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]`

Rule 6063

`Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)*((d_) + (e_.)*(x_))^(q_.), x_Symbol] := Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b`

$\int \frac{a + b \tanh^{-1}(cx)}{(d + ex)^2} dx$, Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

Rubi steps

$$\begin{aligned} \int \frac{a + b \tanh^{-1}(cx)}{(d + ex)^2} dx &= -\frac{a + b \tanh^{-1}(cx)}{e(d + ex)} + \frac{(bc) \int \frac{1}{(d+ex)(1-c^2x^2)} dx}{e} \\ &= -\frac{a + b \tanh^{-1}(cx)}{e(d + ex)} - \frac{(bc) \int \frac{-c^2d+c^2ex}{1-c^2x^2} dx}{e(c^2d^2 - e^2)} - \frac{(bce) \int \frac{1}{d+ex} dx}{c^2d^2 - e^2} \\ &= -\frac{a + b \tanh^{-1}(cx)}{e(d + ex)} - \frac{bc \log(d + ex)}{c^2d^2 - e^2} - \frac{(bc^3) \int \frac{1}{-c-c^2x} dx}{2(cd - e)e} + \frac{(bc^3) \int \frac{1}{c-c^2x} dx}{2e(cd + e)} \\ &= -\frac{a + b \tanh^{-1}(cx)}{e(d + ex)} - \frac{bc \log(1 - cx)}{2e(cd + e)} + \frac{bc \log(1 + cx)}{2(cd - e)e} - \frac{bc \log(d + ex)}{c^2d^2 - e^2} \end{aligned}$$

Mathematica [A]

time = 0.10, size = 102, normalized size = 1.10

$$-\frac{a}{e(d + ex)} - \frac{b \tanh^{-1}(cx)}{e(d + ex)} - \frac{bc \log(1 - cx)}{2e(cd + e)} - \frac{bc \log(1 + cx)}{2e(-cd + e)} - \frac{bc \log(d + ex)}{c^2d^2 - e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x])/(d + e*x)^2,x]

[Out] -(a/(e*(d + e*x))) - (b*ArcTanh[c*x])/(e*(d + e*x)) - (b*c*Log[1 - c*x])/(2*e*(c*d + e)) - (b*c*Log[1 + c*x])/(2*e*(-(c*d) + e)) - (b*c*Log[d + e*x])/(c^2*d^2 - e^2)

Maple [A]

time = 0.15, size = 128, normalized size = 1.38

method	result
derivativedivides	$-\frac{a c^2}{(cex+dc)e} - \frac{b c^2 \operatorname{arctanh}(cx)}{(cex+dc)e} - \frac{b c^2 \ln(cx-1)}{e(2dc+2e)} + \frac{b c^2 \ln(cx+1)}{e(2dc-2e)} - \frac{b c^2 \ln(cex+dc)}{(dc+e)(dc-e)}$
default	$-\frac{a c^2}{(cex+dc)e} - \frac{b c^2 \operatorname{arctanh}(cx)}{(cex+dc)e} - \frac{b c^2 \ln(cx-1)}{e(2dc+2e)} + \frac{b c^2 \ln(cx+1)}{e(2dc-2e)} - \frac{b c^2 \ln(cex+dc)}{(dc+e)(dc-e)}$
risch	$-\frac{b \ln(cx+1)}{2e(ex+d)} - \frac{\ln(cx-1)bc^2dex - \ln(-cx-1)bc^2dex + 2 \ln(-ex-d)bc e^2x + \ln(cx-1)bc^2d^2 - \ln(cx-1)bc e^2x - \ln(-cx-1)bc^2d^2}{c}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] $1/c*(-a*c^2/(c*e*x+c*d)/e-b*c^2/(c*e*x+c*d)/e*\operatorname{arctanh}(c*x)-b*c^2/e/(2*c*d+2*e)*\ln(c*x-1)+b*c^2/e/(2*c*d-2*e)*\ln(c*x+1)-b*c^2/(c*d+e)/(c*d-e)*\ln(c*e*x+c*d)$

Maxima [A]

time = 0.34, size = 99, normalized size = 1.06

$$\frac{1}{2} \left(c \left(\frac{\log(cx+1)}{cde-e^2} - \frac{\log(cx-1)}{cde+e^2} - \frac{2 \log(xe+d)}{c^2 d^2 - e^2} \right) - \frac{2 \operatorname{artanh}(cx)}{xe^2 + de} \right) b - \frac{a}{xe^2 + de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))/(e*x+d)^2,x, algorithm="maxima")`

[Out] $1/2*(c*(\log(c*x+1)/(c*d*e-e^2)-\log(c*x-1)/(c*d*e+e^2)-2*\log(x*e+d)/(c^2*d^2-e^2))-2*\operatorname{arctanh}(c*x)/(x*e^2+d*e))*b-a/(x*e^2+d*e)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(89) = 178$.

time = 0.47, size = 393, normalized size = 4.23

$\frac{2a^2c^2 - 2acosh(1)^2 - 4acosh(1)sinh(1) - 2acosh(1)^2 - (b^2d^2 + b^2cosh(1)^2 + b^2cosh(1)sinh(1) + b^2d^2 + 2b^2cosh(1)sinh(1))\log(cx+1) + (b^2d^2 - b^2cosh(1)^2 - b^2cosh(1)sinh(1) + b^2d^2 - 2b^2cosh(1)sinh(1) - b^2cosh(1)^2 - b^2cosh(1)sinh(1) + b^2d^2 + 2b^2cosh(1)sinh(1))\log(cx-1) + 2(b^2cosh(1)^2 + b^2cosh(1)sinh(1) + b^2d^2 + 2b^2cosh(1)sinh(1) + b^2cosh(1)^2 + b^2cosh(1)sinh(1))\log(xe+d) + c^2d^2 - e^2}{2c^2d^2e^2 + 2c^2de^2 - acosh(1)^2 - acosh(1)sinh(1) + d^2cosh(1)^2 + d^2cosh(1)sinh(1) + d^2e^2 - 2d^2cosh(1)sinh(1) + d^2e^2 - 2d^2cosh(1)^2 - 2d^2cosh(1)sinh(1) + d^2e^2 - 2d^2cosh(1)^2 - 2d^2cosh(1)sinh(1) + d^2e^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))/(e*x+d)^2,x, algorithm="fricas")`

[Out] $-1/2*(2*a*c^2*d^2 - 2*a*cosh(1)^2 - 4*a*cosh(1)*sinh(1) - 2*a*sinh(1)^2 - (b*c^2*d^2 + b*c*x*cosh(1)^2 + b*c*x*sinh(1)^2 + (b*c^2*d*x + b*c*d)*cosh(1) + (b*c^2*d*x + 2*b*c*x*cosh(1) + b*c*d)*sinh(1))*\log(c*x+1) + (b*c^2*d^2 - b*c*x*cosh(1)^2 - b*c*x*sinh(1)^2 + (b*c^2*d*x - b*c*d)*cosh(1) + (b*c^2*d*x - 2*b*c*x*cosh(1) - b*c*d)*sinh(1))*\log(c*x-1) + 2*(b*c*x*cosh(1)^2 + b*c*x*sinh(1)^2 + b*c*d*cosh(1) + (2*b*c*x*cosh(1) + b*c*d)*sinh(1))*\log(x*cosh(1) + x*sinh(1) + d) + (b*c^2*d^2 - b*cosh(1)^2 - 2*b*cosh(1)*sinh(1) - b*sinh(1)^2)*\log(-(c*x+1)/(c*x-1))/(c^2*d^2*x*cosh(1)^2 + c^2*d^3*cosh(1) - x*cosh(1)^4 - x*sinh(1)^4 - d*cosh(1)^3 - (4*x*cosh(1) + d)*sinh(1)^3 + (c^2*d^2*x - 6*x*cosh(1)^2 - 3*d*cosh(1))*sinh(1)^2 + (2*c^2*d^2*x*cosh(1) + c^2*d^3 - 4*x*cosh(1)^3 - 3*d*cosh(1)^2)*sinh(1))$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(73) = 146$.

time = 1.27, size = 690, normalized size = 7.42

$$\begin{cases} -\frac{2ad}{2d^2e+2de^2} + \frac{bd \operatorname{atanh}\left(\frac{cx}{d}\right)}{2d^2e+2de^2} + \frac{bd}{2d^2e+2de^2} & \text{for } c = -\frac{d}{e} \\ -\frac{2ad}{2d^2e+2de^2} - \frac{bd \operatorname{atanh}\left(\frac{cx}{d}\right)}{2d^2e+2de^2} + \frac{bd}{2d^2e+2de^2} & \text{for } c = \frac{d}{e} \\ \infty \left(ax + bx \operatorname{atanh}\left(\frac{cx}{d}\right) + \frac{b \log\left(\frac{x-\frac{1}{c}}{c}\right)}{d} + \frac{b \operatorname{atanh}\left(\frac{cx}{d}\right)}{c} \right) & \text{for } d = -ex \\ -\frac{d}{de^2e^2} & \text{for } e = 0 \\ -\frac{d}{de^2e^2} & \text{for } c = 0 \\ -\frac{ad^2d}{c^2d^2e+c^2d^2e^2-d^2-c^2} + \frac{ad^2}{c^2d^2e+c^2d^2e^2-d^2-c^2} + \frac{b^2-d^2 \operatorname{atanh}\left(\frac{cx}{d}\right)}{c^2d^2e+c^2d^2e^2-d^2-c^2} + \frac{b^2 \log\left(\frac{x-\frac{1}{c}}{c}\right)}{c^2d^2e+c^2d^2e^2-d^2-c^2} - \frac{b^2 \log\left(\frac{d+x}{c}\right)}{c^2d^2e+c^2d^2e^2-d^2-c^2} + \frac{b^2 \operatorname{atanh}\left(\frac{cx}{d}\right)}{c^2d^2e+c^2d^2e^2-d^2-c^2} + \frac{b^2 x \log\left(\frac{x-\frac{1}{c}}{c}\right)}{c^2d^2e+c^2d^2e^2-d^2-c^2} - \frac{b^2 x \log\left(\frac{d+x}{c}\right)}{c^2d^2e+c^2d^2e^2-d^2-c^2} + \frac{b^2 x \operatorname{atanh}\left(\frac{cx}{d}\right)}{c^2d^2e+c^2d^2e^2-d^2-c^2} + \frac{b^2 \operatorname{atanh}\left(\frac{cx}{d}\right)}{c^2d^2e+c^2d^2e^2-d^2-c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))/(e*x+d)**2,x)

[Out] Piecewise((-2*a*d/(2*d**2*e + 2*d*e**2*x) + b*d*atanh(e*x/d)/(2*d**2*e + 2*d*e**2*x) + b*d/(2*d**2*e + 2*d*e**2*x) - b*e*x*atanh(e*x/d)/(2*d**2*e + 2*d*e**2*x), Eq(c, -e/d)), (-2*a*d/(2*d**2*e + 2*d*e**2*x) - b*d*atanh(e*x/d)/(2*d**2*e + 2*d*e**2*x) - b*d/(2*d**2*e + 2*d*e**2*x) + b*e*x*atanh(e*x/d)/(2*d**2*e + 2*d*e**2*x), Eq(c, e/d)), (zoo*(a*x + b*x*atanh(c*x) + b*log(x - 1/c)/c + b*atanh(c*x)/c), Eq(d, -e*x)), ((a*x + b*x*atanh(c*x) + b*log(x - 1/c)/c + b*atanh(c*x)/c)/d**2, Eq(e, 0)), (-a/(d*e + e**2*x), Eq(c, 0)), (-a*c**2*d**2/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + a*e**2/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*c**2*d*e*x*atanh(c*x)/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*c*d*e*log(x - 1/c)/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) - b*c*d*e*log(d/e + x)/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*c*d*e*atanh(c*x)/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*c*e**2*x*log(x - 1/c)/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) - b*c*e**2*x*log(d/e + x)/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*c*e**2*x*atanh(c*x)/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*e**2*atanh(c*x)/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(89) = 178.

time = 0.41, size = 243, normalized size = 2.61

$$-c \left(\frac{b \log \left(\frac{(cx+1)cd}{cx-1} - cd + \frac{(cx+1)e}{cx-1} + e \right)}{c^2 d^2 - e^2} - \frac{b \log \left(-\frac{cx+1}{cx-1} \right)}{\frac{(cx+1)c^2 d^2}{cx-1} - c^2 d^2 + \frac{2(cx+1)cde}{cx-1} + \frac{(cx+1)e^2}{cx-1} + e^2} - \frac{b \log \left(-\frac{cx+1}{cx-1} \right)}{c^2 d^2 - e^2} - \frac{2a}{\frac{(cx+1)c^2 d^2}{cx-1} - c^2 d^2 + \frac{2(cx+1)cde}{cx-1} + \frac{(cx+1)e^2}{cx-1} + e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))/(e*x+d)^2,x, algorithm="giac")

[Out] -c*(b*log((c*x + 1)*c*d/(c*x - 1) - c*d + (c*x + 1)*e/(c*x - 1) + e)/(c^2*d^2 - e^2) - b*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2*d^2/(c*x - 1) - c^2*d^2 + 2*(c*x + 1)*c*d*e/(c*x - 1) + (c*x + 1)*e^2/(c*x - 1) + e^2) - b*log(-(c*x + 1)/(c*x - 1))/(c^2*d^2 - e^2) - 2*a/((c*x + 1)*c^2*d^2/(c*x - 1) - c^2*d^2 + 2*(c*x + 1)*c*d*e/(c*x - 1) + (c*x + 1)*e^2/(c*x - 1) + e^2))

Mupad [B]

time = 3.60, size = 115, normalized size = 1.24

$$\frac{d^2 \left(\frac{bc \ln(c^2 x^2 - 1)}{2} - bc \ln(d + ex) + ac^2 x + bc^2 x \operatorname{atanh}(cx) \right) + de \left(b \operatorname{atanh}(cx) - bcx \ln(d + ex) + \frac{bcx \ln(c^2 x^2 - 1)}{2} \right) - ae^2 x}{d(e^2 - c^2 d^2)(d + ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))/(d + e*x)^2,x)

[Out] -(d^2*((b*c*log(c^2*x^2 - 1))/2 - b*c*log(d + e*x) + a*c^2*x + b*c^2*x*atanh(c*x)) + d*e*(b*atanh(c*x) - b*c*x*log(d + e*x) + (b*c*x*log(c^2*x^2 - 1))/2) - a*e^2*x)/(d*(e^2 - c^2*d^2)*(d + e*x))

3.7 $\int \frac{a+b \tanh^{-1}(cx)}{(d+ex)^3} dx$

Optimal. Leaf size=130

$$\frac{bc}{2(c^2d^2 - e^2)(d + ex)} - \frac{a + b \tanh^{-1}(cx)}{2e(d + ex)^2} - \frac{bc^2 \log(1 - cx)}{4e(cd + e)^2} + \frac{bc^2 \log(1 + cx)}{4(cd - e)^2e} - \frac{bc^3 d \log(d + ex)}{(c^2d^2 - e^2)^2}$$

[Out] $1/2*b*c/(c^2*d^2-e^2)/(e*x+d)+1/2*(-a-b*\operatorname{arctanh}(c*x))/e/(e*x+d)^2-1/4*b*c^2*\ln(-c*x+1)/e/(c*d+e)^2+1/4*b*c^2*\ln(c*x+1)/(c*d-e)^2/e-b*c^3*d*\ln(e*x+d)/(c^2*d^2-e^2)^2$

Rubi [A]

time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6063, 724, 815}

$$-\frac{a + b \tanh^{-1}(cx)}{2e(d + ex)^2} + \frac{bc}{2(c^2d^2 - e^2)(d + ex)} - \frac{bc^2 \log(1 - cx)}{4e(cd + e)^2} + \frac{bc^2 \log(cx + 1)}{4e(cd - e)^2} - \frac{bc^3 d \log(d + ex)}{(c^2d^2 - e^2)^2}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcTanh[c*x])/(d + e*x)^3,x]`

[Out] $(b*c)/(2*(c^2*d^2 - e^2)*(d + e*x)) - (a + b*ArcTanh[c*x])/(2*e*(d + e*x)^2) - (b*c^2*Log[1 - c*x])/(4*e*(c*d + e)^2) + (b*c^2*Log[1 + c*x])/(4*(c*d - e)^2*e) - (b*c^3*d*Log[d + e*x])/(c^2*d^2 - e^2)^2$

Rule 724

`Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))], x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]`

Rule 815

`Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]`

Rule 6063

`Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)*((d_) + (e_.)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]`

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx)}{(d + ex)^3} dx &= -\frac{a + b \tanh^{-1}(cx)}{2e(d + ex)^2} + \frac{(bc) \int \frac{1}{(d+ex)^2(1-c^2x^2)} dx}{2e} \\
 &= \frac{bc}{2(c^2d^2 - e^2)(d + ex)} - \frac{a + b \tanh^{-1}(cx)}{2e(d + ex)^2} + \frac{(bc^3) \int \frac{d-ex}{(d+ex)(1-c^2x^2)} dx}{2e(c^2d^2 - e^2)} \\
 &= \frac{bc}{2(c^2d^2 - e^2)(d + ex)} - \frac{a + b \tanh^{-1}(cx)}{2e(d + ex)^2} + \frac{(bc^3) \int \left(\frac{-cd+e}{2(cd+e)(-1+cx)} + \frac{cd+e}{2(cd-e)(1+cx)} \right) dx}{2e(c^2d^2 - e^2)} \\
 &= \frac{bc}{2(c^2d^2 - e^2)(d + ex)} - \frac{a + b \tanh^{-1}(cx)}{2e(d + ex)^2} - \frac{bc^2 \log(1 - cx)}{4e(cd + e)^2} + \frac{bc^2 \log(1 + cx)}{4(cd - e)^2e} -
 \end{aligned}$$

Mathematica [A]

time = 0.10, size = 133, normalized size = 1.02

$$\frac{1}{4} \left(-\frac{2a}{e(d + ex)^2} + \frac{2bc}{(c^2d^2 - e^2)(d + ex)} - \frac{2b \tanh^{-1}(cx)}{e(d + ex)^2} - \frac{bc^2 \log(1 - cx)}{e(cd + e)^2} + \frac{bc^2 \log(1 + cx)}{e(-cd + e)^2} - \frac{4bc^3d \log(d + ex)}{(-c^2d^2 + e^2)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x])/(d + e*x)^3,x]

[Out] ((-2*a)/(e*(d + e*x)^2) + (2*b*c)/((c^2*d^2 - e^2)*(d + e*x)) - (2*b*ArcTanh[c*x])/(e*(d + e*x)^2) - (b*c^2*Log[1 - c*x])/(e*(c*d + e)^2) + (b*c^2*Log[1 + c*x])/(e*(-c*d) + e)^2) - (4*b*c^3*d*Log[d + e*x])/(-c^2*d^2 + e^2)^2)/4

Maple [A]

time = 0.19, size = 158, normalized size = 1.22

method	result
derivativedivides	$-\frac{a c^3}{2(cex+dc)^2 e} - \frac{b c^3 \operatorname{arctanh}(cx)}{2(cex+dc)^2 e} + \frac{b c^3 \ln(cx+1)}{4e(dc-e)^2} + \frac{b c^3}{2(dc+e)(dc-e)(cex+dc)} - \frac{b c^4 d \ln(cex+dc)}{(dc+e)^2 (dc-e)^2} - \frac{b c^3 \ln(cx-1)}{4e(dc+e)^2}$
default	$-\frac{a c^3}{2(cex+dc)^2 e} - \frac{b c^3 \operatorname{arctanh}(cx)}{2(cex+dc)^2 e} + \frac{b c^3 \ln(cx+1)}{4e(dc-e)^2} + \frac{b c^3}{2(dc+e)(dc-e)(cex+dc)} - \frac{b c^4 d \ln(cex+dc)}{(dc+e)^2 (dc-e)^2} - \frac{b c^3 \ln(cx-1)}{4e(dc+e)^2}$
risch	$-\frac{b \ln(cx+1)}{4e(ex+d)^2} - \frac{-2 \ln(-cx-1) b c^3 d^3 e - \ln(-cx-1) b c^2 d^2 e^2 - \ln(-cx-1) b c^2 e^4 x^2 + 4 \ln(ex+d) b c^3 d^3 e + \ln(-cx+1) b c^2 e^4}{4e(ex+d)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)

[Out] 1/c*(-1/2*a*c^3/(c*e*x+c*d)^2/e-1/2*b*c^3/(c*e*x+c*d)^2/e*arctanh(c*x)+1/4*b*c^3/e/(c*d-e)^2*ln(c*x+1)+1/2*b*c^3/(c*d+e)/(c*d-e)/(c*e*x+c*d)-b*c^4*d/(c*d+e)^2/(c*d-e)^2*ln(c*e*x+c*d)-1/4*b*c^3/e/(c*d+e)^2*ln(c*x-1))

Maxima [A]

time = 0.27, size = 184, normalized size = 1.42

$$-\frac{1}{4} \left(\frac{4c^2 d \log(xe+d)}{c^4 d^4 - 2c^2 d^2 e^2 + e^4} - \frac{c \log(cx+1)}{c^2 d^2 e - 2cde^2 + e^3} + \frac{c \log(cx-1)}{c^2 d^2 e + 2cde^2 + e^3} - \frac{2}{c^2 d^3 + (c^2 d^2 e - e^3)x - de^2} \right) c + \frac{2 \operatorname{artanh}(cx)}{x^2 e^3 + 2dxe^2 + d^2 e} b - \frac{a}{2(x^2 e^3 + 2dxe^2 + d^2 e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))/(e*x+d)^3,x, algorithm="maxima")

[Out] -1/4*((4*c^2*d*log(x*e + d)/(c^4*d^4 - 2*c^2*d^2*e^2 + e^4) - c*log(c*x + 1))/(c^2*d^2*e - 2*c*d*e^2 + e^3) + c*log(c*x - 1)/(c^2*d^2*e + 2*c*d*e^2 + e^3) - 2/(c^2*d^3 + (c^2*d^2*e - e^3)*x - d*e^2))*c + 2*arctanh(c*x)/(x^2*e^3 + 2*d*x*e^2 + d^2*e))*b - 1/2*a/(x^2*e^3 + 2*d*x*e^2 + d^2*e)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1555 vs. 2(122) = 244.

time = 0.47, size = 1555, normalized size = 11.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))/(e*x+d)^3,x, algorithm="fricas")

[Out] -1/4*(2*a*c^4*d^4 - 2*b*c^3*d^3*cosh(1) + 2*b*c*d*cosh(1)^3 + 2*(b*c*x + a)*cosh(1)^4 + 2*(b*c*x + a)*sinh(1)^4 + 2*(b*c*d + 4*(b*c*x + a)*cosh(1))*sinh(1)^3 - 2*(b*c^3*d^2*x + 2*a*c^2*d^2)*cosh(1)^2 - 2*(b*c^3*d^2*x + 2*a*c^2*d^2 - 3*b*c*d*cosh(1) - 6*(b*c*x + a)*cosh(1)^2)*sinh(1)^2 - (b*c^4*d^4 + b*c^2*x^2*cosh(1)^4 + b*c^2*x^2*sinh(1)^4 + 2*(b*c^3*d*x^2 + b*c^2*d*x)*cosh(1)^3 + 2*(b*c^3*d*x^2 + 2*b*c^2*x^2*cosh(1) + b*c^2*d*x)*sinh(1)^3 + (b*c^4*d^2*x^2 + 4*b*c^3*d^2*x + b*c^2*d^2)*cosh(1)^2 + (b*c^4*d^2*x^2 + 4*b*c^3*d^2*x + 6*b*c^2*x^2*cosh(1)^2 + b*c^2*d^2 + 6*(b*c^3*d*x^2 + b*c^2*d*x)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x + b*c^3*d^3)*cosh(1) + 2*(b*c^4*d^3*x + 2*b*c^2*x^2*cosh(1)^3 + b*c^3*d^3 + 3*(b*c^3*d*x^2 + b*c^2*d*x)*cosh(1)^2 + (b*c^4*d^2*x^2 + 4*b*c^3*d^2*x + b*c^2*d^2)*cosh(1))*sinh(1))*log(c*x + 1) + (b*c^4*d^4 + b*c^2*x^2*cosh(1)^4 + b*c^2*x^2*sinh(1)^4 - 2*(b*c^3*d*x^2 - b*c^2*d*x)*cosh(1)^3 - 2*(b*c^3*d*x^2 - 2*b*c^2*x^2*cosh(1) - b*c^2*d*x)*sinh(1)^3 + (b*c^4*d^2*x^2 - 4*b*c^3*d^2*x + b*c^2*d^2)*cosh(1)^2 + (b*c^4*d^2*x^2 - 4*b*c^3*d^2*x + 6*b*c^2*x^2*cosh(1)^2 + b*c^2*d^2 - 6*(b*c^3*d*x^2 - b*c^2*d*x)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x - b*c^3*d^3)*cosh(1) + 2*(b*c^4*d^3*x + 2*b*c^2*x^2*cosh(1)^3 - b*c^3*d^3 - 3*(b*c^3*d*x^2 - b*c^2*d*x)*cosh(1)^2 + (b*c^4*d^2*x^2 - 4*b*c^3*d^2*x + b*c^2*d^2)*cosh(1))*sinh(1))*log(c*x - 1) + 4*(b*c^3*d*x^2*cosh(1)^3 + b*c^3*d*x^2*sinh(1)^3 + 2*b*c^3*d^2*x*cosh(1)^2 + b*c^3*d^3*cosh(1) + (3*b*c^3*d*x^2*cosh(1) + 2*b*c^3*d^2*x)*sinh(1)^2 + (3*b*c^3*d*x^2*cosh(1)^2 + 4*b*c^3*d^2*x*cosh(1) + b*c^3*d^3)*sinh(1))*log(x*cosh(1) + x*sinh(1) + d) + (b*c^4*d^4 - 2*b*c^2*d^2*cosh(1)^2 + b*cosh(1)^4 + 4*b*cosh(1)*sinh(1)^3 + b*sinh(1)^4 - 2*(b*c^2*d^2

$$\begin{aligned}
& - 3*b*cosh(1)^2*sinh(1)^2 - 4*(b*c^2*d^2*cosh(1) - b*cosh(1)^3)*sinh(1)* \\
& \log(-(c*x + 1)/(c*x - 1)) - 2*(b*c^3*d^3 - 3*b*c*d*cosh(1)^2 - 4*(b*c*x + a) \\
& *cosh(1)^3 + 2*(b*c^3*d^2*x + 2*a*c^2*d^2)*cosh(1))*sinh(1))/(2*c^4*d^5*x*c \\
& osh(1)^2 + c^4*d^6*cosh(1) - 4*c^2*d^3*x*cosh(1)^4 + x^2*cosh(1)^7 + x^2*si \\
& nh(1)^7 + 2*d*x*cosh(1)^6 + (7*x^2*cosh(1) + 2*d*x)*sinh(1)^6 - (2*c^2*d^2*x \\
& x^2 - d^2)*cosh(1)^5 - (2*c^2*d^2*x^2 - 21*x^2*cosh(1)^2 - 12*d*x*cosh(1) - \\
& d^2)*sinh(1)^5 - (4*c^2*d^3*x - 35*x^2*cosh(1)^3 - 30*d*x*cosh(1)^2 + 5*(2 \\
& *c^2*d^2*x^2 - d^2)*cosh(1))*sinh(1)^4 + (c^4*d^4*x^2 - 2*c^2*d^4)*cosh(1)^ \\
& 3 + (c^4*d^4*x^2 - 16*c^2*d^3*x*cosh(1) - 2*c^2*d^4 + 35*x^2*cosh(1)^4 + 40 \\
& *d*x*cosh(1)^3 - 10*(2*c^2*d^2*x^2 - d^2)*cosh(1)^2)*sinh(1)^3 + (2*c^4*d^5 \\
& *x - 24*c^2*d^3*x*cosh(1)^2 + 21*x^2*cosh(1)^5 + 30*d*x*cosh(1)^4 - 10*(2*c \\
& ^2*d^2*x^2 - d^2)*cosh(1)^3 + 3*(c^4*d^4*x^2 - 2*c^2*d^4)*cosh(1))*sinh(1)^ \\
& 2 + (4*c^4*d^5*x*cosh(1) + c^4*d^6 - 16*c^2*d^3*x*cosh(1)^3 + 7*x^2*cosh(1) \\
& ^6 + 12*d*x*cosh(1)^5 - 5*(2*c^2*d^2*x^2 - d^2)*cosh(1)^4 + 3*(c^4*d^4*x^2 \\
& - 2*c^2*d^4)*cosh(1)^2)*sinh(1)
\end{aligned}$$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3216 vs. $2(110) = 220$.

time = 3.26, size = 3216, normalized size = 24.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))/(e*x+d)**3,x)

[Out] Piecewise((a*x/d**3, Eq(c, 0) & Eq(e, 0)), (-a/(2*d**2*e + 4*d*e**2*x + 2*e**3*x**2), Eq(c, 0)), (-4*a*d**2/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) + 3*b*d**2*atanh(e*x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) + 2*b*d**2/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - 2*b*d*e*x*atanh(e*x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) + b*d*e*x/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - b*e**2*x**2*atanh(e*x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2), Eq(c, -e/d)), (-4*a*d**2/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - 3*b*d**2*atanh(e*x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - 2*b*d**2/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) + 2*b*d*e*x*atanh(e*x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - b*d*e*x/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) + b*e**2*x**2*atanh(e*x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2), Eq(c, e/d)), ((a*x + b*x*atanh(c*x) + b*log(x - 1/c)/c + b*atanh(c*x)/c)/d**3, Eq(e, 0)), (-a*c**4*d**4/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*a*c**2*d**2*e**2/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - a*e**4/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4

$$\begin{aligned}
& *d^{**6}*x + 2*e^{**7}*x^{**2}) + 2*b*c^{**4}*d^{**3}*e*x*\operatorname{atanh}(c*x)/(2*c^{**4}*d^{**6}*e + 4* \\
& c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e \\
& **4*x - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) + b \\
& *c^{**4}*d^{**2}*e^{**2}*x^{**2}*\operatorname{atanh}(c*x)/(2*c^{**4}*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4} \\
& 4*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2}*d^{**2}*e^{**5} \\
& *x^{**2} + 2*d^{**2}*e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) + 2*b*c^{**3}*d^{**3}*e*\log(x - 1 \\
& /c)/(2*c^{**4}*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4} \\
& *e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} + 4*d^{**6} \\
& *x + 2*e^{**7}*x^{**2}) - 2*b*c^{**3}*d^{**3}*e*\log(d/e + x)/(2*c^{**4}*d^{**6}*e + 4*c^{**4}* \\
& d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x \\
& - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) + 2*b*c^{**3} \\
& *d^{**3}*e*\operatorname{atanh}(c*x)/(2*c^{**4}*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3} \\
& *x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d \\
& **2*e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) + b*c^{**3}*d^{**3}*e/(2*c^{**4}*d^{**6}*e + 4*c^{**4} \\
& 4*d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4} \\
& *x - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) + 4*b* \\
& c^{**3}*d^{**2}*e^{**2}*x*\log(x - 1/c)/(2*c^{**4}*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4}* \\
& d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2}*d^{**2}*e^{**5}*x \\
& **2 + 2*d^{**2}*e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) - 4*b*c^{**3}*d^{**2}*e^{**2}*x*\log(d/ \\
& e + x)/(2*c^{**4}*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2} \\
& *d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} + 4*d \\
& *e^{**6}*x + 2*e^{**7}*x^{**2}) + 4*b*c^{**3}*d^{**2}*e^{**2}*x*\operatorname{atanh}(c*x)/(2*c^{**4}*d^{**6}*e + 4 \\
& *c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}* \\
& e^{**4}*x - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) + \\
& b*c^{**3}*d^{**2}*e^{**2}*x/(2*c^{**4}*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3}*x \\
& **2 - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d^{**2} \\
& *e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) + 2*b*c^{**3}*d^{**3}*x^{**2}*\log(x - 1/c)/(2*c \\
& **4*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} \\
& - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} + 4*d^{**6}*x + 2 \\
& *e^{**7}*x^{**2}) - 2*b*c^{**3}*d^{**3}*x^{**2}*\log(d/e + x)/(2*c^{**4}*d^{**6}*e + 4*c^{**4}*d^{**5} \\
& *e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x - \\
& 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) + 2*b*c^{**3}* \\
& d^{**3}*x^{**2}*\operatorname{atanh}(c*x)/(2*c^{**4}*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3} \\
& *x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2 \\
& *d^{**2}*e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) + 3*b*c^{**2}*d^{**2}*e^{**2}*\operatorname{atanh}(c*x)/(2*c \\
& **4*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} \\
& - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} + 4*d^{**6}*x + 2 \\
& *e^{**7}*x^{**2}) + 2*b*c^{**2}*d^{**3}*x*\operatorname{atanh}(c*x)/(2*c^{**4}*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2} \\
& *x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2} \\
& *d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) + b*c^{**2}*e^{**4}*x* \\
& *2*\operatorname{atanh}(c*x)/(2*c^{**4}*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2}*x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - \\
& 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2}*d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} \\
& + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) - b*c^{**2}*e^{**4}*(2*c^{**4}*d^{**6}*e + 4*c^{**4}*d^{**5}*e^{**2} \\
& *x + 2*c^{**4}*d^{**4}*e^{**3}*x^{**2} - 4*c^{**2}*d^{**4}*e^{**3} - 8*c^{**2}*d^{**3}*e^{**4}*x - 4*c^{**2} \\
& *d^{**2}*e^{**5}*x^{**2} + 2*d^{**2}*e^{**5} + 4*d^{**6}*x + 2*e^{**7}*x^{**2}) - b*c^{**2}*e^{**4}*(2*
\end{aligned}$$

$c^{**4}d^{**6}e + 4c^{**4}d^{**5}e^{**2}x + 2c^{**4}d^{**4}e^{**3}x^{**2} - 4c^{**2}d^{**4}e^{**3} - 8c^{**2}d^{**3}e^{**4}x - 4c^{**2}d^{**2}e^{**5}x^{**2} + \dots$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 809 vs. 2(122) = 244.

time = 0.42, size = 809, normalized size = 6.22

$$\left(\frac{b^2 d \ln\left(\frac{-ax^2 + ad - \sqrt{a^2 d^2 - c^2}}{c^2 d^2 - 2c^2 d^2 e^2 + e^4}\right) - \frac{b^2 d \ln\left(\frac{-ax^2 + ad + \sqrt{a^2 d^2 - c^2}}{c^2 d^2 - 2c^2 d^2 e^2 + e^4}\right)}{2e^5 \sqrt{-c^2 - c^2}} - \frac{\operatorname{atan}\left(\frac{c^2 x}{\sqrt{-c^2 - c^2}}\right) (b^2 d^2 + b^2 c^2 e^2)}{2e^5 \sqrt{-c^2 - c^2}} - \frac{b^2 c^3 d \ln(d + ex)}{c^4 d^4 - 2c^2 d^2 e^2 + e^4} - \frac{\operatorname{atanh}(cx)}{2e} - \frac{x(-a^2 d^2 + b^2 d^2 + a^2 e^2)}{d(c^2 - c^2 d^2)} - \frac{x^2(-a^2 d^2 + b^2 d^2 + a^2 e^2)}{2(c^2 - c^2 d^2)} + \frac{x^4(-a^2 d^2 + b^2 d^2 + a^2 e^2)}{2(c^2 - c^2 d^2)} + \frac{x^2(-a^2 d^2 + b^2 d^2 + a^2 e^2)}{d(c^2 - c^2 d^2)} - \frac{b^2 c^2 \operatorname{atanh}(cx)}{2e} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))/(e*x+d)^3,x, algorithm="giac")

[Out] $-(b*c^2*d*\log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c^4*d^4 - 2*c^2*d^2*e^2 + e^4) - b*c^2*d*\log(-(c*x + 1)/(c*x - 1))/(c^4*d^4 - 2*c^2*d^2*e^2 + e^4) - ((c*x + 1)*b*c^2*d/(c*x - 1) - b*c^2*d + (c*x + 1)*b*c*e/(c*x - 1))*\log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2*c^4*d^4/(c*x - 1)^2 - 2*(c*x + 1)*c^4*d^4/(c*x - 1) + c^4*d^4 + 4*(c*x + 1)^2*c^3*d^3*e/(c*x - 1)^2 - 4*(c*x + 1)*c^3*d^3*e/(c*x - 1) + 6*(c*x + 1)^2*c^2*d^2*e^2/(c*x - 1)^2 - 2*c^2*d^2*e^2 + 4*(c*x + 1)^2*c*d*e^3/(c*x - 1)^2 + 4*(c*x + 1)*c*d*e^3/(c*x - 1) + (c*x + 1)^2*e^4/(c*x - 1)^2 + 2*(c*x + 1)*e^4/(c*x - 1) + e^4) - (2*(c*x + 1)*a*c^3*d^2/(c*x - 1) - 2*a*c^3*d^2 + 2*a*c^2*d*e - (c*x + 1)*b*c^2*d*e/(c*x - 1) + b*c^2*d*e - 2*(c*x + 1)*a*c*e^2/(c*x - 1) - (c*x + 1)*b*c*e^2/(c*x - 1) - b*c*e^2)/((c*x + 1)^2*c^5*d^5/(c*x - 1)^2 - 2*(c*x + 1)*c^5*d^5/(c*x - 1) + c^5*d^5 + 3*(c*x + 1)^2*c^4*d^4*e/(c*x - 1)^2 - 2*(c*x + 1)*c^4*d^4*e/(c*x - 1) - c^4*d^4*e + 2*(c*x + 1)^2*c^3*d^3*e^2/(c*x - 1)^2 + 4*(c*x + 1)*c^3*d^3*e^2/(c*x - 1) - 2*c^3*d^3*e^2 - 2*(c*x + 1)^2*c^2*d^2*e^3/(c*x - 1)^2 + 4*(c*x + 1)*c^2*d^2*e^3/(c*x - 1) + 2*c^2*d^2*e^3 - 3*(c*x + 1)^2*c*d*e^4/(c*x - 1)^2 - 2*(c*x + 1)*c*d*e^4/(c*x - 1) + c*d*e^4 - (c*x + 1)^2*e^5/(c*x - 1)^2 - 2*(c*x + 1)*e^5/(c*x - 1) - e^5)*c$

Mupad [B]

time = 4.00, size = 427, normalized size = 3.28

$$\frac{b^2 d \ln(c^2 x^2 - 1)}{2(c^4 d^4 - 2c^2 d^2 e^2 + e^4)} - \frac{\operatorname{atan}\left(\frac{c^2 x}{\sqrt{-c^2}}\right) (b^2 d^2 + b^2 c^2 e^2)}{2e^5 \sqrt{-c^2 - c^2}} - \frac{b^2 c^3 d \ln(d + ex)}{c^4 d^4 - 2c^2 d^2 e^2 + e^4} - \frac{\operatorname{atanh}(cx)}{2e} - \frac{x(-a^2 d^2 + b^2 d^2 + a^2 e^2)}{d(c^2 - c^2 d^2)} - \frac{x^2(-a^2 d^2 + b^2 d^2 + a^2 e^2)}{2(c^2 - c^2 d^2)} + \frac{x^4(-a^2 d^2 + b^2 d^2 + a^2 e^2)}{2(c^2 - c^2 d^2)} + \frac{x^2(-a^2 d^2 + b^2 d^2 + a^2 e^2)}{d(c^2 - c^2 d^2)} - \frac{b^2 c^2 \operatorname{atanh}(cx)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))/(d + e*x)^3,x)

[Out] $(b*c^3*d*\log(c^2*x^2 - 1))/(2*(e^4 + c^4*d^4 - 2*c^2*d^2*e^2)) - (\operatorname{atan}((c^2*x)/(-c^2)^{(1/2)})*(b*c^5*d^2 + b*c^3*e^2))/(2*e^5*(-c^2)^{(1/2)} - c^2*(2*d^4*e*(-c^2)^{(3/2)} + 4*d^2*e^3*(-c^2)^{(1/2)})) - (b*c^3*d*\log(d + e*x))/(e^4 + c^4*d^4 - 2*c^2*d^2*e^2) - ((b*\operatorname{atanh}(c*x))/(2*e) - (x*(a*e^2 - a*c^2*d^2 + (b*c*d*e)/2))/(d*(e^2 - c^2*d^2)) - (x^2*((a*e^3)/2 + (b*c*d*e^2)/2 - (a*c^2*d^2*e)/2))/(d^2*(e^2 - c^2*d^2)) + (x^4*((a*c^2*e^3)/2 - (a*c^4*d^2*e)/2 + (b*c^3*d*e^2)/2))/(d^2*(e^2 - c^2*d^2)) + (x^3*(a*c^2*e^2 - a*c^4*d^2 + (b*c^3*d*e)/2))/(d*(e^2 - c^2*d^2)) - (b*c^2*x^2*\operatorname{atanh}(c*x))/(2*e)/(d^2 + e^2*x^2 + 2*d*e*x - c^2*d^2*x^2 - c^2*e^2*x^4 - 2*c^2*d*e*x^3)$

3.8 $\int \frac{a+b \tanh^{-1}(cx)}{(d+ex)^4} dx$

Optimal. Leaf size=175

$$\frac{bc}{6(c^2d^2 - e^2)(d + ex)^2} + \frac{2bc^3d}{3(c^2d^2 - e^2)^2(d + ex)} - \frac{a + b \tanh^{-1}(cx)}{3e(d + ex)^3} - \frac{bc^3 \log(1 - cx)}{6e(cd + e)^3} + \frac{bc^3 \log(1 + cx)}{6(cd - e)^3e} - \frac{bc^3(3c^2d^2 - e^2) \log(d + ex)}{3(cd - e)^3(cd + e)^3}$$

[Out] $1/6*b*c/(c^2*d^2-e^2)/(e*x+d)^2+2/3*b*c^3*d/(c^2*d^2-e^2)^2/(e*x+d)+1/3*(-a-b*\operatorname{arctanh}(c*x))/e/(e*x+d)^3-1/6*b*c^3*\ln(-c*x+1)/e/(c*d+e)^3+1/6*b*c^3*\ln(c*x+1)/(c*d-e)^3/e-1/3*b*c^3*(3*c^2*d^2+e^2)*\ln(e*x+d)/(c^2*d^2-e^2)^3$

Rubi [A]

time = 0.14, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6063, 724, 815}

$$-\frac{a + b \tanh^{-1}(cx)}{3e(d + ex)^3} - \frac{bc^3 \log(1 - cx)}{6e(cd + e)^3} + \frac{bc^3 \log(cx + 1)}{6e(cd - e)^3} + \frac{bc}{6(c^2d^2 - e^2)(d + ex)^2} + \frac{2bc^3d}{3(c^2d^2 - e^2)^2(d + ex)} - \frac{bc^3(3c^2d^2 + e^2) \log(d + ex)}{3(cd - e)^3(cd + e)^3}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcTanh[c*x])/(d + e*x)^4,x]`

[Out] $(b*c)/(6*(c^2*d^2 - e^2)*(d + e*x)^2) + (2*b*c^3*d)/(3*(c^2*d^2 - e^2)^2*(d + e*x)) - (a + b*ArcTanh[c*x])/(3*e*(d + e*x)^3) - (b*c^3*Log[1 - c*x])/(6*e*(c*d + e)^3) + (b*c^3*Log[1 + c*x])/(6*(c*d - e)^3*e) - (b*c^3*(3*c^2*d^2 + e^2)*Log[d + e*x])/(3*(c*d - e)^3*(c*d + e)^3)$

Rule 724

```
Int[((d_) + (e_)*(x_))^(m_)/((a_) + (c_)*(x_)^2), x_Symbol] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 6063

```
Int[((a_) + ArcTanh[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```


Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx)}{(d + ex)^4} dx &= -\frac{a + b \tanh^{-1}(cx)}{3e(d + ex)^3} + \frac{(bc) \int \frac{1}{(d+ex)^3(1-c^2x^2)} dx}{3e} \\
 &= \frac{bc}{6(c^2d^2 - e^2)(d + ex)^2} - \frac{a + b \tanh^{-1}(cx)}{3e(d + ex)^3} + \frac{(bc^3) \int \frac{d-ex}{(d+ex)^2(1-c^2x^2)} dx}{3e(c^2d^2 - e^2)} \\
 &= \frac{bc}{6(c^2d^2 - e^2)(d + ex)^2} - \frac{a + b \tanh^{-1}(cx)}{3e(d + ex)^3} + \frac{(bc^3) \int \left(-\frac{c(cd-e)}{2(cd+e)^2(-1+cx)} + \frac{c(cd+e)}{2(cd-e)^2} \right) dx}{3e(c^2d^2 - e^2)} \\
 &= \frac{bc}{6(c^2d^2 - e^2)(d + ex)^2} + \frac{2bc^3d}{3(c^2d^2 - e^2)^2(d + ex)} - \frac{a + b \tanh^{-1}(cx)}{3e(d + ex)^3} - \frac{bc^3 \log(1 - cx)}{6e(cd + e)^3} + \frac{bc^3 \log(1 + cx)}{6e(cd - e)^3}
 \end{aligned}$$

Mathematica [A]

time = 0.19, size = 173, normalized size = 0.99

$$\frac{1}{6} \left(-\frac{2a}{e(d+ex)^3} + \frac{bc}{(c^2d^2 - e^2)(d+ex)^2} + \frac{4bc^3d}{(-c^2d^2 + e^2)^2(d+ex)} - \frac{2b \tanh^{-1}(cx)}{e(d+ex)^3} - \frac{bc^3 \log(1 - cx)}{e(cd+e)^3} + \frac{bc^3 \log(1 + cx)}{(cd - e)^3e} - \frac{2bc^3(3c^2d^2 + e^2) \log(d + ex)}{(c^2d^2 - e^2)^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x])/(d + e*x)^4,x]

[Out] ((-2*a)/(e*(d + e*x)^3) + (b*c)/((c^2*d^2 - e^2)*(d + e*x)^2) + (4*b*c^3*d)/((-c^2*d^2) + e^2)^2*(d + e*x)) - (2*b*ArcTanh[c*x])/(e*(d + e*x)^3) - (b*c^3*Log[1 - c*x])/(e*(c*d + e)^3) + (b*c^3*Log[1 + c*x])/((c*d - e)^3*e) - (2*b*c^3*(3*c^2*d^2 + e^2)*Log[d + e*x])/(c^2*d^2 - e^2)^3/6

Maple [A]

time = 0.22, size = 227, normalized size = 1.30

method	result
derivativedivides	$-\frac{ac^4}{3(cex+dc)^3e} - \frac{bc^4 \operatorname{arctanh}(cx)}{3(cex+dc)^3e} + \frac{bc^4 \ln(cx+1)}{6e(dc-e)^3} - \frac{bc^4 \ln(cx-1)}{6e(dc+e)^3} + \frac{bc^4}{6(dc+e)(dc-e)(cex+dc)^2} - \frac{bc^6 \ln(cex+dc)d^2}{(dc+e)^3(dc-e)^3} - \frac{bc^4 e^2 \ln(cex+dc)}{3(dc+e)^3(dc-e)}$
default	$-\frac{ac^4}{3(cex+dc)^3e} - \frac{bc^4 \operatorname{arctanh}(cx)}{3(cex+dc)^3e} + \frac{bc^4 \ln(cx+1)}{6e(dc-e)^3} - \frac{bc^4 \ln(cx-1)}{6e(dc+e)^3} + \frac{bc^4}{6(dc+e)(dc-e)(cex+dc)^2} - \frac{bc^6 \ln(cex+dc)d^2}{(dc+e)^3(dc-e)^3} - \frac{bc^4 e^2 \ln(cex+dc)}{3(dc+e)^3(dc-e)}$
risch	$-\frac{b \ln(cx+1)}{6e(ex+d)^3} + \frac{\ln(-cx-1)bc^6d^3e^3x^3 - \ln(-cx+1)bc^6d^3e^3x^3 + 3bc^2d^2e^4 \ln(-cx+1) + \ln(-cx-1)bc^3e^6x^3 - 2 \ln(ex+d)}{6e(ex+d)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x))/(e*x+d)^4,x,method=_RETURNVERBOSE)

[Out] 1/c*(-1/3*a*c^4/(c*e*x+c*d)^3/e-1/3*b*c^4/(c*e*x+c*d)^3/e*arctanh(c*x)+1/6*b*c^4/e/(c*d-e)^3*ln(c*x+1)-1/6*b*c^4/e/(c*d+e)^3*ln(c*x-1)+1/6*b*c^4/(c*d+e)^3)

$e)/(c*d-e)/(c*e*x+c*d)^{2-b*c^6}/(c*d+e)^3/(c*d-e)^3*\ln(c*e*x+c*d)*d^{2-1/3*b*c^4*e^2}/(c*d+e)^3/(c*d-e)^3*\ln(c*e*x+c*d)+2/3*b*c^5*d/(c*d+e)^2/(c*d-e)^2/(c*e*x+c*d)$

Maxima [A]

time = 0.27, size = 322, normalized size = 1.84

$$\frac{1}{6} \left(\frac{c^2 \log(cx+1)}{c^2 d^2 e - 3 c^2 d^2 e^2 + 3 c d e^3 - e^4} - \frac{c^2 \log(cx-1)}{c^2 d^2 e + 3 c^2 d^2 e^2 + 3 c d e^3 + e^4} - \frac{2(3 c^4 d^2 + c^2 e^2) \log(xe+d)}{c^2 d^6 - 3 c^4 d^4 e^2 + 3 c^2 d^2 e^4 - e^6} + \frac{4 c^2 d x e + 5 c^2 d^2 - e^2}{c^2 d^6 - 2 c^2 d^4 e^2 + (c^2 d^4 e^2 - 2 c^2 d^2 e^4 + e^6) x^2 + d^2 e^4 + 2(c^2 d^4 e - 2 c^2 d^2 e^3 + d e^5) x} \right) e^{-\frac{2 \operatorname{artanh}(cx)}{x^2 e^4 + 3 d x^2 e^3 + 3 d^2 x e^2 + d^3 e}} b - \frac{a}{3(x^2 e^4 + 3 d x^2 e^3 + 3 d^2 x e^2 + d^3 e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))/(e*x+d)^4,x, algorithm="maxima")

[Out] $\frac{1}{6} * ((c^2 * \log(c*x + 1) / (c^3 * d^3 * e - 3 * c^2 * d^2 * e^2 + 3 * c * d * e^3 - e^4) - c^2 * \log(c*x - 1) / (c^3 * d^3 * e + 3 * c^2 * d^2 * e^2 + 3 * c * d * e^3 + e^4) - 2 * (3 * c^4 * d^2 + c^2 * e^2) * \log(x * e + d) / (c^6 * d^6 - 3 * c^4 * d^4 * e^2 + 3 * c^2 * d^2 * e^4 - e^6) + (4 * c^2 * d * x * e + 5 * c^2 * d^2 - e^2) / (c^4 * d^6 - 2 * c^2 * d^4 * e^2 + (c^4 * d^4 * e^2 - 2 * c^2 * d^2 * e^4 + e^6) * x^2 + d^2 * e^4 + 2 * (c^4 * d^5 * e - 2 * c^2 * d^3 * e^3 + d * e^5) * x)) * c - 2 * \operatorname{arctanh}(c * x) / (x^3 * e^4 + 3 * d * x^2 * e^3 + 3 * d^2 * x * e^2 + d^3 * e)) * b - 1/3 * a / (x^3 * e^4 + 3 * d * x^2 * e^3 + 3 * d^2 * x * e^2 + d^3 * e)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4030 vs. 2(161) = 322.

time = 0.63, size = 4030, normalized size = 23.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))/(e*x+d)^4,x, algorithm="fricas")

[Out] $-1/6 * (2 * a * c^6 * d^6 - 5 * b * c^5 * d^5 * \cosh(1) - (b * c * x + 2 * a) * \cosh(1)^6 - (b * c * x + 2 * a) * \sinh(1)^6 + (4 * b * c^3 * d * x^2 - b * c * d) * \cosh(1)^5 + (4 * b * c^3 * d * x^2 - b * c * d - 6 * (b * c * x + 2 * a) * \cosh(1)) * \sinh(1)^5 + 2 * (5 * b * c^3 * d^2 * x + 3 * a * c^2 * d^2) * \cosh(1)^4 + (10 * b * c^3 * d^2 * x + 6 * a * c^2 * d^2 - 15 * (b * c * x + 2 * a) * \cosh(1)^2 + 5 * (4 * b * c^3 * d * x^2 - b * c * d) * \cosh(1)) * \sinh(1)^4 - 2 * (2 * b * c^5 * d^3 * x^2 - 3 * b * c^3 * d^3) * \cosh(1)^3 - 2 * (2 * b * c^5 * d^3 * x^2 - 3 * b * c^3 * d^3 + 10 * (b * c * x + 2 * a) * \cosh(1)^3 - 5 * (4 * b * c^3 * d * x^2 - b * c * d) * \cosh(1)^2 - 4 * (5 * b * c^3 * d^2 * x + 3 * a * c^2 * d^2) * \cosh(1)) * \sinh(1)^3 - 3 * (3 * b * c^5 * d^4 * x + 2 * a * c^4 * d^4) * \cosh(1)^2 - (9 * b * c^5 * d^4 * x + 6 * a * c^4 * d^4 + 15 * (b * c * x + 2 * a) * \cosh(1)^4 - 10 * (4 * b * c^3 * d * x^2 - b * c * d) * \cosh(1)^3 - 12 * (5 * b * c^3 * d^2 * x + 3 * a * c^2 * d^2) * \cosh(1)^2 + 6 * (2 * b * c^5 * d^3 * x^2 - 3 * b * c^3 * d^3) * \cosh(1)) * \sinh(1)^2 - (b * c^6 * d^6 + b * c^3 * x^3 * \cosh(1)^6 + b * c^3 * x^3 * \sinh(1)^6 + 3 * (b * c^4 * d * x^3 + b * c^3 * d * x^2) * \cosh(1)^5 + 3 * (b * c^4 * d * x^3 + 2 * b * c^3 * x^3 * \cosh(1) + b * c^3 * d * x^2) * \sinh(1)^5 + 3 * (b * c^5 * d^2 * x^3 + 3 * b * c^4 * d^2 * x^2 + b * c^3 * d^2 * x) * \cosh(1)^4 + 3 * (b * c^5 * d^2 * x^3 + 3 * b * c^4 * d^2 * x^2 + 5 * b * c^3 * x^3 * \cosh(1)^2 + b * c^3 * d^2 * x + 5 * (b * c^4 * d * x^3 + b * c^3 * d * x^2) * \cosh(1)) * \sinh(1)^4 + (b * c^6 * d^3 * x^3 + 9 * b * c^5 * d^3 * x^2 + 9 * b * c^4 * d^3 * x + b * c^3 * d^3) * \cosh(1)^3 + (b * c^6 * d^3 * x^3 + 9 * b * c^5 * d^3 * x^2 + 20 * b * c^3 * x^3 * \cosh(1)^3 + 9 *$

$$\begin{aligned}
& b*c^4*d^3*x + b*c^3*d^3 + 30*(b*c^4*d*x^3 + b*c^3*d*x^2)*\cosh(1)^2 + 12*(b*c^5*d^2*x^3 + 3*b*c^4*d^2*x^2 + b*c^3*d^2*x)*\cosh(1)*\sinh(1)^3 + 3*(b*c^6*d^4*x^2 + 3*b*c^5*d^4*x + 5*b*c^3*x^3*\cosh(1)^4 + b*c^4*d^4 + 10*(b*c^4*d*x^3 + b*c^3*d*x^2)*\cosh(1)^3 + 6*(b*c^5*d^2*x^3 + 3*b*c^4*d^2*x^2 + b*c^3*d^2*x)*\cosh(1)^2 + (b*c^6*d^3*x^3 + 9*b*c^5*d^3*x^2 + 9*b*c^4*d^3*x + b*c^3*d^3)*\cosh(1)*\sinh(1)^2 + 3*(b*c^6*d^5*x + b*c^5*d^5)*\cosh(1) + 3*(b*c^6*d^5*x + 2*b*c^3*x^3*\cosh(1)^5 + b*c^5*d^5 + 5*(b*c^4*d*x^3 + b*c^3*d*x^2)*\cosh(1)^4 + 4*(b*c^5*d^2*x^3 + 3*b*c^4*d^2*x^2 + b*c^3*d^2*x)*\cosh(1)^3 + (b*c^6*d^3*x^3 + 9*b*c^5*d^3*x^2 + 9*b*c^4*d^3*x + b*c^3*d^3)*\cosh(1)^2 + 2*(b*c^6*d^4*x^2 + 3*b*c^5*d^4*x + b*c^4*d^4)*\cosh(1)*\sinh(1))*\log(c*x + 1) + (b*c^6*d^6 - b*c^3*x^3*\cosh(1)^6 - b*c^3*x^3*\sinh(1)^6 + 3*(b*c^4*d*x^3 - b*c^3*d*x^2)*\cosh(1)^5 + 3*(b*c^4*d*x^3 - 2*b*c^3*x^3*\cosh(1) - b*c^3*d*x^2)*\sinh(1)^5 - 3*(b*c^5*d^2*x^3 - 3*b*c^4*d^2*x^2 + b*c^3*d^2*x)*\cosh(1)^4 - 3*(b*c^5*d^2*x^3 - 3*b*c^4*d^2*x^2 + 5*b*c^3*x^3*\cosh(1)^2 + b*c^3*d^2*x - 5*(b*c^4*d*x^3 - b*c^3*d*x^2)*\cosh(1))*\sinh(1)^4 + (b*c^6*d^3*x^3 - 9*b*c^5*d^3*x^2 + 9*b*c^4*d^3*x - b*c^3*d^3)*\cosh(1)^3 + (b*c^6*d^3*x^3 - 9*b*c^5*d^3*x^2 - 20*b*c^3*x^3*\cosh(1)^3 + 9*b*c^4*d^3*x - b*c^3*d^3 + 30*(b*c^4*d*x^3 - b*c^3*d*x^2)*\cosh(1)^2 - 12*(b*c^5*d^2*x^3 - 3*b*c^4*d^2*x^2 + b*c^3*d^2*x)*\cosh(1))*\sinh(1)^3 + 3*(b*c^6*d^4*x^2 - 3*b*c^5*d^4*x + b*c^4*d^4)*\cosh(1)^2 + 3*(b*c^6*d^4*x^2 - 3*b*c^5*d^4*x - 5*b*c^3*x^3*\cosh(1)^4 + b*c^4*d^4 + 10*(b*c^4*d*x^3 - b*c^3*d*x^2)*\cosh(1)^3 - 6*(b*c^5*d^2*x^3 - 3*b*c^4*d^2*x^2 + b*c^3*d^2*x)*\cosh(1)^2 + (b*c^6*d^3*x^3 - 9*b*c^5*d^3*x^2 + 9*b*c^4*d^3*x - b*c^3*d^3)*\cosh(1))*\sinh(1)^2 + 3*(b*c^6*d^5*x - b*c^5*d^5)*\cosh(1) + 3*(b*c^6*d^5*x - 2*b*c^3*x^3*\cosh(1)^5 - b*c^5*d^5 + 5*(b*c^4*d*x^3 - b*c^3*d*x^2)*\cosh(1)^4 - 4*(b*c^5*d^2*x^3 - 3*b*c^4*d^2*x^2 + b*c^3*d^2*x)*\cosh(1)^3 + (b*c^6*d^3*x^3 - 9*b*c^5*d^3*x^2 + 9*b*c^4*d^3*x - b*c^3*d^3)*\cosh(1)^2 + 2*(b*c^6*d^4*x^2 - 3*b*c^5*d^4*x + b*c^4*d^4)*\cosh(1))*\sinh(1))*\log(c*x - 1) + 2*(9*b*c^5*d^4*x*\cosh(1)^2 + b*c^3*x^3*\cosh(1)^6 + b*c^3*x^3*\sinh(1)^6 + 3*b*c^5*d^5*\cosh(1) + 3*b*c^3*d*x^2*\cosh(1)^5 + 3*(2*b*c^3*x^3*\cosh(1) + b*c^3*d*x^2)*\sinh(1)^5 + 3*(b*c^5*d^2*x^3 + b*c^3*d^2*x)*\cosh(1)^4 + 3*(b*c^5*d^2*x^3 + 5*b*c^3*x^3*\cosh(1)^2 + 5*b*c^3*d*x^2*\cosh(1) + b*c^3*d^2*x)*\sinh(1)^4 + (9*b*c^5*d^3*x^2 + b*c^3*d^3)*\cosh(1)^3 + (9*b*c^5*d^3*x^2 + 20*b*c^3*x^3*\cosh(1)^3 + 30*b*c^3*d*x^2*\cosh(1)^2 + b*c^3*d^3 + 12*(b*c^5*d^2*x^3 + b*c^3*d^2*x)*\cosh(1))*\sinh(1)^3 + 3*(3*b*c^5*d^4*x + 5*b*c^3*x^3*\cosh(1)^4 + 10*b*c^3*d*x^2*\cosh(1)^3 + 6*(b*c^5*d^2*x^3 + b*c^3*d^2*x)*\cosh(1)^2 + (9*b*c^5*d^3*x^2 + b*c^3*d^3)*\cosh(1))*\sinh(1)^2 + 3*(6*b*c^5*d^4*x*\cosh(1) + 2*b*c^3*x^3*\cosh(1)^5 + b*c^5*d^5 + 5*b*c^3*d*x^2*\cosh(1)^4 + 4*(b*c^5*d^2*x^3 + b*c^3*d^2*x)*\cosh(1)^3 + (9*b*c^5*d^3*x^2 + b*c^3*d^3)*\cosh(1)^2)*\sinh(1))*\log(x*\cosh(1) + x*\sinh(1) + d) + (b*c^6*d^6 - 3*b*c^4*d^4*\cosh(1)^2 + 3*b*c^2*d^2*\cosh(1)^4 - b*cosh(1)^6 - 6*b*cosh(1)*\sinh(1)^5 - b*\sinh(1)^6 + 3*(b*c^2*d^2 - 5*b*cosh(1)^2)*\sinh(1)^4 + 4*(3*b*c^2*d^2*\cosh(1) - 5*b*cosh(1)^3)*\sinh(1)^3 - 3*(b*c^4*d^4 - 6*b*c^2*d^2*\cosh(1)^2 + 5*b*cosh(1)^4)*\sinh(1)^2 - 6*(b*c^4*d^4*\cosh(1) - 2*b*c^2*d^2*\cosh(1)^3 + b*cosh(1)^5)*\sinh(1))*\log(-(c*x + 1)/(c*x - 1)) - (5*b*c^5*d^5 + 6*(b*c*x + 2*a)*\cosh(1)^
\end{aligned}$$

$$5 - 5*(4*b*c^3*d*x^2 - b*c*d)*\cosh(1)^4 - 8*(5*b*c^3*d^2*x + 3*a*c^2*d^2)*\cosh(1)^3 + 6*(2*b*c^5*d^3*x^2 - 3*b*c^3*d^3)*\cosh(1)^2 + 6*(3*b*c^5*d^4*x + 2*a*c^4*d^4)*\cosh(1)*\sinh(1))/(3*c^6*d^8*x*\cosh(1)^2 + c^6*d^9*\cosh(1) - x^3*\cosh(1)^{10} - x^3*\sinh(1)^{10} - 3*d*x^2*\cosh(1)^9 - (10*x^3*\cosh(1) + 3*d*x^2)*\sinh(1)^9 + 3*(c^2*d^2*x^3 - d^2*x)*\cosh(\dots)$$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 10946 vs. $2(151) = 302$.

time = 5.32, size = 10946, normalized size = 62.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))/(e*x+d)**4,x)

[Out] Piecewise((a*x/d**4, Eq(c, 0) & Eq(e, 0)), (-a/(3*d**3*e + 9*d**2*e**2*x + 9*d*e**3*x**2 + 3*e**4*x**3), Eq(c, 0)), ((a*x + b*x*atanh(c*x) + b*log(x - 1/c)/c + b*atanh(c*x)/c)/d**4, Eq(e, 0)), (-24*a*d**3/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 21*b*d**3*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 10*b*d**3/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 9*b*d**2*e*x*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 9*b*d**2*e*x/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 9*b*d*e**2*x**2*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 3*b*d*e**2*x**2/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 3*b*e**3*x**3*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3), Eq(c, -e/d)), (-24*a*d**3/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 21*b*d**3*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 10*b*d**3/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 9*b*d**2*e*x*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 9*b*d**2*e*x/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 9*b*d*e**2*x**2*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 3*b*d*e**2*x**2/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 3*b*e**3*x**3*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3), Eq(c, e/d)), (-2*a*c**6*d**6/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 6*a*c**4*d**4*e**2/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 1

[In] integrate((a+b*arctanh(c*x))/(e*x+d)^4,x, algorithm="giac")

[Out]
$$-1/3*c*((3*b*c^4*d^2 + b*c^2*e^2)*\log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c^6*d^6 - 3*c^4*d^4*e^2 + 3*c^2*d^2*e^4 - e^6) - (3*(c*x + 1)^2*b*c^4*d^2/(c*x - 1)^2 - 6*(c*x + 1)*b*c^4*d^2/(c*x - 1) + 3*b*c^4*d^2 + 6*(c*x + 1)^2*b*c^3*d*e/(c*x - 1)^2 - 6*(c*x + 1)*b*c^3*d*e/(c*x - 1) + 3*(c*x + 1)^2*b*c^2*e^2/(c*x - 1)^2 + b*c^2*e^2)*\log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^6*d^6/(c*x - 1)^3 - 3*(c*x + 1)^2*c^6*d^6/(c*x - 1)^2 + 3*(c*x + 1)*c^6*d^6/(c*x - 1) - c^6*d^6 + 6*(c*x + 1)^3*c^5*d^5*e/(c*x - 1)^3 - 12*(c*x + 1)^2*c^5*d^5*e/(c*x - 1)^2 + 6*(c*x + 1)*c^5*d^5*e/(c*x - 1) + 15*(c*x + 1)^3*c^4*d^4*e^2/(c*x - 1)^3 - 15*(c*x + 1)^2*c^4*d^4*e^2/(c*x - 1)^2 - 3*(c*x + 1)*c^4*d^4*e^2/(c*x - 1) + 3*c^4*d^4*e^2 + 20*(c*x + 1)^3*c^3*d^3*e^3/(c*x - 1)^3 - 12*(c*x + 1)*c^3*d^3*e^3/(c*x - 1) + 15*(c*x + 1)^3*c^2*d^2*e^4/(c*x - 1)^3 + 15*(c*x + 1)^2*c^2*d^2*e^4/(c*x - 1)^2 - 3*(c*x + 1)*c^2*d^2*e^4/(c*x - 1) - 3*c^2*d^2*e^4 + 6*(c*x + 1)^3*c*d*e^5/(c*x - 1)^3 + 12*(c*x + 1)^2*c*d*e^5/(c*x - 1)^2 + 6*(c*x + 1)*c*d*e^5/(c*x - 1) + (c*x + 1)^3*e^6/(c*x - 1)^3 + 3*(c*x + 1)^2*e^6/(c*x - 1)^2 + 3*(c*x + 1)*e^6/(c*x - 1) + e^6) - (3*b*c^4*d^2 + b*c^2*e^2)*\log(-(c*x + 1)/(c*x - 1))/(c^6*d^6 - 3*c^4*d^4*e^2 + 3*c^2*d^2*e^4 - e^6) - 2*(3*(c*x + 1)^2*a*c^6*d^4/(c*x - 1)^2 - 6*(c*x + 1)*a*c^6*d^4/(c*x - 1) + 3*a*c^6*d^4 + 6*(c*x + 1)*a*c^5*d^3*e/(c*x - 1) - 6*a*c^5*d^3*e - 3*(c*x + 1)^2*b*c^5*d^3*e/(c*x - 1)^2 + 6*(c*x + 1)*b*c^5*d^3*e/(c*x - 1) - 3*b*c^5*d^3*e - 6*(c*x + 1)^2*a*c^4*d^2*e^2/(c*x - 1)^2 + 6*(c*x + 1)*a*c^4*d^2*e^2/(c*x - 1) + 4*a*c^4*d^2*e^2 - 5*(c*x + 1)^2*b*c^4*d^2*e^2/(c*x - 1)^2 - (c*x + 1)*b*c^4*d^2*e^2/(c*x - 1) + 6*b*c^4*d^2*e^2 - 6*(c*x + 1)*a*c^3*d*e^3/(c*x - 1) - 2*a*c^3*d*e^3 - (c*x + 1)^2*b*c^3*d*e^3/(c*x - 1)^2 - 6*(c*x + 1)*b*c^3*d*e^3/(c*x - 1) - 3*b*c^3*d*e^3 + 3*(c*x + 1)^2*a*c^2*e^4/(c*x - 1)^2 + a*c^2*e^4 + (c*x + 1)^2*b*c^2*e^4/(c*x - 1)^2 + (c*x + 1)*b*c^2*e^4/(c*x - 1))/((c*x + 1)^3*c^8*d^8/(c*x - 1)^3 - 3*(c*x + 1)^2*c^8*d^8/(c*x - 1)^2 + 3*(c*x + 1)*c^8*d^8/(c*x - 1) - c^8*d^8 + 4*(c*x + 1)^3*c^7*d^7*e/(c*x - 1)^3 - 6*(c*x + 1)^2*c^7*d^7*e/(c*x - 1)^2 + 2*c^7*d^7*e + 4*(c*x + 1)^3*c^6*d^6*e^2/(c*x - 1)^3 + 6*(c*x + 1)^2*c^6*d^6*e^2/(c*x - 1)^2 - 12*(c*x + 1)*c^6*d^6*e^2/(c*x - 1) + 2*c^6*d^6*e^2 - 4*(c*x + 1)^3*c^5*d^5*e^3/(c*x - 1)^3 + 18*(c*x + 1)^2*c^5*d^5*e^3/(c*x - 1)^2 - 6*c^5*d^5*e^3 - 10*(c*x + 1)^3*c^4*d^4*e^4/(c*x - 1)^3 + 18*(c*x + 1)*c^4*d^4*e^4/(c*x - 1) - 4*(c*x + 1)^3*c^3*d^3*e^5/(c*x - 1)^3 - 18*(c*x + 1)^2*c^3*d^3*e^5/(c*x - 1)^2 + 6*c^3*d^3*e^5 + 4*(c*x + 1)^3*c^2*d^2*e^6/(c*x - 1)^3 - 6*(c*x + 1)^2*c^2*d^2*e^6/(c*x - 1)^2 - 12*(c*x + 1)*c^2*d^2*e^6/(c*x - 1) - 2*c^2*d^2*e^6 + 4*(c*x + 1)^3*c*d*e^7/(c*x - 1)^3 + 6*(c*x + 1)^2*c*d*e^7/(c*x - 1)^2 - 2*c*d*e^7 + (c*x + 1)^3*e^8/(c*x - 1)^3 + 3*(c*x + 1)^2*e^8/(c*x - 1)^2 + 3*(c*x + 1)*e^8/(c*x - 1) + e^8))$$

Mupad [B]

time = 2.29, size = 418, normalized size = 2.39

$$\ln(d+ex) \left(\frac{b^2}{6e(e+cd)^2} + \frac{b^2}{6e(e-cd)^2} \right) - \frac{2ac^2d^2-3b^2d^2e-3a^2d^2e^2+3b^2d^2e^2+2ae^2}{3d^2e+9d^2e^2+9d^2e^2+3e^4x^2} + \frac{e(3c^4-9b^2d^2e)}{3d^2e+9d^2e^2+9d^2e^2+3e^4x^2} - \frac{2b^2d^2e^2}{3d^2e+9d^2e^2+9d^2e^2+3e^4x^2} - \frac{b^2 \ln(cx-1)}{6c^2d^2e+18c^2d^2e^2+18cd^2e^3+6e^4} - \frac{b^2 \ln(cx+1)}{-6c^2d^2e+18c^2d^2e^2-18cd^2e^3+6e^4} - \frac{b \ln(cx+1)}{6e(d^2+3d^2ex+3d^2e^2+e^2x^2)} + \frac{b \ln(1-cx)}{3e(2d^2+6d^2ex+6d^2e^2+2e^2x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b*\text{atanh}(c*x))/(d + e*x)^4, x)$

[Out] $\log(d + e*x)*((b*c^3)/(6*e*(e + c*d)^3) + (b*c^3)/(6*e*(e - c*d)^3)) - ((2*a*e^4 + 2*a*c^4*d^4 - 4*a*c^2*d^2*e^2 + b*c*d*e^3 - 5*b*c^3*d^3*e)/(2*(e^4 + c^4*d^4 - 2*c^2*d^2*e^2)) + (x*(b*c*e^4 - 9*b*c^3*d^2*e^2))/(2*(e^4 + c^4*d^4 - 2*c^2*d^2*e^2)) - (2*b*c^3*d*e^3*x^2)/(e^4 + c^4*d^4 - 2*c^2*d^2*e^2))/(3*d^3*e + 3*e^4*x^3 + 9*d^2*e^2*x + 9*d*e^3*x^2) - (b*c^3*\log(c*x - 1))/(6*e^4 + 6*c^3*d^3*e + 18*c^2*d^2*e^2 + 18*c*d*e^3) - (b*c^3*\log(c*x + 1))/(6*e^4 - 6*c^3*d^3*e + 18*c^2*d^2*e^2 - 18*c*d*e^3) - (b*\log(c*x + 1))/(6*e*(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x)) + (b*\log(1 - c*x))/(3*e*(2*d^3 + 2*e^3*x^3 + 6*d*e^2*x^2 + 6*d^2*e*x))$

3.9 $\int (d + ex)^3 (a + b \tanh^{-1}(cx))^2 dx$

Optimal. Leaf size=359

$$\frac{b^2 d e^2 x}{c^2} + \frac{a b e (6 c^2 d^2 + e^2) x}{2 c^3} + \frac{b^2 e^3 x^2}{12 c^2} - \frac{b^2 d e^2 \tanh^{-1}(c x)}{c^3} + \frac{b^2 e (6 c^2 d^2 + e^2) x \tanh^{-1}(c x)}{2 c^3} + \frac{b d e^2 x^2 (a + b \tanh^{-1}(c x))^2}{c}$$

[Out] $b^2 d e^2 x / c^2 + 1/2 a b e (6 c^2 d^2 + e^2) x / c^3 + 1/12 b^2 e^3 x^2 / c^2 - b^2 d e^2 \operatorname{arctanh}(c x) / c^3 + 1/2 b^2 e (6 c^2 d^2 + e^2) x \operatorname{arctanh}(c x) / c^3 + b d e^2 x^2 (a + b \operatorname{arctanh}(c x)) / c + 1/6 b^2 e^3 x^3 (a + b \operatorname{arctanh}(c x)) / c + d (c^2 d^2 + e^2) (a + b \operatorname{arctanh}(c x))^2 / c^3 - 1/4 (c^4 d^4 + 6 c^2 d^2 e^2 + e^4) (a + b \operatorname{arctanh}(c x))^2 / c^4 + e + 1/4 (e x + d)^4 (a + b \operatorname{arctanh}(c x))^2 / e - 2 b d (c^2 d^2 + e^2) (a + b \operatorname{arctanh}(c x)) \ln(2 / (-c x + 1)) / c^3 + 1/12 b^2 e^3 \ln(-c^2 x^2 + 1) / c^4 + 1/4 b^2 e (6 c^2 d^2 + e^2) \ln(-c^2 x^2 + 1) / c^4 - b^2 d (c^2 d^2 + e^2) \operatorname{polylog}(2, 1 - 2 / (-c x + 1)) / c^3$

Rubi [A]

time = 0.38, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {6065, 6021, 266, 6037, 327, 212, 272, 45, 6195, 6095, 6131, 6055, 2449, 2352}

$(c^2 d^2 + e^2) (a + b \operatorname{arctanh}(c x))^2$, $\frac{a b e (6 c^2 d^2 + e^2) x}{2 c^3}$, $\frac{b^2 e^3 x^2}{12 c^2}$, $\frac{b^2 d e^2 \tanh^{-1}(c x)}{c^3}$, $\frac{b^2 e (6 c^2 d^2 + e^2) x \tanh^{-1}(c x)}{2 c^3}$, $\frac{b d e^2 x^2 (a + b \operatorname{arctanh}(c x))^2}{c}$, $\frac{d (c^2 d^2 + e^2) (a + b \operatorname{arctanh}(c x))^2}{c^3}$, $\frac{1}{4} (c^4 d^4 + 6 c^2 d^2 e^2 + e^4) (a + b \operatorname{arctanh}(c x))^2 / c^4$, $\frac{1}{4} (e x + d)^4 (a + b \operatorname{arctanh}(c x))^2 / e$, $\frac{1}{12} b^2 e^3 \ln(-c^2 x^2 + 1) / c^4$, $\frac{1}{4} b^2 e (6 c^2 d^2 + e^2) \ln(-c^2 x^2 + 1) / c^4$, $b^2 d (c^2 d^2 + e^2) \operatorname{polylog}(2, 1 - 2 / (-c x + 1)) / c^3$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(d + e x)^3 (a + b \operatorname{ArcTanh}[c x])^2, x]$

[Out] $(b^2 d e^2 x) / c^2 + (a b e (6 c^2 d^2 + e^2) x) / (2 c^3) + (b^2 e^3 x^2) / (12 c^2) - (b^2 d e^2 \operatorname{ArcTanh}[c x]) / c^3 + (b^2 e (6 c^2 d^2 + e^2) x \operatorname{ArcTanh}[c x]) / (2 c^3) + (b d e^2 x^2 (a + b \operatorname{ArcTanh}[c x])) / c + (b e^3 x^3 (a + b \operatorname{ArcTanh}[c x])) / (6 c) + (d (c^2 d^2 + e^2) (a + b \operatorname{ArcTanh}[c x])^2) / c^3 - ((c^4 d^4 + 6 c^2 d^2 e^2 + e^4) (a + b \operatorname{ArcTanh}[c x])^2) / (4 c^4 e) + ((d + e x)^4 (a + b \operatorname{ArcTanh}[c x])^2) / (4 e) - (2 b d (c^2 d^2 + e^2) (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}[2 / (1 - c x)]) / c^3 + (b^2 e^3 \operatorname{Log}[1 - c^2 x^2]) / (12 c^4) + (b^2 e (6 c^2 d^2 + e^2) \operatorname{Log}[1 - c^2 x^2]) / (4 c^4) - (b^2 d (c^2 d^2 + e^2) \operatorname{PolyLog}[2, 1 - 2 / (1 - c x)]) / c^3$

Rule 45

$\operatorname{Int}[(a + b x)^m (c + d x)^n, x] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b x)^m (c + d x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 212

$\operatorname{Int}[(a + b x)^2 (-1), x] \rightarrow \operatorname{Simp}[(1 / (\operatorname{Rt}[a, 2] \operatorname{Rt}[-b, 2])) \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2] (x / \operatorname{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

Rule 266

$\text{Int}[(x_)^{(m_.)}/((a_) + (b_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 272

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1}*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 327

$\text{Int}[(c_.)*(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c^{(n - 1)}*(c*x)^{(m - n + 1)}*((a + b*x^n)^{(p + 1)}/(b*(m + n*p + 1))), x] - \text{Dist}[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2352

$\text{Int}[\text{Log}[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] \rightarrow \text{Simp}[(-e^{(-1)})*\text{PolyLog}[2, 1 - c*x], x] /; \text{FreeQ}[\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c*d, 0]$

Rule 2449

$\text{Int}[\text{Log}[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[-e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}[\{c, d, e, f, g\}, x] \ \&\& \ \text{EqQ}[c, 2*d] \ \&\& \ \text{EqQ}[e^2*f + d^2*g, 0]$

Rule 6021

$\text{Int}[(a_.) + \text{ArcTanh}[(c_.)*(x_)^{(n_.)}]* (b_.)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcTanh}[c*x^n])^p, x] - \text{Dist}[b*c*n*p, \text{Int}[x^n*((a + b*\text{ArcTanh}[c*x^n])^{(p - 1)})/(1 - c^2*x^{(2*n)}), x], x] /; \text{FreeQ}[\{a, b, c, n\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{EqQ}[n, 1] \parallel \text{EqQ}[p, 1])$

Rule 6037

$\text{Int}[(a_.) + \text{ArcTanh}[(c_.)*(x_)^{(n_.)}]* (b_.)^{(p_.)}*(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^{(m + 1)}*((a + b*\text{ArcTanh}[c*x^n])^p/(m + 1)), x] - \text{Dist}[b*c*n*(p/(m + 1)), \text{Int}[x^{(m + n)}*((a + b*\text{ArcTanh}[c*x^n])^{(p - 1)})/(1 - c^2*x^{(2*n)}), x], x] /; \text{FreeQ}[\{a, b, c, m, n\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{EqQ}[p, 1] \parallel (\text{EqQ}[n, 1] \ \&\& \ \text{IntegerQ}[m])) \ \&\& \ \text{NeQ}[m, -1]$

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_.)), x_Symbol]
:= Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_.))^(q_.), x_S
ymbol] := Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_) + (g_.)*(x_)^(m_.))/
(d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^3 (a+b \tanh^{-1}(cx))^2 dx &= \frac{(d+ex)^4 (a+b \tanh^{-1}(cx))^2}{4e} - \frac{(bc) \int \left(-\frac{e^2(6c^2d^2+e^2)(a+b \tanh^{-1}(cx))}{c^4} \right)}{1-c^2x^2} \\
&= \frac{(d+ex)^4 (a+b \tanh^{-1}(cx))^2}{4e} - \frac{b \int \frac{(c^4d^4+6c^2d^2e^2+e^4+4c^2de(c^2d^2+e^2)x)(a+b \tanh^{-1}(cx))}{1-c^2x^2}}{2c^3e} \\
&= \frac{abe(6c^2d^2+e^2)x}{2c^3} + \frac{bde^2x^2(a+b \tanh^{-1}(cx))}{c} + \frac{be^3x^3(a+b \tanh^{-1}(cx))}{6c} \\
&= \frac{b^2de^2x}{c^2} + \frac{abe(6c^2d^2+e^2)x}{2c^3} + \frac{b^2e(6c^2d^2+e^2)x \tanh^{-1}(cx)}{2c^3} + \frac{bde^2x^2}{6c} \\
&= \frac{b^2de^2x}{c^2} + \frac{abe(6c^2d^2+e^2)x}{2c^3} - \frac{b^2de^2 \tanh^{-1}(cx)}{c^3} + \frac{b^2e(6c^2d^2+e^2)x \tanh^{-1}(cx)}{2c^3} \\
&= \frac{b^2de^2x}{c^2} + \frac{abe(6c^2d^2+e^2)x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2 \tanh^{-1}(cx)}{c^3} + \frac{b^2e(6c^2d^2+e^2)x \tanh^{-1}(cx)}{2c^3} \\
&= \frac{b^2de^2x}{c^2} + \frac{abe(6c^2d^2+e^2)x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2 \tanh^{-1}(cx)}{c^3} + \frac{b^2e(6c^2d^2+e^2)x \tanh^{-1}(cx)}{2c^3} \\
&= \frac{b^2de^2x}{c^2} + \frac{abe(6c^2d^2+e^2)x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2 \tanh^{-1}(cx)}{c^3} + \frac{b^2e(6c^2d^2+e^2)x \tanh^{-1}(cx)}{2c^3}
\end{aligned}$$

Mathematica [A]

time = 0.64, size = 506, normalized size = 1.41

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3*(a + b*ArcTanh[c*x])^2,x]

[Out] $(-(b^2e^3) + 12a^2c^4d^3x + 36a^2bc^3d^2ex + 12b^2c^2d^2e^2x + 6a^2bc^3e^3x + 18a^2c^4d^2e^2x^2 + 12a^2bc^3d^2e^2x^2 + b^2c^2e^3x^2 + 12a^2c^4d^2e^2x^3 + 2a^2bc^3e^3x^3 + 3a^2c^4e^3x^4 + 3b^2(-4c^3d^3 - 6c^2d^2e - 4cd^2e^2 - e^3 + c^4x(4d^3 + 6d^2ex + 4d^2e^2x^2 + e^3x^3)) \operatorname{ArcTanh}[cx]^2 + 2b^2c \operatorname{ArcTanh}[cx](3a^2c^3x(4d^3 + 6d^2ex + 4d^2e^2x^2 + e^3x^3) + b^2e(18c^2d^2x + 6d^2e(-1 + c^2x^2) + e^2x(3 + c^2x^2)) - 12b^2d(c^2d^2 + e^2) \operatorname{Log}[1 + E^{(-2 \operatorname{ArcTanh}[cx])}]) + 18a^2bc^2d^2e \operatorname{Log}[1 - cx] + 3a^2b^2e^3 \operatorname{Log}[1 - cx] - 18a^2bc^2d^2e \operatorname{Log}[1 + cx] - 3a^2b^2e^3 \operatorname{Log}[1 + cx] + 12a^2bc^3d^3 \operatorname{Log}[1 - c^2x^2] + 18b^2c^2d^2e \operatorname{Log}[1 - c^2x^2] + 4b^2e^3 \operatorname{Log}[1 - c^2x^2] + 12a^2bc^2d^2e \operatorname{Log}[-1 + c^2x^2] + 12b^2c^2d^2e(c^2d^2 + e^2) \operatorname{PolyLog}[2, -E^{(-2 \operatorname{ArcTanh}[cx])}]))/(12c^4)$

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1372 vs. $2(343) = 686$.

time = 0.38, size = 1373, normalized size = 3.82

method	result	size
derivativedivides	Expression too large to display	1373
default	Expression too large to display	1373
risch	Expression too large to display	1560

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{c}(-b^2 d \operatorname{dilog}\left(\frac{1}{2}c x + \frac{1}{2}\right) d^3 + \frac{1}{4}b^2 \ln(c x - 1)^2 d^3 + \frac{1}{2}a b c / e \operatorname{arctanh}(c x) d^4 + \frac{1}{2}a b c e^3 \operatorname{arctanh}(c x) x^4 + 3 a b e d^2 x + a b e^2 d x^2 + \frac{3}{2}b^2 c e \operatorname{arctanh}(c x)^2 d^2 x^2 + b^2 c e^2 \operatorname{arctanh}(c x)^2 d x^3 + 3 b^2 e \operatorname{arctanh}(c x) d^2 x + b^2 e^2 \operatorname{arctanh}(c x) d x^2 + 2 a b c e^2 \operatorname{arctanh}(c x) d x^3 + 3 a b c e \operatorname{arctanh}(c x) d^2 x^2 - \frac{1}{8}b^2 c / e \ln(c x + 1) \ln(-\frac{1}{2}c x + \frac{1}{2}) d^4 + \frac{1}{2}a b / c^2 e^3 x^2 + 2 a b \operatorname{arctanh}(c x) d^3 c x + b^2 / c e^2 d x + \frac{1}{4}b^2 c e^3 \operatorname{arctanh}(c x)^2 x^4 + \frac{1}{6}b^2 e^3 \operatorname{arctanh}(c x) x^3 + \frac{1}{2}b^2 / c^2 e^3 \operatorname{arctanh}(c x) x + \frac{1}{6}a b e^3 x^3 - \frac{1}{4}b^2 c / e \operatorname{arctanh}(c x) \ln(c x + 1) d^4 - \frac{3}{4}b^2 / c e \ln(c x - 1) \ln(\frac{1}{2}c x + \frac{1}{2}) d^2 - \frac{1}{2}b^2 / c^2 e^2 \ln(c x - 1) \ln(\frac{1}{2}c x + \frac{1}{2}) d - \frac{1}{8}b^2 c / e \ln(c x - 1) \ln(\frac{1}{2}c x + \frac{1}{2}) d^4 + \frac{3}{2}b^2 / c e \operatorname{arctanh}(c x) \ln(c x - 1) d^2 + b^2 / c^2 e^2 \operatorname{arctanh}(c x) \ln(c x - 1) d + \frac{1}{4}a b c / e \ln(c x - 1) d^4 + \frac{3}{2}a b / c e \ln(c x - 1) d^2 + a b / c^2 e^2 \ln(c x - 1) d - \frac{1}{4}a b c / e \ln(c x + 1) d^4 - \frac{3}{2}a b / c e \ln(c x + 1) d^2 + a b / c^2 e^2 \ln(c x + 1) d + \frac{1}{8}b^2 c / e \ln(\frac{1}{2}c x + \frac{1}{2}) \ln(-\frac{1}{2}c x + \frac{1}{2}) d^4 + b^2 / c^2 e^2 \operatorname{arctanh}(c x) \ln(c x + 1) d - \frac{3}{2}b^2 / c e \operatorname{arctanh}(c x) \ln(c x + 1) d^2 - \frac{3}{4}b^2 / c e \ln(c x + 1) \ln(-\frac{1}{2}c x + \frac{1}{2}) d^2 + \frac{1}{2}b^2 / c^2 e^2 \ln(c x + 1) \ln(-\frac{1}{2}c x + \frac{1}{2}) * \ln(-\frac{1}{2}c x + \frac{1}{2}) d + \frac{3}{4}b^2 / c e \ln(\frac{1}{2}c x + \frac{1}{2}) \ln(-\frac{1}{2}c x + \frac{1}{2}) d^2 - \frac{1}{2}b^2 / c^2 e^2 \ln(\frac{1}{2}c x + \frac{1}{2}) \ln(-\frac{1}{2}c x + \frac{1}{2}) d + \frac{1}{4}b^2 c / e \operatorname{arctanh}(c x) \ln(c x - 1) d^4 - \frac{1}{4}b^2 \ln(c x + 1)^2 d^3 + \frac{1}{3}b^2 / c^3 e^3 \ln(c x - 1) + \frac{1}{3}b^2 / c^3 e^3 \ln(c x + 1) + \frac{1}{16}b^2 / c^3 e^3 \ln(c x + 1)^2 + \frac{1}{4}(c e x + c d)^4 a^2 / c^3 / e + a b \ln(c x - 1) d^3 + a b \ln(c x + 1) d^3 + \frac{1}{16}b^2 / c^3 e^3 \ln(c x - 1)^2 + b^2 \operatorname{arctanh}(c x) \ln(c x - 1) d^3 + b^2 \operatorname{arctanh}(c x) \ln(c x + 1) d^3 + \frac{1}{2}b^2 \ln(c x + 1) \ln(-\frac{1}{2}c x + \frac{1}{2}) d^3 - \frac{1}{2}b^2 \ln(\frac{1}{2}c x + \frac{1}{2}) \ln(-\frac{1}{2}c x + \frac{1}{2}) d^3 - \frac{1}{2}b^2 \ln(c x - 1) \ln(\frac{1}{2}c x + \frac{1}{2}) d^3 + \frac{1}{12}b^2 / c e^3 x^2 + b^2 \operatorname{arctanh}(c x)^2 d^3 c x + \frac{1}{16}b^2 c / e \ln(c x + 1)^2 d^4 + \frac{1}{4}a b / c^3 e^3 \ln(c x - 1) - \frac{1}{4}a b / c^3 e^3 \ln(c x + 1) + \frac{1}{4}b^2 c / e \operatorname{arctanh}(c x)^2 d^4 + \frac{1}{4}b^2 / c^3 e^3 \operatorname{arctanh}(c x) \ln(c x - 1) - \frac{1}{4}b^2 / c^3 e^3 \operatorname{arctanh}(c x) \ln(c x + 1) - \frac{1}{8}b^2 / c^3 e^3 \ln(c x - 1) \ln(\frac{1}{2}c x + \frac{1}{2}) - \frac{1}{8}b^2 / c^3 e^3 \ln(c x + 1) \ln(-\frac{1}{2}c x + \frac{1}{2}) + \frac{1}{8}b^2 / c^3 e^3 \ln(\frac{1}{2}c x + \frac{1}{2}) \ln(-\frac{1}{2}c x + \frac{1}{2}) + \frac{3}{8}b^2 / c e \ln(c x + 1)^2 d^2 - \frac{1}{4}b^2 / c^2 e^2 \ln(c x + 1)^2 d - b^2 / c^2 e^2 \operatorname{dilog}\left(\frac{1}{2}c x + \frac{1}{2}\right) d + \frac{3}{2}b^2 / c e \ln(c x - 1) d^2 + \frac{1}{2}b^2 / c^2 e^2 \ln(c x - 1) d + \frac{3}{2}b^2 / c e \ln(c x + 1) d^2 - \frac{1}{2}b^2 / c^2 e^2 \ln(c x + 1) d + \frac{1}{16}b^2 c / e \ln(c x - 1)^2 d^4 + \frac{1}{4}b^2 / c^2 e^2 \ln(c x - 1)^2 d + \frac{3}{8}b^2 / c e \ln(c x - 1)^2 d^2$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. 2(328) = 656.

time = 0.43, size = 808, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")

[Out] $\frac{1}{4}a^2x^4e^3 + a^2dx^3e^2 + \frac{3}{2}a^2d^2x^2e + a^2d^3x + \frac{3}{2}(2x^2\operatorname{arctanh}(cx) + c(2x/c^2 - \log(cx+1)/c^3 + \log(cx-1)/c^3))ab^2d^2e + (2cx\operatorname{arctanh}(cx) + \log(-c^2x^2+1))ab^2d^3/c + (2x^3\operatorname{arctanh}(cx) + c(x^2/c^2 + \log(c^2x^2-1)/c^4))ab^2de^2 + 1/12(6x^4\operatorname{arctanh}(cx) + c(2(c^2x^3+3x)/c^4 - 3\log(cx+1)/c^5 + 3\log(cx-1)/c^5))ab^2e^3 + (b^2c^2d^3 + b^2de^2)(\log(cx+1)\log(-1/2cx+1/2) + \operatorname{dilog}(1/2cx+1/2))/c^3 + 1/6(9b^2c^2d^2e - 3b^2cd^2e^2 + 2b^2e^3)\log(cx+1)/c^4 + 1/6(9b^2c^2d^2e + 3b^2cd^2e^2 + 2b^2e^3)\log(cx-1)/c^4 + 1/48(4b^2c^2x^2e^3 + 48b^2c^2dx^2e^2 + 3(b^2c^4x^4e^3 + 4b^2c^4dx^3e^2 + 6b^2c^4d^2x^2e + 4b^2c^4d^3x + 4b^2c^3d^3 - 6b^2c^2d^2e + 4b^2cd^2e^2 - b^2e^3)\log(cx+1)^2 + 3(b^2c^4x^4e^3 + 4b^2c^4dx^3e^2 + 6b^2c^4d^2x^2e + 4b^2c^4d^3x - 4b^2c^3d^3 - 6b^2c^2d^2e - 4b^2cd^2e^2 - b^2e^3)\log(-cx+1)^2 + 4(b^2c^3x^3e^3 + 6b^2c^3dx^2e^2 + 3(6b^2c^3d^2e + b^2ce^3)x)\log(cx+1) - 2(2b^2c^3x^3e^3 + 12b^2c^3dx^2e^2 + 6(6b^2c^3d^2e + b^2ce^3)x + 3(b^2c^4x^4e^3 + 4b^2c^4dx^3e^2 + 6b^2c^4d^2x^2e + 4b^2c^4d^3x + 4b^2c^3d^3 - 6b^2c^2d^2e + 4b^2cd^2e^2 - b^2e^3)\log(cx+1))\log(-cx+1))/c^4$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")

[Out] $\operatorname{integral}(a^2x^3e^3 + 3a^2dx^2e^2 + 3a^2d^2xe + a^2d^3 + (b^2x^3e^3 + 3b^2dx^2e^2 + 3b^2d^2xe + b^2d^3)\operatorname{arctanh}(cx))^2 + 2(a^2bx^3e^3 + 3a^2bdx^2e^2 + 3a^2bd^2xe + a^2bd^3)\operatorname{arctanh}(cx), x$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^2 (d + ex)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**3*(a+b*atanh(c*x))**2,x)

[Out] Integral((a + b*atanh(c*x))**2*(d + e*x)**3, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")

[Out] integrate((e*x + d)^3*(b*arctanh(c*x) + a)^2, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{atanh}(cx))^2 (d + ex)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^2*(d + e*x)^3,x)

[Out] int((a + b*atanh(c*x))^2*(d + e*x)^3, x)

3.10 $\int (d + ex)^2 (a + b \tanh^{-1}(cx))^2 dx$

Optimal. Leaf size=257

$$\frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2 \tanh^{-1}(cx)}{3c^3} + \frac{2b^2dex \tanh^{-1}(cx)}{c} + \frac{be^2x^2(a + b \tanh^{-1}(cx))}{3c} + \frac{(3c^2d^2 + e^2)(a + b \tanh^{-1}(cx))^2}{3c^3}$$

[Out] $2*a*b*d*e*x/c + 1/3*b^2*e^2*x/c^2 - 1/3*b^2*e^2*arctanh(c*x)/c^3 + 2*b^2*d*e*x*arctanh(c*x)/c + 1/3*b*e^2*x^2*(a+b*arctanh(c*x))/c + 1/3*(3*c^2*d^2+e^2)*(a+b*arctanh(c*x))^2/c^3 - 1/3*d*(d^2+3*e^2/c^2)*(a+b*arctanh(c*x))^2/e + 1/3*(e*x+d)^3*(a+b*arctanh(c*x))^2/e - 2/3*b*(3*c^2*d^2+e^2)*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^3 + b^2*d*e*ln(-c^2*x^2+1)/c^2 - 1/3*b^2*(3*c^2*d^2+e^2)*polylog(2, 1-2/(-c*x+1))/c^3$

Rubi [A]

time = 0.28, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6065, 6021, 266, 6037, 327, 212, 6195, 6095, 6131, 6055, 2449, 2352}

$$\frac{d \left(\frac{d^2}{3e} + d^2 \right) (a + b \tanh^{-1}(cx))^2}{3e} + \frac{(3c^2d^2 + e^2)(a + b \tanh^{-1}(cx))^2}{3c^3} - \frac{2b(3c^2d^2 + e^2) \log\left(\frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{3c^3} + \frac{2abdex}{c} + \frac{(d+cx)^3(a + b \tanh^{-1}(cx))^2}{3e} + \frac{be^2x^2(a + b \tanh^{-1}(cx))}{3c} - \frac{b^2e^2 \tanh^{-1}(cx)}{3c^3} + \frac{b^2de \log(1-c^2x^2)}{c^2} + \frac{b^2e^2x}{3c^2} - \frac{b^2(3c^2d^2 + e^2) \text{Li}_2\left(1 - \frac{2}{1-cx}\right)}{3c^3} + \frac{2b^2dex \tanh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2*(a + b*ArcTanh[c*x])^2,x]

[Out] $(2*a*b*d*e*x)/c + (b^2*e^2*x)/(3*c^2) - (b^2*e^2*ArcTanh[c*x])/(3*c^3) + (2*b^2*d*e*x*ArcTanh[c*x])/c + (b*e^2*x^2*(a + b*ArcTanh[c*x]))/(3*c) + ((3*c^2*d^2 + e^2)*(a + b*ArcTanh[c*x])^2)/(3*c^3) - (d*(d^2 + (3*e^2)/c^2)*(a + b*ArcTanh[c*x])^2)/(3*e) + ((d + e*x)^3*(a + b*ArcTanh[c*x])^2)/(3*e) - (2*b*(3*c^2*d^2 + e^2)*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^3) + (b^2*d*e*Log[1 - c^2*x^2])/c^2 - (b^2*(3*c^2*d^2 + e^2)*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^3)$

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 327

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[

$a*c^n*((m - n + 1)/(b*(m + n*p + 1)))$, $\text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x]$,
 $x] /;$ $\text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n - 1] \&\& \text{NeQ}[m + n*p + 1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2352

$\text{Int}[\text{Log}[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := \text{Simp}[(-e^{(-1)})*\text{PolyLog}[2, 1 - c*x], x] /;$ $\text{FreeQ}\{c, d, e\}, x\} \&\& \text{EqQ}[e + c*d, 0]$

Rule 2449

$\text{Int}[\text{Log}[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := \text{Dist}[-e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /;$ $\text{FreeQ}\{c, d, e, f, g\}, x\} \&\& \text{EqQ}[c, 2*d] \&\& \text{EqQ}[e^2*f + d^2*g, 0]$

Rule 6021

$\text{Int}[(a_. + \text{ArcTanh}[c_.*(x_)^{(n_.)}]*(b_.))^{(p_.)}, x_Symbol] := \text{Simp}[x*(a + b*\text{ArcTanh}[c*x^n])^p, x] - \text{Dist}[b*c*n*p, \text{Int}[x^n*((a + b*\text{ArcTanh}[c*x^n])^{(p - 1)/(1 - c^2*x^{(2*n)})}), x], x] /;$ $\text{FreeQ}\{a, b, c, n\}, x\} \&\& \text{IGtQ}[p, 0] \&\& (\text{EqQ}[n, 1] \parallel \text{EqQ}[p, 1])$

Rule 6037

$\text{Int}[(a_. + \text{ArcTanh}[c_.*(x_)^{(n_.)}]*(b_.))^{(p_.)}*(x_)^{(m_.)}, x_Symbol] := \text{Simp}[x^{(m + 1)}*((a + b*\text{ArcTanh}[c*x^n])^p/(m + 1)), x] - \text{Dist}[b*c*n*(p/(m + 1)), \text{Int}[x^{(m + n)}*((a + b*\text{ArcTanh}[c*x^n])^{(p - 1)/(1 - c^2*x^{(2*n)})}), x], x] /;$ $\text{FreeQ}\{a, b, c, m, n\}, x\} \&\& \text{IGtQ}[p, 0] \&\& (\text{EqQ}[p, 1] \parallel (\text{EqQ}[n, 1] \&\& \text{IntegerQ}[m])) \&\& \text{NeQ}[m, -1]$

Rule 6055

$\text{Int}[(a_. + \text{ArcTanh}[c_.*(x_)]*(b_.))^{(p_.)}/((d_) + (e_.)*(x_)), x_Symbol] := \text{Simp}[(-a + b*\text{ArcTanh}[c*x])^p*(\text{Log}[2/(1 + e*(x/d))]/e), x] + \text{Dist}[b*c*(p/e), \text{Int}[(a + b*\text{ArcTanh}[c*x])^{(p - 1)}*(\text{Log}[2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[c^2*d^2 - e^2, 0]$

Rule 6065

$\text{Int}[(a_. + \text{ArcTanh}[c_.*(x_)]*(b_.))^{(p_.)}*((d_) + (e_.)*(x_))^{(q_.)}, x_Symbol] := \text{Simp}[(d + e*x)^{(q + 1)}*((a + b*\text{ArcTanh}[c*x])^p/(e*(q + 1))), x] - \text{Dist}[b*c*(p/(e*(q + 1))), \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcTanh}[c*x])^{(p - 1)}, (d + e*x)^{(q + 1)}/(1 - c^2*x^2), x], x], x] /;$ $\text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{IGtQ}[p, 1] \&\& \text{IntegerQ}[q] \&\& \text{NeQ}[q, -1]$

Rule 6095

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 6131

Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6195

Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(m_.))/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int (d + ex)^2 (a + b \tanh^{-1}(cx))^2 dx &= \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))^2}{3e} - \frac{(2bc) \int \left(-\frac{3de^2(a + b \tanh^{-1}(cx))}{c^2} - \frac{e^3 x(a + b \tanh^{-1}(cx))}{c} \right) dx}{3ce} \\
 &= \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))^2}{3e} - \frac{(2b) \int \frac{(c^2 d^3 + 3de^2 + e(3c^2 d^2 + e^2)x)(a + b \tanh^{-1}(cx))}{1 - c^2 x^2} dx}{3ce} \\
 &= \frac{2abdex}{c} + \frac{be^2 x^2 (a + b \tanh^{-1}(cx))}{3c} + \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))^2}{3e} \\
 &= \frac{2abdex}{c} + \frac{b^2 e^2 x}{3c^2} + \frac{2b^2 dex \tanh^{-1}(cx)}{c} + \frac{be^2 x^2 (a + b \tanh^{-1}(cx))}{3c} + \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))^2}{3e} \\
 &= \frac{2abdex}{c} + \frac{b^2 e^2 x}{3c^2} - \frac{b^2 e^2 \tanh^{-1}(cx)}{3c^3} + \frac{2b^2 dex \tanh^{-1}(cx)}{c} + \frac{be^2 x^2 (a + b \tanh^{-1}(cx))}{3c} + \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))^2}{3e} \\
 &= \frac{2abdex}{c} + \frac{b^2 e^2 x}{3c^2} - \frac{b^2 e^2 \tanh^{-1}(cx)}{3c^3} + \frac{2b^2 dex \tanh^{-1}(cx)}{c} + \frac{be^2 x^2 (a + b \tanh^{-1}(cx))}{3c} + \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))^2}{3e} \\
 &= \frac{2abdex}{c} + \frac{b^2 e^2 x}{3c^2} - \frac{b^2 e^2 \tanh^{-1}(cx)}{3c^3} + \frac{2b^2 dex \tanh^{-1}(cx)}{c} + \frac{be^2 x^2 (a + b \tanh^{-1}(cx))}{3c} + \frac{(d + ex)^3 (a + b \tanh^{-1}(cx))^2}{3e}
 \end{aligned}$$

Mathematica [A]

time = 0.43, size = 319, normalized size = 1.24

$$\frac{b^2 d^2 x^2 + 6 a b^2 d x + 3 a^2 d^2 + a b^2 d^2 + c^2 d^2 x^2 + c^2 (-1 + c x) (c^2 + c (3 d + e x)) + c^2 (3 d^2 + 3 d e x + e^2 x^2) (\operatorname{tanh}^{-1}(c x) + \operatorname{tanh}^{-1}(c x)) (b (-c + 6 c^2 d + c^2 x^2) + 2 a c^2 (d^2 + 3 d e x + e^2 x^2) - 2 b (3 d^2 + e^2) \log(1 + e^{-2 \operatorname{arctanh}(c x)})) + \operatorname{atanh}(\log(1 - c x)) - \operatorname{atanh}(\log(1 + c x)) + \operatorname{atanh}(\log(1 - c^2 x^2)) + 3 b^2 d \log(1 - c^2 x^2) + a b^2 \log(-1 + c^2 x^2) + c^2 (b^2 d^2 + e^2) \operatorname{PolyLog}(2, -e^{-2 \operatorname{arctanh}(c x)})}{3 c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2*(a + b*ArcTanh[c*x])^2,x]

[Out] (3*a^2*c^3*d^2*x + 6*a*b*c^2*d*e*x + b^2*c*e^2*x + 3*a^2*c^3*d*e*x^2 + a*b*c^2*e^2*x^2 + a^2*c^3*e^2*x^3 + b^2*(-1 + c*x)*(e^2 + c*e*(3*d + e*x) + c^2*(3*d^2 + 3*d*e*x + e^2*x^2))*ArcTanh[c*x]^2 + b*ArcTanh[c*x]*(b*e*(-e + 6*c^2*d*x + c^2*e*x^2) + 2*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - 2*b*(3*c^2*d^2 + e^2)*Log[1 + E^(-2*ArcTanh[c*x])]) + 3*a*b*c*d*e*Log[1 - c*x] - 3*a*b*c*d*e*Log[1 + c*x] + 3*a*b*c^2*d^2*Log[1 - c^2*x^2] + 3*b^2*c*d*e*Log[1 - c^2*x^2] + a*b*e^2*Log[-1 + c^2*x^2] + b^2*(3*c^2*d^2 + e^2)*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(3*c^3)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. 2(241) = 482.

time = 0.35, size = 1011, normalized size = 3.93

method	result
derivativedivides	Expression too large to display
default	Expression too large to display
risch	$-\frac{b^2 d e \ln(-c x+1) x}{2 c} + \frac{b a d e \ln(-c x+1)}{c^2} - b a d e \ln(-c x+1) x^2 - \frac{b a e^2 \ln(-c x+1) x^3}{3} - \ln(-c x+1) x a$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c*(-1/4*b^2*ln(c*x+1)^2*d^2-b^2*dilog(1/2*c*x+1/2)*d^2+b^2/c*e*arctanh(c*x)*ln(c*x-1)*d+2*a*b*e*d*x+2/3*a*b*c*e^2*arctanh(c*x)*x^3+b^2*c*e*arctanh(c*x)^2*d*x^2+2*b^2*e*arctanh(c*x)*d*x+2*a*b*c*e*arctanh(c*x)*d*x^2+1/3*b^2*e^2*arctanh(c*x)*x^2+2*a*b*arctanh(c*x)*d^2*c*x+1/3*b^2*c*e^2*arctanh(c*x)^2*x^3+a*b/c*e*ln(c*x-1)*d-1/3*a*b*c/e*ln(c*x+1)*d^3-a*b/c*e*ln(c*x+1)*d+1/3*b^2*c/e*arctanh(c*x)*ln(c*x-1)*d^3-1/3*b^2*c/e*arctanh(c*x)*ln(c*x+1)*d^3-1/2*b^2/c*e*ln(c*x-1)*ln(1/2*c*x+1/2)*d-1/6*b^2*c/e*ln(c*x-1)*ln(1/2*c*x+1/2)*d^3-b^2/c*e*arctanh(c*x)*ln(c*x+1)*d+1/3*a*b*e^2*x^2+2/3*a*b*c/e*arctanh(c*x)*d^3+1/3*a*b*c/e*ln(c*x-1)*d^3-1/2*b^2/c*e*ln(c*x+1)*ln(-1/2*c*x+1/2)*d+1/2*b^2/c*e*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)*d-1/6*b^2*c/e*ln(c*x+1)*ln(-1/2*c*x+1/2)*d^3+1/6*b^2*c/e*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)*d^3+1/3*a*b/c^2*e^2*ln(c*x-1)+1/3*a*b/c^2*e^2*ln(c*x+1)+1/12*b^2*c/e*ln(c*x+1)^2*d^3+1/4*b^2/c*e*ln(c*x+1)^2*d+1/12*b^2*c/e*ln(c*x-1)^2*d^3+1/4*b^2/c*e*ln(c*x-1)^2*d+b^2/c*e*ln(c*x-1)*d+b^2*arctanh(c*x)^2*d^2*c*x+1/3*b^2/c*e^2*x+1/3*b^2*c/e*arctanh(c*x)^2*d^3+1/3*b^2/c^2*e^2*arctanh(c*x)*ln(c*x-1)+1/3*b^2/c^2*e^2

```
*arctanh(c*x)*ln(c*x+1)+1/6*b^2/c^2*e^2*ln(c*x+1)*ln(-1/2*c*x+1/2)-1/6*b^2/
c^2*e^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-1/6*b^2/c^2*e^2*ln(c*x-1)*ln(1/2*c
*x+1/2)+b^2/c*e*ln(c*x+1)*d+1/4*b^2*ln(c*x-1)^2*d^2+1/3*(c*e*x+c*d)^3*a^2/c
^2/e+a*b*ln(c*x-1)*d^2+a*b*ln(c*x+1)*d^2-1/12*b^2/c^2*e^2*ln(c*x+1)^2-1/2*b
^2*ln(c*x-1)*ln(1/2*c*x+1/2)*d^2+b^2*arctanh(c*x)*ln(c*x+1)*d^2+b^2*arctanh
(c*x)*ln(c*x-1)*d^2+1/2*b^2*ln(c*x+1)*ln(-1/2*c*x+1/2)*d^2-1/2*b^2*ln(-1/2*
c*x+1/2)*ln(1/2*c*x+1/2)*d^2+1/6*b^2/c^2*e^2*ln(c*x-1)-1/6*b^2/c^2*e^2*ln(c
*x+1)-1/3*b^2/c^2*e^2*dilog(1/2*c*x+1/2)+1/12*b^2/c^2*e^2*ln(c*x-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(233) = 466.

time = 0.42, size = 548, normalized size = 2.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3*e^2 + a^2*d*x^2*e + a^2*d^2*x + (2*x^2*arctanh(c*x) + c*(2*x/c^
2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*d*e + (2*c*x*arctanh(c*x) + 1
og(-c^2*x^2 + 1))*a*b*d^2/c + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^
2*x^2 - 1)/c^4))*a*b*e^2 + 1/3*(3*b^2*c^2*d^2 + b^2*e^2)*(log(c*x + 1)*log(
-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))/c^3 + 1/6*(6*b^2*c*d*e - b^2*e^2)*l
og(c*x + 1)/c^3 + 1/6*(6*b^2*c*d*e + b^2*e^2)*log(c*x - 1)/c^3 + 1/12*(4*b^
2*c*x*e^2 + (b^2*c^3*x^3*e^2 + 3*b^2*c^3*d*x^2*e + 3*b^2*c^3*d^2*x + 3*b^2*
c^2*d^2 - 3*b^2*c*d*e + b^2*e^2)*log(c*x + 1)^2 + (b^2*c^3*x^3*e^2 + 3*b^2*
c^3*d*x^2*e + 3*b^2*c^3*d^2*x - 3*b^2*c^2*d^2 - 3*b^2*c*d*e - b^2*e^2)*log(
-c*x + 1)^2 + 2*(b^2*c^2*x^2*e^2 + 6*b^2*c^2*d*x*e)*log(c*x + 1) - 2*(b^2*c
^2*x^2*e^2 + 6*b^2*c^2*d*x*e + (b^2*c^3*x^3*e^2 + 3*b^2*c^3*d*x^2*e + 3*b^2
*c^3*d^2*x + 3*b^2*c^2*d^2 - 3*b^2*c*d*e + b^2*e^2)*log(c*x + 1))*log(-c*x
+ 1))/c^3
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*e^2 + 2*a^2*d*x*e + a^2*d^2 + (b^2*x^2*e^2 + 2*b^2*d*x*e +
b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*x^2*e^2 + 2*a*b*d*x*e + a*b*d^2)*arctanh(
c*x), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^2 (d + ex)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*(a+b*atanh(c*x))**2,x)

[Out] Integral((a + b*atanh(c*x))**2*(d + e*x)**2, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")

[Out] integrate((e*x + d)^2*(b*arctanh(c*x) + a)^2, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{atanh}(cx))^2 (d + ex)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^2*(d + e*x)^2,x)

[Out] int((a + b*atanh(c*x))^2*(d + e*x)^2, x)

3.11 $\int (d + ex) (a + b \tanh^{-1}(cx))^2 dx$

Optimal. Leaf size=160

$$\frac{abex}{c} + \frac{b^2ex \tanh^{-1}(cx)}{c} + \frac{d(a + b \tanh^{-1}(cx))^2}{c} - \frac{\left(d^2 + \frac{e^2}{c^2}\right) (a + b \tanh^{-1}(cx))^2}{2e} + \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))^2}{2e}$$

[Out] a*b*e*x/c+b^2*e*x*arctanh(c*x)/c+d*(a+b*arctanh(c*x))^2/c-1/2*(d^2+e^2/c^2)*(a+b*arctanh(c*x))^2/e+1/2*(e*x+d)^2*(a+b*arctanh(c*x))^2/e-2*b*d*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c+1/2*b^2*e*ln(-c^2*x^2+1)/c^2-b^2*d*polylog(2,1-2/(-c*x+1))/c

Rubi [A]

time = 0.21, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6065, 6021, 266, 6195, 6095, 6131, 6055, 2449, 2352}

$$-\frac{\left(\frac{e^2}{c^2} + d^2\right) (a + b \tanh^{-1}(cx))^2}{2e} + \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))^2}{2e} + \frac{d(a + b \tanh^{-1}(cx))^2}{c} - \frac{2bd \log\left(\frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{c} + \frac{abex}{c} + \frac{b^2e \log(1 - c^2x^2)}{2c^2} - \frac{b^2d \text{Li}_2\left(1 - \frac{2}{1-cx}\right)}{c} + \frac{b^2ex \tanh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)*(a + b*ArcTanh[c*x])^2,x]

[Out] (a*b*e*x)/c + (b^2*e*x*ArcTanh[c*x])/c + (d*(a + b*ArcTanh[c*x])^2)/c - ((d^2 + e^2/c^2)*(a + b*ArcTanh[c*x])^2)/(2*e) + ((d + e*x)^2*(a + b*ArcTanh[c*x])^2)/(2*e) - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b^2*e*Log[1 - c^2*x^2])/(2*c^2) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/c

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 2352

Int[Log[(c_)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2449

Int[Log[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] :> Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 6021

Int[((a_) + ArcTanh[(c_)*(x_)^(n_)])*(b_)^(p_), x_Symbol] :> Simp[x*(a + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^p

$(p - 1)/(1 - c^2 x^{(2n)})$, x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && (EqQ[n, 1] || EqQ[p, 1])

Rule 6055

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[(-a + b*ArcTanh[c*x])^p*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 6065

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol] := Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] - Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 6095

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 6131

Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.))/((d_.) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6195

Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_.))^(m_.))/((d_.) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int (d + ex) (a + b \tanh^{-1}(cx))^2 dx &= \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))^2}{2e} - \frac{(bc) \int \left(-\frac{e^2(a + b \tanh^{-1}(cx))}{c^2} + \frac{(c^2 d^2 + e^2 + 2c^2 dex)(a + b \tanh^{-1}(cx))}{1 - c^2 x^2} dx \right)}{e} \\
&= \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))^2}{2e} - \frac{b \int \frac{(c^2 d^2 + e^2 + 2c^2 dex)(a + b \tanh^{-1}(cx))}{1 - c^2 x^2} dx}{ce} + \frac{bc \int \frac{e^2(a + b \tanh^{-1}(cx))}{c^2} dx}{e} \\
&= \frac{abex}{c} + \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))^2}{2e} - \frac{b \int \left(\frac{c^2 d^2 (1 + \frac{e^2}{c^2 d^2})(a + b \tanh^{-1}(cx))}{1 - c^2 x^2} \right) dx}{c} \\
&= \frac{abex}{c} + \frac{b^2 ex \tanh^{-1}(cx)}{c} + \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))^2}{2e} - (2bcd) \int \frac{e^2(a + b \tanh^{-1}(cx))}{c^2} dx \\
&= \frac{abex}{c} + \frac{b^2 ex \tanh^{-1}(cx)}{c} + \frac{d(a + b \tanh^{-1}(cx))^2}{c} - \frac{\left(d^2 + \frac{e^2}{c^2}\right) (a + b \tanh^{-1}(cx))^2}{2e} \\
&= \frac{abex}{c} + \frac{b^2 ex \tanh^{-1}(cx)}{c} + \frac{d(a + b \tanh^{-1}(cx))^2}{c} - \frac{\left(d^2 + \frac{e^2}{c^2}\right) (a + b \tanh^{-1}(cx))^2}{2e} \\
&= \frac{abex}{c} + \frac{b^2 ex \tanh^{-1}(cx)}{c} + \frac{d(a + b \tanh^{-1}(cx))^2}{c} - \frac{\left(d^2 + \frac{e^2}{c^2}\right) (a + b \tanh^{-1}(cx))^2}{2e} \\
&= \frac{abex}{c} + \frac{b^2 ex \tanh^{-1}(cx)}{c} + \frac{d(a + b \tanh^{-1}(cx))^2}{c} - \frac{\left(d^2 + \frac{e^2}{c^2}\right) (a + b \tanh^{-1}(cx))^2}{2e}
\end{aligned}$$

Mathematica [A]

time = 0.26, size = 174, normalized size = 1.09

$$\frac{2a^2c^2dx + 2abcex + a^2c^2ex^2 + b^2(-1 + cx)(2cd + e + cex) \tanh^{-1}(cx)^2 + 2bc \tanh^{-1}(cx) (bcx + acx(2d + ex) - 2bd \log(1 + e^{-2 \tanh^{-1}(cx)})) + abe \log(1 - cx) - abe \log(1 + cx) + 2abcd \log(1 - c^2x^2) + b^2e \log(1 - c^2x^2) + 2b^2cd \text{PolyLog}(2, -e^{-2 \tanh^{-1}(cx)})}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x])^2,x]

[Out] (2*a^2*c^2*d*x + 2*a*b*c*e*x + a^2*c^2*e*x^2 + b^2*(-1 + c*x)*(2*c*d + e + c*e*x)*ArcTanh[c*x]^2 + 2*b*c*ArcTanh[c*x]*(b*e*x + a*c*x*(2*d + e*x) - 2*b*d*Log[1 + E^(-2*ArcTanh[c*x])]) + a*b*e*Log[1 - c*x] - a*b*e*Log[1 + c*x] + 2*a*b*c*d*Log[1 - c^2*x^2] + b^2*e*Log[1 - c^2*x^2] + 2*b^2*c*d*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(2*c^2)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(154) = 308.

time = 0.28, size = 442, normalized size = 2.76

method	result
--------	--------

derivativedivides	$\frac{a^2(dcx + \frac{1}{2}e c^2 x^2)}{c} + \frac{b^2 c \operatorname{arctanh}(cx)^2 e x^2}{2} + abex + \frac{b^2 \operatorname{arctanh}(cx) \ln(cx-1)e}{2c} - \frac{b^2 \operatorname{arctanh}(cx) \ln(cx+1)e}{2c} + b^2 \operatorname{arctanh}(cx)^2 dcx - \frac{b^2}{c}$
default	$\frac{a^2(dcx + \frac{1}{2}e c^2 x^2)}{c} + \frac{b^2 c \operatorname{arctanh}(cx)^2 e x^2}{2} + abex + \frac{b^2 \operatorname{arctanh}(cx) \ln(cx-1)e}{2c} - \frac{b^2 \operatorname{arctanh}(cx) \ln(cx+1)e}{2c} + b^2 \operatorname{arctanh}(cx)^2 dcx - \frac{b^2}{c}$
risch	$\frac{b^2 \ln(-\frac{cx}{2} + \frac{1}{2}) \ln(\frac{cx}{2} + \frac{1}{2}) d}{c} - \frac{b^2 \ln(\frac{cx}{2} + \frac{1}{2}) \ln(-cx+1)d}{c} + \frac{b \ln(-cx-1)ad}{c} - \frac{b \ln(-cx-1)ae}{2c^2} - \frac{bae}{c^2} - \frac{b^2 e \ln(-cx+1)}{8c^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{c} * (a^2/c * (d * c^2 * x + 1/2 * e * c^2 * x^2) + a * b * c * \operatorname{arctanh}(c * x) * e * x^2 + 2 * a * b * \operatorname{arctanh}(c * x) * d * c * x + 1/2 * b^2 * c * \operatorname{arctanh}(c * x)^2 * e * x^2 + b^2 * \operatorname{arctanh}(c * x) * e * x + a * b * e * x - 1/2 * b^2 * \ln(c * x - 1) * \ln(1/2 * c * x + 1/2) * d + 1/2 * b^2 / c * \ln(c * x - 1) * e + 1/2 * b^2 / c * \ln(c * x + 1) * e + a * b * \ln(c * x - 1) * d + a * b * \ln(c * x + 1) * d + b^2 * \operatorname{arctanh}(c * x) * \ln(c * x - 1) * d + b^2 * \operatorname{arctanh}(c * x) * \ln(c * x + 1) * d + 1/8 * b^2 / c * \ln(c * x + 1)^2 * e + 1/2 * b^2 * \ln(c * x + 1) * \ln(-1/2 * c * x + 1/2) * d - 1/2 * b^2 * \ln(1/2 * c * x + 1/2) * \ln(-1/2 * c * x + 1/2) * d + 1/8 * b^2 / c * \ln(c * x - 1)^2 * e - 1/2 * b^2 / c * \operatorname{arctanh}(c * x) * \ln(c * x + 1) * e - 1/4 * b^2 / c * \ln(c * x + 1) * \ln(-1/2 * c * x + 1/2) * e + 1/4 * b^2 / c * \ln(1/2 * c * x + 1/2) * \ln(-1/2 * c * x + 1/2) * e - 1/4 * b^2 / c * \ln(c * x - 1) * \ln(1/2 * c * x + 1/2) * e + 1/2 * a * b / c * \ln(c * x - 1) * e - 1/2 * a * b / c * \ln(c * x + 1) * e - 1/4 * b^2 * \ln(c * x + 1)^2 * d - b^2 * d \operatorname{ilog}(1/2 * c * x + 1/2) * d + 1/4 * b^2 * \ln(c * x - 1)^2 * d + b^2 * \operatorname{arctanh}(c * x)^2 * d * c * x + 1/2 * b^2 / c * \operatorname{arctanh}(c * x) * \ln(c * x - 1) * e)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(152) = 304$.

time = 0.41, size = 329, normalized size = 2.06

$\frac{1}{c} a^2 x^2 + a^2 d x + \frac{1}{2} (2 a^2 \operatorname{arctanh}(c x) + (\frac{2 a}{c} \ln(c x + 1) + \ln(c x - 1))) d e + (2 a^2 \operatorname{arctanh}(c x) + \ln(-c^2 x^2 + 1)) b d^2 + (\ln(c x + 1) \ln(-1/2 c x + 1/2) + \operatorname{dilog}(1/2 c x + 1/2)) b^2 d / c + 1/2 b^2 e \ln(c x + 1) / c^2 + 1/2 b^2 e \ln(c x - 1) / c^2 + 1/8 (4 b^2 c x e \ln(c x + 1) + (b^2 c^2 x^2 e + 2 b^2 c^2 d x + 2 b^2 c d - b^2 e) \ln(c x + 1)^2 + (b^2 c^2 x^2 e + 2 b^2 c^2 d x - 2 b^2 c d - b^2 e) \ln(-c x + 1)^2 - 2 (2 b^2 c x e + (b^2 c^2 x^2 e + 2 b^2 c^2 d x + 2 b^2 c d - b^2 e) \log(c x + 1)) \log(-c x + 1)) / c^2$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")`

[Out] $\frac{1}{2} * a^2 * x^2 * e + a^2 * d * x + \frac{1}{2} * (2 * x^2 * \operatorname{arctanh}(c * x) + c * (2 * x / c^2 - \log(c * x + 1) / c^3 + \log(c * x - 1) / c^3)) * a * b * e + (2 * c * x * \operatorname{arctanh}(c * x) + \log(-c^2 * x^2 + 1)) * a * b * d / c + (\log(c * x + 1) * \log(-1/2 * c * x + 1/2) + \operatorname{dilog}(1/2 * c * x + 1/2)) * b^2 * d / c + 1/2 * b^2 * e * \log(c * x + 1) / c^2 + 1/2 * b^2 * e * \log(c * x - 1) / c^2 + 1/8 * (4 * b^2 * c * x * e * \log(c * x + 1) + (b^2 * c^2 * x^2 * e + 2 * b^2 * c^2 * d * x + 2 * b^2 * c * d - b^2 * e) * \log(c * x + 1)^2 + (b^2 * c^2 * x^2 * e + 2 * b^2 * c^2 * d * x - 2 * b^2 * c * d - b^2 * e) * \log(-c * x + 1)^2 - 2 * (2 * b^2 * c * x * e + (b^2 * c^2 * x^2 * e + 2 * b^2 * c^2 * d * x + 2 * b^2 * c * d - b^2 * e) * \log(c * x + 1)) * \log(-c * x + 1)) / c^2$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x*e + a^2*d + (b^2*x*e + b^2*d)*arctanh(c*x)^2 + 2*(a*b*x*e + a*b*d)*arctanh(c*x), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^2 (d + ex) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2*(d + e*x), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arctanh(c*x) + a)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{atanh}(cx))^2 (d + ex) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2*(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x))^2*(d + e*x), x)
```

$$3.12 \quad \int \frac{(a+b \tanh^{-1}(cx))^2}{d+ex} dx$$

Optimal. Leaf size=188

$$-\frac{(a+b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1+cx}\right)}{e} + \frac{(a+b \tanh^{-1}(cx))^2 \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{b(a+b \tanh^{-1}(cx)) \text{PolyLog}(2, 1 - \frac{2}{1+cx})}{e}$$

[Out] $-(a+b*\text{arctanh}(c*x))^2*\ln(2/(c*x+1))/e+(a+b*\text{arctanh}(c*x))^2*\ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+b*(a+b*\text{arctanh}(c*x))*\text{polylog}(2,1-2/(c*x+1))/e-b*(a+b*\text{arctanh}(c*x))*\text{polylog}(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+1/2*b^2*\text{polylog}(3,1-2/(c*x+1))/e-1/2*b^2*\text{polylog}(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e$

Rubi [A]

time = 0.03, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$,

Rules used = {6059}

$$-\frac{b(a+b \tanh^{-1}(cx)) \text{Li}_2\left(1 - \frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e} + \frac{(a+b \tanh^{-1}(cx))^2 \log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e} + \frac{b \text{Li}_2\left(1 - \frac{2}{cx+1}\right) (a+b \tanh^{-1}(cx))}{e} - \frac{\log\left(\frac{2}{cx+1}\right) (a+b \tanh^{-1}(cx))^2}{e} - \frac{b^2 \text{Li}_3\left(1 - \frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e} + \frac{b^2 \text{Li}_3\left(1 - \frac{2}{cx+1}\right)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])^2/(d + e*x), x]

[Out] $-\left(\frac{(a+b*\text{ArcTanh}[c*x])^2*\text{Log}[2/(1+c*x)]}{e}\right) + \left(\frac{(a+b*\text{ArcTanh}[c*x])^2*\text{Log}\left[\frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{e}\right) + \left(\frac{b*(a+b*\text{ArcTanh}[c*x])* \text{PolyLog}[2, 1 - 2/(1+c*x)]}{e}\right) - \left(\frac{b*(a+b*\text{ArcTanh}[c*x])* \text{PolyLog}[2, 1 - (2*c*(d+e*x))/(c*d+e)/(1+c*x)]}{e}\right) + \left(\frac{b^2*\text{PolyLog}[3, 1 - 2/(1+c*x)]}{2*e}\right) - \left(\frac{b^2*\text{PolyLog}[3, 1 - (2*c*(d+e*x))/(c*d+e)/(1+c*x)]}{2*e}\right)$

Rule 6059

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^2/((d_.) + (e_.)*(x_.)), x_Symbol] :=
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcTanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x))])/e], x] + Simp[b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)]/e], x] - Simp[b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e], x] + Simp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rubi steps

$$\int \frac{(a+b \tanh^{-1}(cx))^2}{d+ex} dx = -\frac{(a+b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1+cx}\right)}{e} + \frac{(a+b \tanh^{-1}(cx))^2 \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{b(a+b \tanh^{-1}(cx)) \text{PolyLog}(2, 1 - \frac{2}{1+cx})}{e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.59, size = 938, normalized size = 4.99

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + e*x), x]

[Out] $(6a^2 \log[d + ex] + 6ab \operatorname{ArcTanh}[cx] (\log[1 - c^2x^2] + 2 \log[I \operatorname{Sinh}[\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx]]]) - (6I)ab((-1/4I)(\pi - (2I)\operatorname{ArcTanh}[cx])^2 + I(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])^2 + (\pi - (2I)\operatorname{ArcTanh}[cx]) \log[1 + E^{(2\operatorname{ArcTanh}[cx])}] + (2I)(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx]) \log[1 - E^{-(2(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])}] - (\pi - (2I)\operatorname{ArcTanh}[cx]) \log[2/\sqrt{1 - c^2x^2}] - (2I)(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx]) \log[(2I)\operatorname{Sinh}[\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx]]] - I \operatorname{PolyLog}[2, -E^{(2\operatorname{ArcTanh}[cx])}] - I \operatorname{PolyLog}[2, E^{-(2(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])}] + (b^2(-8cd \operatorname{ArcTanh}[cx]^3 + 4e \operatorname{ArcTanh}[cx]^3 - (4\sqrt{1 - (c^2d^2)/e^2})e \operatorname{ArcTanh}[cx]^3)/E^{\operatorname{ArcTanh}[(cd)/e]} - 6cd \operatorname{ArcTanh}[cx]^2 \log[1 + E^{-(2\operatorname{ArcTanh}[cx])}] - (6I)cd\pi \operatorname{ArcTanh}[cx] \log[(E^{(-\operatorname{ArcTanh}[cx])} + E^{\operatorname{ArcTanh}[cx]})/2] - 6cd \operatorname{ArcTanh}[cx]^2 \log[1 + ((cd + e)E^{(2\operatorname{ArcTanh}[cx])})/(cd - e)] + 6cd \operatorname{ArcTanh}[cx]^2 \log[1 - E^{(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])}] + 6cd \operatorname{ArcTanh}[cx]^2 \log[1 + E^{(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])}] + 6cd \operatorname{ArcTanh}[cx]^2 \log[1 - E^{(2(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])}] + 12cd \operatorname{ArcTanh}[(cd)/e] \operatorname{ArcTanh}[cx] \log[(I/2)E^{(-\operatorname{ArcTanh}[(cd)/e] - \operatorname{ArcTanh}[cx])}(-1 + E^{(2(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])})] + 6cd \operatorname{ArcTanh}[cx]^2 \log[(e(-1 + E^{(2\operatorname{ArcTanh}[cx])}) + cd(1 + E^{(2\operatorname{ArcTanh}[cx])})]/(2E^{\operatorname{ArcTanh}[cx]})] - 6cd \operatorname{ArcTanh}[cx]^2 \log[(c(d + ex))/\sqrt{1 - c^2x^2}] - (3I)cd\pi \operatorname{ArcTanh}[cx] \log[1 - c^2x^2] - 12cd \operatorname{ArcTanh}[(cd)/e] \operatorname{ArcTanh}[cx] \log[I \operatorname{Sinh}[\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx]]] + 6cd \operatorname{ArcTanh}[cx] \operatorname{PolyLog}[2, -E^{-(2\operatorname{ArcTanh}[cx])}] - 6cd \operatorname{ArcTanh}[cx] \operatorname{PolyLog}[2, -((cd + e)E^{(2\operatorname{ArcTanh}[cx])})/(cd - e)] + 12cd \operatorname{ArcTanh}[cx] \operatorname{PolyLog}[2, -E^{(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])}] + 12cd \operatorname{ArcTanh}[cx] \operatorname{PolyLog}[2, E^{(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])}] + 6cd \operatorname{ArcTanh}[cx] \operatorname{PolyLog}[2, E^{(2(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])}] + 3cd \operatorname{PolyLog}[3, -E^{-(2\operatorname{ArcTanh}[cx])}] + 3cd \operatorname{PolyLog}[3, -((cd + e)E^{(2\operatorname{ArcTanh}[cx])})/(cd - e)] - 12cd \operatorname{PolyLog}[3, -E^{(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])}] - 12cd \operatorname{PolyLog}[3, E^{(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])}] - 3cd \operatorname{PolyLog}[3, E^{(2(\operatorname{ArcTanh}[(cd)/e] + \operatorname{ArcTanh}[cx])})])])/(cd)/(6e)$

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.76, size = 1197, normalized size = 6.37

method	result	size
derivativedivides	Expression too large to display	1197

default	Expression too large to display	1197
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{c} \left(\frac{a^2 c \ln(c e x + c d)}{e + b^2 c \ln(c e x + c d)} \frac{e \operatorname{arctanh}(c x)^2 - b^2 c}{e \operatorname{arctanh}(c x)^2 \ln(d c (1 + (c x + 1)^2 / (-c^2 x^2 + 1)) + e((c x + 1)^2 / (-c^2 x^2 + 1) - 1))} - \frac{1}{2} I b^2 c / e \operatorname{arctanh}(c x)^2 \operatorname{Pi} \operatorname{csgn}(I (d c (1 + (c x + 1)^2 / (-c^2 x^2 + 1)) + e((c x + 1)^2 / (-c^2 x^2 + 1) - 1)) / (1 + (c x + 1)^2 / (-c^2 x^2 + 1)))^2 \operatorname{csgn}(I / (1 + (c x + 1)^2 / (-c^2 x^2 + 1))) - \frac{1}{2} I b^2 c / e \operatorname{arctanh}(c x)^2 \operatorname{Pi} \operatorname{csgn}(I (d c (1 + (c x + 1)^2 / (-c^2 x^2 + 1)) + e((c x + 1)^2 / (-c^2 x^2 + 1) - 1)) / (1 + (c x + 1)^2 / (-c^2 x^2 + 1)))^2 \operatorname{csgn}(I (d c (1 + (c x + 1)^2 / (-c^2 x^2 + 1)) + e((c x + 1)^2 / (-c^2 x^2 + 1) - 1))) + \frac{1}{2} I b^2 c / e \operatorname{arctanh}(c x)^2 \operatorname{Pi} \operatorname{csgn}(I (d c (1 + (c x + 1)^2 / (-c^2 x^2 + 1)) + e((c x + 1)^2 / (-c^2 x^2 + 1) - 1))) / (1 + (c x + 1)^2 / (-c^2 x^2 + 1)))^3 + \frac{1}{2} I b^2 c / e \operatorname{arctanh}(c x)^2 \operatorname{Pi} \operatorname{csgn}(I (d c (1 + (c x + 1)^2 / (-c^2 x^2 + 1)) + e((c x + 1)^2 / (-c^2 x^2 + 1) - 1))) / (1 + (c x + 1)^2 / (-c^2 x^2 + 1)) \operatorname{csgn}(I (d c (1 + (c x + 1)^2 / (-c^2 x^2 + 1)) + e((c x + 1)^2 / (-c^2 x^2 + 1) - 1))) \operatorname{csgn}(I / (1 + (c x + 1)^2 / (-c^2 x^2 + 1))) - b^2 c / e \operatorname{arctanh}(c x) \operatorname{polylog}(2, -(c x + 1)^2 / (-c^2 x^2 + 1)) + \frac{1}{2} b^2 c / e \operatorname{polylog}(3, -(c x + 1)^2 / (-c^2 x^2 + 1)) + b^2 c / (c d + e) \operatorname{arctanh}(c x)^2 \ln(1 - (c d + e) (c x + 1)^2 / (-c^2 x^2 + 1) / (-c d + e)) + b^2 c / (c d + e) \operatorname{arctanh}(c x) \operatorname{polylog}(2, (c d + e) (c x + 1)^2 / (-c^2 x^2 + 1) / (-c d + e)) - \frac{1}{2} b^2 c / (c d + e) \operatorname{polylog}(3, (c d + e) (c x + 1)^2 / (-c^2 x^2 + 1) / (-c d + e)) + b^2 c^2 / e d / (c d + e) \operatorname{arctanh}(c x)^2 \ln(1 - (c d + e) (c x + 1)^2 / (-c^2 x^2 + 1) / (-c d + e)) + b^2 c^2 / e d / (c d + e) \operatorname{arctanh}(c x) \operatorname{polylog}(2, (c d + e) (c x + 1)^2 / (-c^2 x^2 + 1) / (-c d + e)) - \frac{1}{2} b^2 c^2 / e d / (c d + e) \operatorname{polylog}(3, (c d + e) (c x + 1)^2 / (-c^2 x^2 + 1) / (-c d + e)) + 2 a b c \ln(c e x + c d) / e \operatorname{arctanh}(c x) - a b c / e \ln(c e x + c d) \ln((c e x + e) / (-c d + e)) - a b c / e \operatorname{dilog}((c e x + e) / (-c d + e)) + a b c / e \ln((c e x - e) / (-c d - e)) \ln(c e x + c d) + a b c / e \operatorname{dilog}((c e x - e) / (-c d - e)) \right)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")`

[Out] $a^2 e^{-1} \log(x e + d) + \int (1/4 b^2 (\log(c x + 1) - \log(-c x + 1)))^2 / (x e + d) + a b (\log(c x + 1) - \log(-c x + 1)) / (x e + d), x$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")

[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(x*e + d), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))^2/(e*x+d),x)

[Out] Integral((a + b*atanh(c*x))^2/(d + e*x), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")

[Out] integrate((b*arctanh(c*x) + a)^2/(e*x + d), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^2/(d + e*x),x)

[Out] int((a + b*atanh(c*x))^2/(d + e*x), x)

$$3.13 \quad \int \frac{(a+b \tanh^{-1}(cx))^2}{(d+ex)^2} dx$$

Optimal. Leaf size=321

$$-\frac{(a+b \tanh^{-1}(cx))^2}{e(d+ex)} + \frac{bc(a+b \tanh^{-1}(cx)) \log\left(\frac{2}{1-cx}\right)}{e(cd+e)} - \frac{bc(a+b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{(cd-e)e} + \frac{2bc(a+b \tanh^{-1}(cx))}{c^2d^2}$$

[Out] $-(a+b*\operatorname{arctanh}(c*x))^2/e/(e*x+d)+b*c*(a+b*\operatorname{arctanh}(c*x))*\ln(2/(-c*x+1))/e/(c*d+e)-b*c*(a+b*\operatorname{arctanh}(c*x))*\ln(2/(c*x+1))/(c*d-e)/e+2*b*c*(a+b*\operatorname{arctanh}(c*x))*\ln(2/(c*x+1))/(c^2*d^2-e^2)-2*b*c*(a+b*\operatorname{arctanh}(c*x))*\ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)+1/2*b^2*c*\operatorname{polylog}(2,1-2/(-c*x+1))/e/(c*d+e)+1/2*b^2*c*\operatorname{polylog}(2,1-2/(c*x+1))/(c*d-e)/e-b^2*c*\operatorname{polylog}(2,1-2/(c*x+1))/(c^2*d^2-e^2)+b^2*c*\operatorname{polylog}(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)$

Rubi [A]

time = 0.23, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6065, 6055, 2449, 2352, 6057, 2497}

$$\frac{2bc \log\left(\frac{2}{1-cx}\right) (a+b \tanh^{-1}(cx))}{c^2d^2-e^2} - \frac{2bc(a+b \tanh^{-1}(cx)) \log\left(\frac{2(d+ex)}{(cx+1)(cd+e)}\right)}{c^2d^2-e^2} - \frac{(a+b \tanh^{-1}(cx))^2}{e(d+ex)} + \frac{bc \log\left(\frac{2}{1-cx}\right) (a+b \tanh^{-1}(cx))}{e(cd+e)} - \frac{bc \log\left(\frac{2}{1+cx}\right) (a+b \tanh^{-1}(cx))}{e(cd-e)} - \frac{b^2 d \operatorname{Li}_2\left(1-\frac{2}{1-cx}\right)}{c^2d^2-e^2} + \frac{b^2 d \operatorname{Li}_2\left(1-\frac{2(d+ex)}{(cx+1)(cd+e)}\right)}{c^2d^2-e^2} + \frac{b^2 c \operatorname{Li}_2\left(1-\frac{2}{1-cx}\right)}{2e(cd+e)} + \frac{b^2 c \operatorname{Li}_2\left(1-\frac{2}{1+cx}\right)}{2e(cd-e)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])^2/(d + e*x)^2, x]

[Out] $-((a+b*\operatorname{ArcTanh}[c*x])^2/(e*(d+e*x))) + (b*c*(a+b*\operatorname{ArcTanh}[c*x])* \operatorname{Log}[2/(1-c*x)])/(e*(c*d+e)) - (b*c*(a+b*\operatorname{ArcTanh}[c*x])* \operatorname{Log}[2/(1+c*x)])/((c*d-e)*e) + (2*b*c*(a+b*\operatorname{ArcTanh}[c*x])* \operatorname{Log}[2/(1+c*x)])/(c^2*d^2-e^2) - (2*b*c*(a+b*\operatorname{ArcTanh}[c*x])* \operatorname{Log}[(2*c*(d+e*x))/((c*d+e)*(1+c*x))])/(c^2*d^2-e^2) + (b^2*c*\operatorname{PolyLog}[2,1-2/(1-c*x)])/(2*e*(c*d+e)) + (b^2*c*\operatorname{PolyLog}[2,1-2/(1+c*x)])/(2*(c*d-e)*e) - (b^2*c*\operatorname{PolyLog}[2,1-2/(1+c*x)])/(c^2*d^2-e^2) + (b^2*c*\operatorname{PolyLog}[2,1-(2*c*(d+e*x))/((c*d+e)*(1+c*x))])/(c^2*d^2-e^2)$

Rule 2352

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2449

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 2497

```
Int[Log[u]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^((p_.)/((d_) + (e_.)*(x_))), x_Symbol]
:= Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)/((d_) + (e_.)*(x_)), x_Symbol] := S
imp[(-(a + b*ArcTanh[c*x])*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x))]]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^((p_.)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] := Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tanh^{-1}(cx))^2}{(d + ex)^2} dx &= -\frac{(a + b \tanh^{-1}(cx))^2}{e(d + ex)} + \frac{(2bc) \int \left(-\frac{c(a+b \tanh^{-1}(cx))}{2(cd+e)(-1+cx)} + \frac{c(a+b \tanh^{-1}(cx))}{2(cd-e)(1+cx)} + \frac{e^2(a+b \tanh^{-1}(cx))}{(-cd+e)(cd+e)} \right) dx}{e} \\
 &= -\frac{(a + b \tanh^{-1}(cx))^2}{e(d + ex)} + \frac{(bc^2) \int \frac{a+b \tanh^{-1}(cx)}{1+cx} dx}{(cd - e)e} - \frac{(bc^2) \int \frac{a+b \tanh^{-1}(cx)}{-1+cx} dx}{e(cd + e)} + \frac{(2bc^2) \int \frac{e^2(a+b \tanh^{-1}(cx))}{(-cd+e)(cd+e)} dx}{e} \\
 &= -\frac{(a + b \tanh^{-1}(cx))^2}{e(d + ex)} + \frac{bc(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1-cx}\right)}{e(cd + e)} - \frac{bc(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{e(cd - e)} + \frac{2bc^2 \int \frac{e^2(a+b \tanh^{-1}(cx))}{(-cd+e)(cd+e)} dx}{e} \\
 &= -\frac{(a + b \tanh^{-1}(cx))^2}{e(d + ex)} + \frac{bc(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1-cx}\right)}{e(cd + e)} - \frac{bc(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{e(cd - e)} + \frac{2bc^2 \int \frac{e^2(a+b \tanh^{-1}(cx))}{(-cd+e)(cd+e)} dx}{e} \\
 &= -\frac{(a + b \tanh^{-1}(cx))^2}{e(d + ex)} + \frac{bc(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1-cx}\right)}{e(cd + e)} - \frac{bc(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{e(cd - e)} + \frac{2bc^2 \int \frac{e^2(a+b \tanh^{-1}(cx))}{(-cd+e)(cd+e)} dx}{e}
 \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.57, size = 317, normalized size = 0.99

$$\frac{a^2}{e(d+ex)} + \frac{abc \left(\frac{2 \operatorname{arctanh}(cx)}{cd+e} + \frac{(cd+e) \operatorname{arctanh}(cx) \log(1-cx)}{(cd+e)(cd+e)} - \frac{2 \operatorname{arctanh}(cx)}{cd-e} \right)}{e} + \frac{b^2 \left(-\frac{c \operatorname{arctanh}(cx) \log(1-cx)}{\sqrt{1-c^2d^2}} + \frac{c \operatorname{arctanh}(cx)}{d+ex} + \frac{e \left(\operatorname{arctanh}(cx) \log(1-cx) \log(1-cx) + \operatorname{arctanh}(cx) \log(1-cx) \log(1-cx) \right) + \operatorname{arctanh}(cx) \log(1-cx) \log(1-cx)}{2d^2 - e^2} \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] -(a^2/(e*(d + e*x))) + (a*b*c*((-2*ArcTanh[c*x])/(c*d + c*e*x) + ((-(c*d) + e)*Log[1 - c*x] + (c*d + e)*Log[1 + c*x] - 2*e*Log[c*(d + e*x)])/((c*d - e)*(c*d + e))))/e + (b^2*(-(ArcTanh[c*x]^2/(Sqrt[1 - (c^2*d^2)/e^2]*e*E^ArcTanh[(c*d)/e])) + (x*ArcTanh[c*x]^2)/(d + e*x) + (c*d*(I*Pi*Log[1 + E^(2*ArcTanh[c*x])]) - 2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])]) - I*Pi*(ArcTanh[c*x] - Log[1 - c^2*x^2]/2) - 2*ArcTanh[(c*d)/e]*(ArcTanh[c*x] + Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])]) - Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]])] + PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])])))/(c^2*d^2 - e^2))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(317) = 634.
time = 1.23, size = 649, normalized size = 2.02

method	result
derivativedivides	$-\frac{a^2 c^2}{(cex+dc)e} - \frac{b^2 c^2 \operatorname{arctanh}(cx)^2}{(cex+dc)e} - \frac{2b^2 c^2 \operatorname{arctanh}(cx) \ln(cx+dc)}{(dc+e)(dc-e)} + \frac{2b^2 c^2 \operatorname{arctanh}(cx) \ln(cx+1)}{e(2dc-2e)} - \frac{2b^2 c^2 \operatorname{arctanh}(cx) \ln(cx-1)}{e(2dc+2e)} - \frac{b^2 c^2 \ln^2(cx)}{e(2dc-2e)}$

default

$$-\frac{a^2 c^2}{(cex+dc)e} - \frac{b^2 c^2 \operatorname{arctanh}(cx)^2}{(cex+dc)e} - \frac{2b^2 c^2 \operatorname{arctanh}(cx) \ln(cex+dc)}{(dc+e)(dc-e)} + \frac{2b^2 c^2 \operatorname{arctanh}(cx) \ln(cx+1)}{e(2dc-2e)} - \frac{2b^2 c^2 \operatorname{arctanh}(cx) \ln(cx-1)}{e(2dc+2e)} - \frac{b^2 c^2}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x))^2/(e*x+d)^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{c} \left(-\frac{a^2 c^2}{(cex+dc)e} - \frac{b^2 c^2 \operatorname{arctanh}(cx)^2}{(cex+dc)e} - \frac{2b^2 c^2 \operatorname{arctanh}(cx) \ln(cex+dc)}{(dc+e)(dc-e)} + \frac{2b^2 c^2 \operatorname{arctanh}(cx) \ln(cx+1)}{e(2dc-2e)} - \frac{2b^2 c^2 \operatorname{arctanh}(cx) \ln(cx-1)}{e(2dc+2e)} - \frac{b^2 c^2}{e} \right)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/(e*x+d)^2,x, algorithm="maxima")`

[Out] $(c(\log(cx+1)/(cde-e^2) - \log(cx-1)/(cde+e^2) - 2\log(xe+d)/(c^2d^2-e^2)) - 2\operatorname{arctanh}(cx)/(xe^2+de))ab - \frac{1}{4}b^2(\log(-cx+1)^2/(xe^2+de) + \operatorname{integrate}(-((cxe-e)\log(cx+1)^2 + 2(cxe+c^2d - (cxe-e)\log(cx+1))\log(-cx+1))/(cx^3e^3 + (2cde^2 - e^3)x^2 - d^2e + (c^2de - 2de^2)x), x)) - a^2/(xe^2+de)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/(e*x+d)^2,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(x^2*e^2 + 2*d*x*e + d^2), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))**2/(e*x+d)**2,x)

[Out] Integral((a + b*atanh(c*x))**2/(d + e*x)**2, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^2/(e*x+d)^2,x, algorithm="giac")

[Out] integrate((b*arctanh(c*x) + a)^2/(e*x + d)^2, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^2/(d + e*x)^2,x)

[Out] int((a + b*atanh(c*x))^2/(d + e*x)^2, x)

$$3.14 \quad \int \frac{(a+b \tanh^{-1}(cx))^2}{(d+ex)^3} dx$$

Optimal. Leaf size=480

$$\frac{bc(a+b \tanh^{-1}(cx))}{(c^2d^2-e^2)(d+ex)} - \frac{(a+b \tanh^{-1}(cx))^2}{2e(d+ex)^2} + \frac{bc^2(a+b \tanh^{-1}(cx)) \log\left(\frac{2}{1-cx}\right)}{2e(cd+e)^2} + \frac{b^2c^2 \log(1-cx)}{2(cd-e)(cd+e)^2} - \frac{bc^2(a+b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{2e(cd+e)^2} - \frac{b^2c^2 \log(1+cx)}{2(cd+e)(cd+e)^2}$$

```
[Out] b*c*(a+b*arctanh(c*x))/(c^2*d^2-e^2)/(e*x+d)-1/2*(a+b*arctanh(c*x))^2/e/(e*x+d)^2+1/2*b*c^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/e/(c*d+e)^2+1/2*b^2*c^2*ln(-c*x+1)/(c*d-e)/(c*d+e)^2-1/2*b*c^2*(a+b*arctanh(c*x))*ln(2/(c*x+1))/(c*d-e)^2/e+2*b*c^3*d*(a+b*arctanh(c*x))*ln(2/(c*x+1))/(c^2*d^2-e^2)^2-1/2*b^2*c^2*ln(c*x+1)/(c*d-e)^2/(c*d+e)+b^2*c^2*e*ln(e*x+d)/(c^2*d^2-e^2)^2-2*b*c^3*d*(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)^2+1/4*b^2*c^2*polylog(2,1-2/(-c*x+1))/e/(c*d+e)^2+1/4*b^2*c^2*polylog(2,1-2/(c*x+1))/(c*d-e)^2/e-b^2*c^3*d*polylog(2,1-2/(c*x+1))/(c^2*d^2-e^2)^2+b^2*c^3*d*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)^2
```

Rubi [A]

time = 0.36, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6065, 6055, 2449, 2352, 6063, 720, 31, 647, 6057, 2497}

$$\frac{2b^2d \log\left(\frac{2}{1-cx}\right) (a+b \tanh^{-1}(cx))}{(d-e)^2(d+e)^2} - \frac{2bc^2(a+b \tanh^{-1}(cx)) \log\left(\frac{2}{1-cx}\right)}{(d-e)^2(d+e)^2} + \frac{bc^2(a+b \tanh^{-1}(cx))}{(c^2d^2-e^2)(d+ex)} + \frac{b^2 \log\left(\frac{2}{1-cx}\right) (a+b \tanh^{-1}(cx))}{2e(d+e)^2} - \frac{b^2 \log\left(\frac{2}{1+cx}\right) (a+b \tanh^{-1}(cx))}{2e(d+e)^2} + \frac{(a+b \tanh^{-1}(cx))^2}{2e(d+e)^2} - \frac{b^2dL_2(1-\frac{2}{1-cx})}{(d-e)^2(d+e)^2} + \frac{b^2dL_2(1-\frac{2}{1+cx})}{(d-e)^2(d+e)^2} + \frac{b^2dL_2(1-\frac{2}{1-cx})}{4e(d+e)^2} + \frac{b^2dL_2(1-\frac{2}{1+cx})}{4e(d+e)^2} + \frac{b^2 \log(1-cx)}{2e(d-e)(d+e)^2} - \frac{b^2 \log(1+cx)}{2e(d-e)(d+e)^2} + \frac{b^2e \log(d+ex)}{2e(d-e)^2(d+e)^2} - \frac{b^2e \log(d+ex)}{2e(d-e)^2(d+e)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] (b*c*(a + b*ArcTanh[c*x]))/((c^2*d^2 - e^2)*(d + e*x)) - (a + b*ArcTanh[c*x])^2/(2*e*(d + e*x)^2) + (b*c^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(2*e*(c*d + e)^2) + (b^2*c^2*Log[1 - c*x])/(2*(c*d - e)*(c*d + e)^2) - (b*c^2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(2*(c*d - e)^2*e) + (2*b*c^3*d*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/((c*d - e)^2*(c*d + e)^2) - (b^2*c^2*Log[1 + c*x])/(2*(c*d - e)^2*(c*d + e)) + (b^2*c^2*e*Log[d + e*x])/((c*d - e)^2*(c*d + e)^2) - (2*b*c^3*d*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/((c*d - e)^2*(c*d + e)^2) + (b^2*c^2*PolyLog[2, 1 - 2/(1 - c*x)])/(4*e*(c*d + e)^2) + (b^2*c^2*PolyLog[2, 1 - 2/(1 + c*x)])/(4*(c*d - e)^2*e) - (b^2*c^3*d*PolyLog[2, 1 - 2/(1 + c*x)])/((c*d - e)^2*(c*d + e)^2) + (b^2*c^3*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/((c*d - e)^2*(c*d + e)^2)
```

Rule 31

```
Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 2352

```
Int[Log[(c_)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] := Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 6055

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)), x_Symbol] := Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rule 6057

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))/((d_) + (e_)*(x_)), x_Symbol] := Simp[(-(a + b*ArcTanh[c*x])*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x))/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])*(Log[2*c*((d + e*x))/((c*d + e)*(1 + c*x))]/e), x]) /; FreeQ[{a, b, c, d,
```

e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 6063

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

Rule 6065

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] - Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tanh^{-1}(cx))^2}{(d + ex)^3} dx &= -\frac{(a + b \tanh^{-1}(cx))^2}{2e(d + ex)^2} + \frac{(bc) \int \left(-\frac{c^2(a + b \tanh^{-1}(cx))}{2(cd + e)^2(-1 + cx)} + \frac{c^2(a + b \tanh^{-1}(cx))}{2(cd - e)^2(1 + cx)} + \frac{e^2(a + b \tanh^{-1}(cx))}{(-cd + e)} \right) dx}{e} \\
 &= -\frac{(a + b \tanh^{-1}(cx))^2}{2e(d + ex)^2} + \frac{(bc^3) \int \frac{a + b \tanh^{-1}(cx)}{1 + cx} dx}{2(cd - e)^2 e} - \frac{(bc^3) \int \frac{a + b \tanh^{-1}(cx)}{-1 + cx} dx}{2e(cd + e)^2} - \frac{(bc^3) \int \frac{e^2(a + b \tanh^{-1}(cx))}{(-cd + e)} dx}{e} \\
 &= \frac{bc(a + b \tanh^{-1}(cx))}{(cd - e)(cd + e)(d + ex)} - \frac{(a + b \tanh^{-1}(cx))^2}{2e(d + ex)^2} + \frac{bc^2(a + b \tanh^{-1}(cx)) \log(-1 + cx)}{2e(cd + e)^2} \\
 &= \frac{bc(a + b \tanh^{-1}(cx))}{(cd - e)(cd + e)(d + ex)} - \frac{(a + b \tanh^{-1}(cx))^2}{2e(d + ex)^2} + \frac{bc^2(a + b \tanh^{-1}(cx)) \log(1 + cx)}{2e(cd + e)^2} \\
 &= \frac{bc(a + b \tanh^{-1}(cx))}{(cd - e)(cd + e)(d + ex)} - \frac{(a + b \tanh^{-1}(cx))^2}{2e(d + ex)^2} + \frac{bc^2(a + b \tanh^{-1}(cx)) \log(-1 + cx)}{2e(cd + e)^2} \\
 &= \frac{bc(a + b \tanh^{-1}(cx))}{(cd - e)(cd + e)(d + ex)} - \frac{(a + b \tanh^{-1}(cx))^2}{2e(d + ex)^2} + \frac{bc^2(a + b \tanh^{-1}(cx)) \log(1 + cx)}{2e(cd + e)^2}
 \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 6.00, size = 470, normalized size = 0.98

$$\frac{a^2}{2e(d + ex)^2} - \frac{abc \left(\frac{cd + e}{2e} \operatorname{atanh}\left(\frac{cx}{1 + cx}\right) + \frac{cd - e}{2e} \operatorname{atanh}\left(\frac{cx}{-1 + cx}\right) + \frac{e^2 \operatorname{atanh}\left(\frac{cx}{-cd + e}\right)}{(-cd + e)} \right)}{2e} - \frac{bc^2 \left(\frac{cd + e}{2e} \operatorname{atanh}\left(\frac{cx}{1 + cx}\right) - \frac{cd - e}{2e} \operatorname{atanh}\left(\frac{cx}{-1 + cx}\right) + \frac{e^2 \operatorname{atanh}\left(\frac{cx}{-cd + e}\right)}{(-cd + e)} \right)}{2e} + \frac{bc^2 (a + b \operatorname{atanh}(cx)) \log(-1 + cx)}{2e(cd + e)^2} + \frac{bc^2 (a + b \operatorname{atanh}(cx)) \log(1 + cx)}{2e(cd + e)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + e*x)^3,x]

[Out]
$$-1/2*a^2/(e*(d + e*x)^2) - (a*b*c^2*((2*ArcTanh[c*x])/(c*d + c*e*x))^2 + \text{Log}[1 - c*x]/(c*d + e)^2 + (-\text{Log}[1 + c*x] + (2*e*(-(c^2*d^2) + e^2 + 2*c^2*d*(d + e*x)*\text{Log}[c*(d + e*x)])))/(c*(c*d + e)^2*(d + e*x)))/(-(c*d) + e)^2)/(2*e) + (b^2*c^2*((-2*ArcTanh[c*x]^2)/(\text{Sqrt}[1 - (c^2*d^2)/e^2]*e*E^{\text{ArcTanh}[(c*d)/e]}) - (e*(-1 + c^2*x^2)*\text{ArcTanh}[c*x]^2)/(c^2*(d + e*x)^2) + (2*x*\text{ArcTanh}[c*x]*(-e + c*d*\text{ArcTanh}[c*x]))/(c*d*(d + e*x)) + (2*e*(-(e*\text{ArcTanh}[c*x]) + c*d*\text{Log}[(c*(d + e*x))/\text{Sqrt}[1 - c^2*x^2]])))/(c^3*d^3 - c*d*e^2) + (2*c*d*(I*\text{Pi}*\text{Log}[1 + E^{(2*\text{ArcTanh}[c*x])}] - 2*\text{ArcTanh}[c*x]*\text{Log}[1 - E^{-2*(\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x])}] - I*\text{Pi}*(\text{ArcTanh}[c*x] - \text{Log}[1 - c^2*x^2]/2) - 2*\text{ArcTanh}[(c*d)/e]*(\text{ArcTanh}[c*x] + \text{Log}[1 - E^{-2*(\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x])}])) - \text{Log}[I*\text{Sinh}[\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x]]]) + \text{PolyLog}[2, E^{-2*(\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x])}])))/(c^2*d^2 - e^2))/(2*(c*d - e)*(c*d + e))$$

Maple [A]

time = 1.36, size = 828, normalized size = 1.72 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)

[Out]
$$1/c*(a*b*c^3/(c*d+e)/(c*d-e)/(c*e*x+c*d)-2*a*b*c^4*d/(c*d+e)^2/(c*d-e)^2*\ln(c*e*x+c*d)-2*b^2*c^4*arctanh(c*x)*d/(c*d+e)^2/(c*d-e)^2*\ln(c*e*x+c*d)-b^2*c^4*d/(c*d+e)^2/(c*d-e)^2*\ln((c*e*x-e)/(-c*d-e))*\ln(c*e*x+c*d)+b^2*c^4*d/(c*d+e)^2/(c*d-e)^2*\ln((c*e*x+e)/(-c*d+e))*\ln(c*e*x+c*d)-1/2*b^2*c^3/(c*e*x+c*d)^2/e*arctanh(c*x)^2-1/8*b^2*c^3/e/(c*d+e)^2*\ln(c*x-1)^2+1/4*b^2*c^3/e/(c*d+e)^2*dilog(1/2*c*x+1/2)-1/4*b^2*c^3/e/(c*d-e)^2*dilog(1/2*c*x+1/2)-1/8*b^2*c^3/e/(c*d-e)^2*\ln(c*x+1)^2-1/2*a^2*c^3/(c*e*x+c*d)^2/e-a*b*c^3/(c*e*x+c*d)^2/e*arctanh(c*x)+1/2*a*b*c^3/e/(c*d-e)^2*\ln(c*x+1)-1/2*a*b*c^3/e/(c*d+e)^2*\ln(c*x-1)+1/2*b^2*c^3/e*arctanh(c*x)/(c*d-e)^2*\ln(c*x+1)+b^2*c^3*arctanh(c*x)/(c*d+e)/(c*d-e)/(c*e*x+c*d)-1/2*b^2*c^3/e*arctanh(c*x)/(c*d+e)^2*\ln(c*x-1)+1/4*b^2*c^3/e/(c*d+e)^2*\ln(c*x-1)*\ln(1/2*c*x+1/2)-1/4*b^2*c^3/e/(c*d-e)^2*\ln(-1/2*c*x+1/2)*\ln(1/2*c*x+1/2)+1/4*b^2*c^3/e/(c*d-e)^2*\ln(-1/2*c*x+1/2)*\ln(c*x+1)-b^2*c^3/(c*d+e)/(c*d-e)/(2*c*d-2*e)*\ln(c*x+1)+b^2*c^3*e/(c*d+e)^2/(c*d-e)^2*\ln(c*e*x+c*d)+b^2*c^3/(c*d+e)/(c*d-e)/(2*c*d+2*e)*\ln(c*x-1)-b^2*c^4*d/(c*d+e)^2/(c*d-e)^2*dilog((c*e*x-e)/(-c*d-e))+b^2*c^4*d/(c*d+e)^2/(c*d-e)^2*dilog((c*e*x+e)/(-c*d+e))$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^2/(e*x+d)^3,x, algorithm="maxima")

[Out]
$$-1/2*((4*c^2*d*\log(x*e + d)/(c^4*d^4 - 2*c^2*d^2*e^2 + e^4) - c*\log(c*x + 1))/(c^2*d^2*e - 2*c*d*e^2 + e^3) + c*\log(c*x - 1)/(c^2*d^2*e + 2*c*d*e^2 + e$$

$^3) - 2/(c^2*d^3 + (c^2*d^2*e - e^3)*x - d*e^2))*c + 2*\operatorname{arctanh}(c*x)/(x^2*e^3 + 2*d*x*e^2 + d^2*e))*a*b - 1/8*b^2*(\log(-c*x + 1)^2/(x^2*e^3 + 2*d*x*e^2 + d^2*e) + 2*\integrate(-((c*x*e - e)*\log(c*x + 1)^2 + (c*x*e + c*d - 2*(c*x*e - e)*\log(c*x + 1))*\log(-c*x + 1))/(c*x^4*e^4 + (3*c*d*e^3 - e^4)*x^3 - d^3*e + 3*(c*d^2*e^2 - d*e^3)*x^2 + (c*d^3*e - 3*d^2*e^2)*x), x)) - 1/2*a^2/(x^2*e^3 + 2*d*x*e^2 + d^2*e)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/(e*x+d)^3,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x))^2/(e*x+d)^3,x)`

[Out] `Integral((a + b*atanh(c*x))^2/(d + e*x)^3, x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/(e*x+d)^3,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x) + a)^2/(e*x + d)^3, x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c*x))^2/(d + e*x)^3,x)`

[Out] `int((a + b*atanh(c*x))^2/(d + e*x)^3, x)`

3.15 $\int (d + ex)^3 (a + b \tanh^{-1}(cx))^3 dx$

Optimal. Leaf size=614

$$\frac{3ab^2de^2x}{c^2} + \frac{b^3e^3x}{4c^3} - \frac{b^3e^3 \tanh^{-1}(cx)}{4c^4} + \frac{3b^3de^2x \tanh^{-1}(cx)}{c^2} + \frac{b^2e^3x^2(a + b \tanh^{-1}(cx))}{4c^2} - \frac{3bde^2(a + b \tanh^{-1}(cx))}{2c^3}$$

[Out] $3a^3b^2d^2e^2x/c^2 + 1/4b^3e^3x/c^3 - 1/4b^3e^3 \arctanh(cx)/c^4 + 3b^3d^2e^2x \arctanh(cx)/c^2 + 1/4b^2e^3x^2(a + b \arctanh(cx))/c^2 - 3/2b^2d^2e^2(a + b \arctanh(cx))^2/c^3 + 1/4b^3e^3(a + b \arctanh(cx))^2/c^4 + 3/4b^3e^3(6c^2d^2 + e^2)(a + b \arctanh(cx))^2/c^4 + 3/4b^3e^3(6c^2d^2 + e^2)x(a + b \arctanh(cx))^2/c^3 + 3/2b^2d^2e^2x^2(a + b \arctanh(cx))^2/c^4 + 1/4b^3e^3x^3(a + b \arctanh(cx))^2/c^4 + d(c^2d^2 + e^2)(a + b \arctanh(cx))^3/c^3 - 1/4(c^4d^4 + 6c^2d^2e^2 + e^4)(a + b \arctanh(cx))^3/c^4 + 1/4(e^2x + d)(a + b \arctanh(cx))^3/e - 1/2b^2e^3(a + b \arctanh(cx)) \ln(2/(-cx + 1))/c^4 - 3/2b^2e^3(6c^2d^2 + e^2)(a + b \arctanh(cx)) \ln(2/(-cx + 1))/c^4 - 3b^2d^2e^2(c^2d^2 + e^2)(a + b \arctanh(cx))^2 \ln(2/(-cx + 1))/c^3 + 3/2b^3d^2e^2 \ln(-c^2x^2 + 1)/c^3 - 1/4b^3e^3 \text{polylog}(2, 1 - 2/(-cx + 1))/c^4 - 3/4b^3e^3(6c^2d^2 + e^2) \text{polylog}(2, 1 - 2/(-cx + 1))/c^4 - 3b^2d^2e^2(c^2d^2 + e^2)(a + b \arctanh(cx)) \text{polylog}(2, 1 - 2/(-cx + 1))/c^3 + 3/2b^3d^2e^2(c^2d^2 + e^2) \text{polylog}(3, 1 - 2/(-cx + 1))/c^3$

Rubi [A]

time = 0.96, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 15, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6065, 6021, 6131, 6055, 2449, 2352, 6037, 6127, 266, 6095, 327, 212, 6195, 6205, 6745}

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3*(a + b*ArcTanh[c*x])^3,x]

[Out] $(3a^3b^2d^2e^2x)/c^2 + (b^3e^3x)/(4c^3) - (b^3e^3 \text{ArcTanh}[c*x])/(4c^4) + (3b^3d^2e^2x \text{ArcTanh}[c*x])/c^2 + (b^2e^3x^2(a + b \text{ArcTanh}[c*x]))/(4c^2) - (3b^2d^2e^2(a + b \text{ArcTanh}[c*x])^2)/(2c^3) + (b^2e^3(a + b \text{ArcTanh}[c*x])^2)/(4c^4) + (3b^2e^3(6c^2d^2 + e^2)(a + b \text{ArcTanh}[c*x])^2)/(4c^4) + (3b^2e^3(6c^2d^2 + e^2)x(a + b \text{ArcTanh}[c*x])^2)/(4c^3) + (3b^2d^2e^2x^2(a + b \text{ArcTanh}[c*x])^2)/(2c) + (b^2e^3x^3(a + b \text{ArcTanh}[c*x])^2)/(4c) + (d(c^2d^2 + e^2)(a + b \text{ArcTanh}[c*x])^3)/c^3 - ((c^4d^4 + 6c^2d^2e^2 + e^4)(a + b \text{ArcTanh}[c*x])^3)/(4c^4e) + ((d + e*x)^4(a + b \text{ArcTanh}[c*x])^3)/(4e) - (b^2e^3(a + b \text{ArcTanh}[c*x]) \text{Log}[2/(1 - c*x)])/(2c^4) - (3b^2e^3(6c^2d^2 + e^2)(a + b \text{ArcTanh}[c*x]) \text{Log}[2/(1 - c*x)])/(2c^4) - (3b^2d^2e^2(c^2d^2 + e^2)(a + b \text{ArcTanh}[c*x])^2 \text{Log}[2/(1 - c*x)])/c^3 + (3b^3d^2e^2 \text{Log}[1 - c^2x^2])/(2c^3) - (b^3e^3 \text{PolyLog}[2, 1 - 2/(1 - c*x)])/(4c^4) - (3b^3e^3(6c^2d^2 + e^2) \text{PolyLog}[2, 1 - 2/(1 - c*x)])/(4c^4)$

$$- (3*b^2*d*(c^2*d^2 + e^2)*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] / c^3 + (3*b^3*d*(c^2*d^2 + e^2)*PolyLog[3, 1 - 2/(1 - c*x)]) / (2*c^3)$$

Rule 212

$$\text{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))* \text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 266

$$\text{Int}[(x^m)/((a + b*x^n)^n), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}\{a, b, m, n, x\} \ \&\& \ \text{EqQ}[m, n - 1]$$

Rule 327

$$\text{Int}[(c*x^m)*((a + b*x^n)^p), x_Symbol] \rightarrow \text{Simp}[c^{(n-1)}*(c*x)^{(m-n+1)}*((a + b*x^n)^{(p+1)}/(b*(m+n*p+1))), x] - \text{Dist}[a*c^n*((m-n+1)/(b*(m+n*p+1))), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$$

Rule 2352

$$\text{Int}[\text{Log}[(c*x)/(d + e*x)], x_Symbol] \rightarrow \text{Simp}[(-e^{-1})*\text{PolyLog}[2, 1 - c*x], x] /; \text{FreeQ}\{c, d, e, x\} \ \&\& \ \text{EqQ}[e + c*d, 0]$$

Rule 2449

$$\text{Int}[\text{Log}[(c)/(d + e*x)]/((f + g*x^2), x_Symbol] \rightarrow \text{Dist}[-e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g, x\} \ \&\& \ \text{EqQ}[c, 2*d] \ \&\& \ \text{EqQ}[e^2*f + d^2*g, 0]$$

Rule 6021

$$\text{Int}[(a + \text{ArcTanh}[c*x^n])^p, x_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcTanh}[c*x^n])^p, x] - \text{Dist}[b*c*n*p, \text{Int}[x^n*((a + b*\text{ArcTanh}[c*x^n])^{(p-1)})/(1 - c^2*x^{(2*n)}), x], x] /; \text{FreeQ}\{a, b, c, n, x\} \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{EqQ}[n, 1] \ || \ \text{EqQ}[p, 1])$$

Rule 6037

$$\text{Int}[(a + \text{ArcTanh}[c*x^n])^p*(x^m), x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}*((a + b*\text{ArcTanh}[c*x^n])^p/(m+1)), x] - \text{Dist}[b*c*n*(p/(m+1)), \text{Int}[x^{(m+n)}*((a + b*\text{ArcTanh}[c*x^n])^{(p-1)})/(1 - c^2*x^{(2*n)}), x], x] /; \text{FreeQ}\{a, b, c, m, n, x\} \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{EqQ}[p, 1] \ || \ (\text{EqQ}[n, 1] \ \&\& \ \text{IntegerQ}[m])) \ \&\& \ \text{NeQ}[m, -1]$$

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_.)), x_Symbol]
:= Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_.))^(q_.), x_S
ymbol] := Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_)^(m_)))/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_) + (g_.)*(x_)^(m_.)))/
((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int (d + ex)^3 (a + b \tanh^{-1}(cx))^3 dx &= \frac{(d + ex)^4 (a + b \tanh^{-1}(cx))^3}{4e} - \frac{(3bc) \int \left(-\frac{e^2(6c^2d^2 + e^2)(a + b \tanh^{-1}(cx))^2}{c^4} \right)}{4c^3e} \\
&= \frac{(d + ex)^4 (a + b \tanh^{-1}(cx))^3}{4e} - \frac{(3b) \int \frac{(c^4d^4 + 6c^2d^2e^2 + e^4 + 4c^2de(c^2d^2 + e^2)x)}{1 - c^2x^2}}{4c^3e} \\
&= \frac{3be(6c^2d^2 + e^2)x(a + b \tanh^{-1}(cx))^2}{4c^3} + \frac{3bde^2x^2(a + b \tanh^{-1}(cx))^2}{2c} \\
&= \frac{3be(6c^2d^2 + e^2)(a + b \tanh^{-1}(cx))^2}{4c^4} + \frac{3be(6c^2d^2 + e^2)x(a + b \tanh^{-1}(cx))}{4c^3} \\
&= \frac{3ab^2de^2x}{c^2} + \frac{b^2e^3x^2(a + b \tanh^{-1}(cx))}{4c^2} - \frac{3bde^2(a + b \tanh^{-1}(cx))^2}{2c^3} + \\
&= \frac{3ab^2de^2x}{c^2} + \frac{b^3e^3x}{4c^3} + \frac{3b^3de^2x \tanh^{-1}(cx)}{c^2} + \frac{b^2e^3x^2(a + b \tanh^{-1}(cx))}{4c^2} \\
&= \frac{3ab^2de^2x}{c^2} + \frac{b^3e^3x}{4c^3} - \frac{b^3e^3 \tanh^{-1}(cx)}{4c^4} + \frac{3b^3de^2x \tanh^{-1}(cx)}{c^2} + \frac{b^2e^3x^2}{c^2} \\
&= \frac{3ab^2de^2x}{c^2} + \frac{b^3e^3x}{4c^3} - \frac{b^3e^3 \tanh^{-1}(cx)}{4c^4} + \frac{3b^3de^2x \tanh^{-1}(cx)}{c^2} + \frac{b^2e^3x^2}{c^2}
\end{aligned}$$

Mathematica [A]

time = 1.40, size = 830, normalized size = 1.35

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3*(a + b*ArcTanh[c*x])^3,x]

[Out] (2*a^2*c*(4*a*c^3*d^3 + 3*b*e*(6*c^2*d^2 + e^2))*x + 12*a^2*c^3*d*e*(a*c*d + b*e)*x^2 + 2*a^2*c^3*e^2*(4*a*c*d + b*e)*x^3 + 2*a^3*c^4*e^3*x^4 + 6*a^2*b*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcTanh[c*x] + 3*a^2*b*(4*c^3*d^3 + 6*c^2*d^2*e + 4*c*d*e^2 + e^3)*Log[1 - c*x] + 3*a^2*b*(4*c^3*d^3 - 6*c^2*d^2*e + 4*c*d*e^2 - e^3)*Log[1 + c*x] + 36*a*b^2*c^2*d^2*e*(2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + Log[1 - c^2*x^2]) + 2*a*b^2*e^3*(-1 + c^2*x^2 + 2*c*x*(3 + c^2*x^2)*ArcTanh[c*x] + 3*(-1 + c^4*x^4)*ArcTanh[c*x]^2 + 4*Log[1 - c^2*x^2]) - 12*b^3*c^2*d^2*e*(ArcTanh[c*x]*((3 - 3*c*x)*ArcTanh[c*x] + (1 - c^2*x^2)*ArcTanh[c*x]^2 + 6*Log[1 + E^(-2*ArcTanh[c*x])])) - 3*PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 24*a*b^2*c*d*e^2*(c*x + (-1 + c^3*x^3)*ArcTanh[c*x]^2 + ArcTanh[c*x]*(-1 + c^2*x^2 - 2*Log[1 + E^(-2*ArcTanh[c*x])])) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 24*a*b^2*c^3*d^3*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])])) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 2*b^3*e^3*(c*x + (-4 + 3*c*x + c^3*x^3)*ArcTanh[c*x]^2 + (-1 + c^4*x^4)*ArcTanh[c*x]^3 + ArcTanh[c*x]*(-1 + c^2*x^2 - 8*Log[1 + E^(-2*ArcTanh[c*x])])) + 4*PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 8*b^3*c^3*d^3*(ArcTanh[c*x]^2*((-1 + c*x)*ArcTanh[c*x] - 3*Log[1 + E^(-2*ArcTanh[c*x])])) + 3*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])]) + (3*PolyLog[3, -E^(-2*ArcTanh[c*x])])/2 + 4*b^3*c*d*e^2*(6*c*x*ArcTanh[c*x] - 3*ArcTanh[c*x]^2 + 3*c^2*x^2*ArcTanh[c*x]^2 - 2*ArcTanh[c*x]^3 + 2*c^3*x^3*ArcTanh[c*x]^3 - 6*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])]) + 3*Log[1 - c^2*x^2] + 6*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 3*PolyLog[3, -E^(-2*ArcTanh[c*x])]))/(8*c^4)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 27.53, size = 6004, normalized size = 9.78

method	result	size
derivativedivides	Expression too large to display	6004
default	Expression too large to display	6004

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)

[Out] result too large to display

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^3,x, algorithm="maxima")

```
[Out] 1/4*a^3*x^4*e^3 + a^3*d*x^3*e^2 + 3/2*a^3*d^2*x^2*e + a^3*d^3*x + 9/4*(2*x^
2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a^2*b*d
^2*e + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a^2*b*d^3/c + 3/2*(2*x^
3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a^2*b*d*e^2 + 1/8*(6*x
^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x
- 1)/c^5))*a^2*b*e^3 - 1/32*((b^3*c^4*x^4*e^3 + 4*b^3*c^4*d*x^3*e^2 + 6*b^
3*c^4*d^2*x^2*e + 4*b^3*c^4*d^3*x - 4*b^3*c^3*d^3 - 6*b^3*c^2*d^2*e - 4*b^3
*c*d*e^2 - b^3*e^3)*log(-c*x + 1)^3 - (6*a*b^2*c^4*x^4*e^3 + 2*(12*a*b^2*c^
4*d*e^2 + b^3*c^3*e^3)*x^3 + 12*(3*a*b^2*c^4*d^2*e + b^3*c^3*d*e^2)*x^2 + 6
*(4*a*b^2*c^4*d^3 + 6*b^3*c^3*d^2*e + b^3*c^3*e^3)*x + 3*(b^3*c^4*x^4*e^3 + 4
*b^3*c^4*d*x^3*e^2 + 6*b^3*c^4*d^2*x^2*e + 4*b^3*c^4*d^3*x + 4*b^3*c^3*d^3
- 6*b^3*c^2*d^2*e + 4*b^3*c*d*e^2 - b^3*e^3)*log(c*x + 1))*log(-c*x + 1)^2
/c^4 - integrate(-1/16*(2*(b^3*c^4*x^4*e^3 - b^3*c^3*d^3 + (3*b^3*c^4*d*e^2
- b^3*c^3*e^3)*x^3 + 3*(b^3*c^4*d^2*e - b^3*c^3*d*e^2)*x^2 + (b^3*c^4*d^3
- 3*b^3*c^3*d^2*e)*x)*log(c*x + 1)^3 + 12*(a*b^2*c^4*x^4*e^3 - a*b^2*c^3*d^
3 + (3*a*b^2*c^4*d*e^2 - a*b^2*c^3*e^3)*x^3 + 3*(a*b^2*c^4*d^2*e - a*b^2*c^
3*d*e^2)*x^2 + (a*b^2*c^4*d^3 - 3*a*b^2*c^3*d^2*e)*x)*log(c*x + 1)^2 - (6*a
*b^2*c^4*x^4*e^3 + 2*(12*a*b^2*c^4*d*e^2 + b^3*c^3*e^3)*x^3 + 12*(3*a*b^2*c^
4*d^2*e + b^3*c^3*d*e^2)*x^2 + 6*(b^3*c^4*x^4*e^3 - b^3*c^3*d^3 + (3*b^3*c^
4*d*e^2 - b^3*c^3*e^3)*x^3 + 3*(b^3*c^4*d^2*e - b^3*c^3*d*e^2)*x^2 + (b^3*
c^4*d^3 - 3*b^3*c^3*d^2*e)*x)*log(c*x + 1)^2 + 6*(4*a*b^2*c^4*d^3 + 6*b^3*c^
^3*d^2*e + b^3*c^3*e^3)*x - 3*(8*a*b^2*c^3*d^3 - 4*b^3*c^3*d^3 + 6*b^3*c^2*d^
2*e - (8*a*b^2*c^4 + b^3*c^4)*x^4*e^3 - 4*b^3*c*d*e^2 + 4*(2*a*b^2*c^3*e^3
- (6*a*b^2*c^4*d + b^3*c^4*d)*e^2)*x^3 + b^3*e^3 + 6*(4*a*b^2*c^3*d*e^2 - (
4*a*b^2*c^4*d^2 + b^3*c^4*d^2)*e)*x^2 - 4*(2*a*b^2*c^4*d^3 + b^3*c^4*d^3 -
6*a*b^2*c^3*d^2*e)*x)*log(c*x + 1))*log(-c*x + 1))/(c^4*x - c^3), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*x^3*e^3 + 3*a^3*d*x^2*e^2 + 3*a^3*d^2*x*e + a^3*d^3 + (b^3*x^3
*e^3 + 3*b^3*d*x^2*e^2 + 3*b^3*d^2*x*e + b^3*d^3)*arctanh(c*x)^3 + 3*(a*b^2
*x^3*e^3 + 3*a*b^2*d*x^2*e^2 + 3*a*b^2*d^2*x*e + a*b^2*d^3)*arctanh(c*x)^2
+ 3*(a^2*b*x^3*e^3 + 3*a^2*b*d*x^2*e^2 + 3*a^2*b*d^2*x*e + a^2*b*d^3)*arcta
nh(c*x), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^3 (d + ex)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**3*(a+b*atanh(c*x))**3,x)

[Out] Integral((a + b*atanh(c*x))**3*(d + e*x)**3, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^3,x, algorithm="giac")

[Out] integrate((e*x + d)^3*(b*arctanh(c*x) + a)^3, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{atanh}(cx))^3 (d + ex)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^3*(d + e*x)^3,x)

[Out] int((a + b*atanh(c*x))^3*(d + e*x)^3, x)

3.16 $\int (d + ex)^2 (a + b \tanh^{-1}(cx))^3 dx$

Optimal. Leaf size=387

$$\frac{ab^2e^2x}{c^2} + \frac{b^3e^2x \tanh^{-1}(cx)}{c^2} + \frac{3bde(a + b \tanh^{-1}(cx))^2}{c^2} - \frac{be^2(a + b \tanh^{-1}(cx))^2}{2c^3} + \frac{3bdex(a + b \tanh^{-1}(cx))^2}{c}$$

```
[Out] a*b^2*e^2*x/c^2+b^3*e^2*x*arctanh(c*x)/c^2+3*b*d*e*(a+b*arctanh(c*x))^2/c^2
-1/2*b*e^2*(a+b*arctanh(c*x))^2/c^3+3*b*d*e*x*(a+b*arctanh(c*x))^2/c+1/2*b*
e^2*x^2*(a+b*arctanh(c*x))^2/c+1/3*(3*c^2*d^2+e^2)*(a+b*arctanh(c*x))^3/c^3
-1/3*d*(d^2+3*e^2/c^2)*(a+b*arctanh(c*x))^3/e+1/3*(e*x+d)^3*(a+b*arctanh(c*
x))^3/e-6*b^2*d*e*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^2-b*(3*c^2*d^2+e^2)*(
a+b*arctanh(c*x))^2*ln(2/(-c*x+1))/c^3+1/2*b^3*e^2*ln(-c^2*x^2+1)/c^3-3*b^3
*d*e*polylog(2,1-2/(-c*x+1))/c^2-b^2*(3*c^2*d^2+e^2)*(a+b*arctanh(c*x))*pol
ylog(2,1-2/(-c*x+1))/c^3+1/2*b^3*(3*c^2*d^2+e^2)*polylog(3,1-2/(-c*x+1))/c^
3
```

Rubi [A]

time = 0.55, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6065, 6021, 6131, 6055, 2449, 2352, 6037, 6127, 266, 6095, 6195, 6205, 6745}

$\frac{6065 \log\left(\frac{d+ex}{c}\right) (a+b \tanh^{-1}(cx))}{c^2} - \frac{b^3 e^2 x \operatorname{arctanh}(cx)}{c^2} + \frac{3 b d e (a+b \tanh^{-1}(cx))^2}{c^2} - \frac{b e^2 (a+b \tanh^{-1}(cx))^2}{2 c^3} + \frac{3 b d e x (a+b \tanh^{-1}(cx))^2}{c}$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2*(a + b*ArcTanh[c*x])^3,x]

```
[Out] (a*b^2*e^2*x)/c^2 + (b^3*e^2*x*ArcTanh[c*x])/c^2 + (3*b*d*e*(a + b*ArcTanh[
c*x])^2)/c^2 - (b*e^2*(a + b*ArcTanh[c*x])^2)/(2*c^3) + (3*b*d*e*x*(a + b*A
rcTanh[c*x])^2)/c + (b*e^2*x^2*(a + b*ArcTanh[c*x])^2)/(2*c) + ((3*c^2*d^2
+ e^2)*(a + b*ArcTanh[c*x])^3)/(3*c^3) - (d*(d^2 + (3*e^2)/c^2)*(a + b*ArcT
anh[c*x])^3)/(3*e) + ((d + e*x)^3*(a + b*ArcTanh[c*x])^3)/(3*e) - (6*b^2*d*
e*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)]/c^2 - (b*(3*c^2*d^2 + e^2)*(a + b*
ArcTanh[c*x])^2*Log[2/(1 - c*x)]/c^3 + (b^3*e^2*Log[1 - c^2*x^2])/(2*c^3)
- (3*b^3*d*e*PolyLog[2, 1 - 2/(1 - c*x)]/c^2 - (b^2*(3*c^2*d^2 + e^2)*(a +
b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)]/c^3 + (b^3*(3*c^2*d^2 + e^2)*
PolyLog[3, 1 - 2/(1 - c*x)]/(2*c^3)
```

Rule 266

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 2352

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2449

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 6021

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && (EqQ[n, 1] || EqQ[p, 1])

Rule 6037

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m + 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]

Rule 6055

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-a + b*ArcTanh[c*x])^p*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 6065

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.), x_Symbol] := Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] - Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 6095

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((f_) + (g_.)*(x_))^(m_))/((
d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6205

```
Int[(Log[u]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^2 (a+b \tanh^{-1}(cx))^3 dx &= \frac{(d+ex)^3 (a+b \tanh^{-1}(cx))^3}{3e} - \frac{(bc) \int \left(-\frac{3de^2(a+b \tanh^{-1}(cx))^2}{c^2} - \frac{e^3x(a+b \tanh^{-1}(cx))}{c} \right) dx}{ce} \\
&= \frac{(d+ex)^3 (a+b \tanh^{-1}(cx))^3}{3e} - \frac{b \int \frac{(c^2d^3+3de^2+e(3c^2d^2+e^2)x)(a+b \tanh^{-1}(cx))}{1-c^2x^2} dx}{ce} \\
&= \frac{3bdex(a+b \tanh^{-1}(cx))^2}{c} + \frac{be^2x^2(a+b \tanh^{-1}(cx))^2}{2c} + \frac{(d+ex)^3 (a+b \tanh^{-1}(cx))^3}{3e} \\
&= \frac{3bde(a+b \tanh^{-1}(cx))^2}{c^2} + \frac{3bdex(a+b \tanh^{-1}(cx))^2}{c} + \frac{be^2x^2(a+b \tanh^{-1}(cx))^2}{2c} \\
&= \frac{ab^2e^2x}{c^2} + \frac{3bde(a+b \tanh^{-1}(cx))^2}{c^2} - \frac{be^2(a+b \tanh^{-1}(cx))^2}{2c^3} + \frac{3bdex}{c} \\
&= \frac{ab^2e^2x}{c^2} + \frac{b^3e^2x \tanh^{-1}(cx)}{c^2} + \frac{3bde(a+b \tanh^{-1}(cx))^2}{c^2} - \frac{be^2(a+b \tanh^{-1}(cx))^2}{2c} \\
&= \frac{ab^2e^2x}{c^2} + \frac{b^3e^2x \tanh^{-1}(cx)}{c^2} + \frac{3bde(a+b \tanh^{-1}(cx))^2}{c^2} - \frac{be^2(a+b \tanh^{-1}(cx))^2}{2c} \\
&= \frac{ab^2e^2x}{c^2} + \frac{b^3e^2x \tanh^{-1}(cx)}{c^2} + \frac{3bde(a+b \tanh^{-1}(cx))^2}{c^2} - \frac{be^2(a+b \tanh^{-1}(cx))^2}{2c}
\end{aligned}$$

Mathematica [A]

time = 1.28, size = 591, normalized size = 1.53

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2*(a + b*ArcTanh[c*x])^3,x]

[Out] (6*a^2*c^2*d*(a*c*d + 3*b*e)*x + 3*a^2*c^2*e*(2*a*c*d + b*e)*x^2 + 2*a^3*c^3*e^2*x^3 + 6*a^2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcTanh[c*x] + 3*a^2*b*(3*c^2*d^2 + 3*c*d*e + e^2)*Log[1 - c*x] + 3*a^2*b*(3*c^2*d^2 - 3*c*d*e + e^2)*Log[1 + c*x] + 18*a*b^2*c*d*e*(2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + Log[1 - c^2*x^2]) - 6*b^3*c*d*e*(ArcTanh[c*x]*((3 - 3*c*x)*ArcTanh[c*x] + (1 - c^2*x^2)*ArcTanh[c*x]^2 + 6*Log[1 + E^(-2*ArcTanh[c*x])]) - 3*PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 6*a*b^2*e^2*(c*x + (-1 + c^3*x^3)*ArcTanh[c*x]^2 + ArcTanh[c*x]*(-1 + c^2*x^2 - 2*Log[1 + E^(-2*ArcTanh[c*x])])) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 18*a*b^2*c^2*d^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 6*b^3*c^2*d^2*(ArcTanh[c*x]^2*(-1 + c*x)*ArcTanh[c*x]

$$- 3*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c*x])}] + 3*\text{ArcTanh}[c*x]*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c*x])}] + (3*\text{PolyLog}[3, -E^{(-2*\text{ArcTanh}[c*x])}])/2 + b^3*e^{2*(6*c*x*\text{ArcTanh}[c*x] - 3*\text{ArcTanh}[c*x]^2 + 3*c^2*x^2*\text{ArcTanh}[c*x]^2 - 2*\text{ArcTanh}[c*x]^3 + 2*c^3*x^3*\text{ArcTanh}[c*x]^3 - 6*\text{ArcTanh}[c*x]^2*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c*x])}] + 3*\text{Log}[1 - c^2*x^2] + 6*\text{ArcTanh}[c*x]*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c*x])}] + 3*\text{PolyLog}[3, -E^{(-2*\text{ArcTanh}[c*x])}]))/(6*c^3)$$

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 16.24, size = 4520, normalized size = 11.68

method	result	size
derivativedivides	Expression too large to display	4520
default	Expression too large to display	4520

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{c} \left(3a^2b^3e^{2d}x + a^2b^3e^{2d}x^2 + a^2b^3e^{2d}x^3 + 3a^2b^3e^{2d}x^4 + 3a^2b^3e^{2d}x^5 + 3a^2b^3e^{2d}x^6 + 3a^2b^3e^{2d}x^7 + 3a^2b^3e^{2d}x^8 + 3a^2b^3e^{2d}x^9 + 3a^2b^3e^{2d}x^{10} + 3a^2b^3e^{2d}x^{11} + 3a^2b^3e^{2d}x^{12} + 3a^2b^3e^{2d}x^{13} + 3a^2b^3e^{2d}x^{14} + 3a^2b^3e^{2d}x^{15} + 3a^2b^3e^{2d}x^{16} + 3a^2b^3e^{2d}x^{17} + 3a^2b^3e^{2d}x^{18} + 3a^2b^3e^{2d}x^{19} + 3a^2b^3e^{2d}x^{20} + 3a^2b^3e^{2d}x^{21} + 3a^2b^3e^{2d}x^{22} + 3a^2b^3e^{2d}x^{23} + 3a^2b^3e^{2d}x^{24} + 3a^2b^3e^{2d}x^{25} + 3a^2b^3e^{2d}x^{26} + 3a^2b^3e^{2d}x^{27} + 3a^2b^3e^{2d}x^{28} + 3a^2b^3e^{2d}x^{29} + 3a^2b^3e^{2d}x^{30} + 3a^2b^3e^{2d}x^{31} + 3a^2b^3e^{2d}x^{32} + 3a^2b^3e^{2d}x^{33} + 3a^2b^3e^{2d}x^{34} + 3a^2b^3e^{2d}x^{35} + 3a^2b^3e^{2d}x^{36} + 3a^2b^3e^{2d}x^{37} + 3a^2b^3e^{2d}x^{38} + 3a^2b^3e^{2d}x^{39} + 3a^2b^3e^{2d}x^{40} + 3a^2b^3e^{2d}x^{41} + 3a^2b^3e^{2d}x^{42} + 3a^2b^3e^{2d}x^{43} + 3a^2b^3e^{2d}x^{44} + 3a^2b^3e^{2d}x^{45} + 3a^2b^3e^{2d}x^{46} + 3a^2b^3e^{2d}x^{47} + 3a^2b^3e^{2d}x^{48} + 3a^2b^3e^{2d}x^{49} + 3a^2b^3e^{2d}x^{50} + 3a^2b^3e^{2d}x^{51} + 3a^2b^3e^{2d}x^{52} + 3a^2b^3e^{2d}x^{53} + 3a^2b^3e^{2d}x^{54} + 3a^2b^3e^{2d}x^{55} + 3a^2b^3e^{2d}x^{56} + 3a^2b^3e^{2d}x^{57} + 3a^2b^3e^{2d}x^{58} + 3a^2b^3e^{2d}x^{59} + 3a^2b^3e^{2d}x^{60} + 3a^2b^3e^{2d}x^{61} + 3a^2b^3e^{2d}x^{62} + 3a^2b^3e^{2d}x^{63} + 3a^2b^3e^{2d}x^{64} + 3a^2b^3e^{2d}x^{65} + 3a^2b^3e^{2d}x^{66} + 3a^2b^3e^{2d}x^{67} + 3a^2b^3e^{2d}x^{68} + 3a^2b^3e^{2d}x^{69} + 3a^2b^3e^{2d}x^{70} + 3a^2b^3e^{2d}x^{71} + 3a^2b^3e^{2d}x^{72} + 3a^2b^3e^{2d}x^{73} + 3a^2b^3e^{2d}x^{74} + 3a^2b^3e^{2d}x^{75} + 3a^2b^3e^{2d}x^{76} + 3a^2b^3e^{2d}x^{77} + 3a^2b^3e^{2d}x^{78} + 3a^2b^3e^{2d}x^{79} + 3a^2b^3e^{2d}x^{80} + 3a^2b^3e^{2d}x^{81} + 3a^2b^3e^{2d}x^{82} + 3a^2b^3e^{2d}x^{83} + 3a^2b^3e^{2d}x^{84} + 3a^2b^3e^{2d}x^{85} + 3a^2b^3e^{2d}x^{86} + 3a^2b^3e^{2d}x^{87} + 3a^2b^3e^{2d}x^{88} + 3a^2b^3e^{2d}x^{89} + 3a^2b^3e^{2d}x^{90} + 3a^2b^3e^{2d}x^{91} + 3a^2b^3e^{2d}x^{92} + 3a^2b^3e^{2d}x^{93} + 3a^2b^3e^{2d}x^{94} + 3a^2b^3e^{2d}x^{95} + 3a^2b^3e^{2d}x^{96} + 3a^2b^3e^{2d}x^{97} + 3a^2b^3e^{2d}x^{98} + 3a^2b^3e^{2d}x^{99} + 3a^2b^3e^{2d}x^{100} \right) \cdot \text{arctanh}(c*x)^3$$

$$\begin{aligned}
& 2*x^2+1)^{(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^{2+3/4}*I*b^3*arctanh(c*x)^2*Pi \\
& *d^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(\\
& I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^{-3}*a*b^2/c*e*arctanh(c*x \\
&)*\ln(c*x+1)*d+3/2*a*b^2/c*e*\ln(-1/2*c*x+1/2)*\ln(1/2*c*x+1/2)*d-1/2*a*b^2*c/ \\
& e*\ln(c*x-1)*\ln(1/2*c*x+1/2)*d^3-1/2*a*b^2*c/e*\ln(c*x+1)*\ln(-1/2*c*x+1/2)*d^ \\
& 3+1/2*a*b^2*c/e*\ln(-1/2*c*x+1/2)*\ln(1/2*c*x+1/2)*d^3+a*b^2*c/e*arctanh(c*x) \\
& *\ln(c*x-1)*d^3-a*b^2*c/e*arctanh(c*x)*\ln(c*x+1)*d^3-3/2*a*b^2/c*e*\ln(c*x-1) \\
& *\ln(1/2*c*x+1/2)*d-3/2*a*b^2/c*e*\ln(c*x+1)*\ln(-1/2*c*x+1/2)*d+3*a*b^2/c*e*a \\
& rctanh(c*x)*\ln(c*x-1)*d+1/4*I*b^3*c/e*Pi*d^3*csgn(I*(c*x+1)/(-c^2*x^2+1))^{(1 \\
& /2))^{2}*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2+1/4*I*b^3*c/e*arctanh(c \\
& *x)^2*Pi*d^3*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1) \\
&)/(1+(c*x+1)^2/(-c^2*x^2+1)))^{2+3/4}*I*b^3/c*e*arctanh(c*x)^2*Pi*d*csgn(I/(1 \\
& +(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x \\
& ^2+1)))^{2+3/4}*I*b^3/c*e*arctanh(c*x)^2*Pi*d*csgn(I*(c*x+1)/(-c^2*x^2+1))^{(1/ \\
& 2))^{2}*csgn(I*(c*x+1)^2/(c^2*x^2-1))-1/4*I*b^3*c/e*arctanh(c*x)^2*Pi*d^3*csg \\
& n(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2* \\
& x^2+1)))^{2-3/4}*I*b^3/c*e*arctanh(c*x)^2*Pi*d*csgn(I*(c*x+1)^2/(c^2*x^2-1))* \\
& csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^{2+3/2}*I*b^3/c*e*ar \\
& ctanh(c*x)^2*Pi*d*csgn(I*(c*x+1)/(-c^2*x^2+1))^{(1/2))*csgn(I*(c*x+1)^2/(c^2* \\
& x^2-1))^{2+1/2}*I*b^3*c/e*arctanh(c*x)^2*Pi*d^3*csgn(I*(c*x+1)/(-c^2*x^2+1))^{(\\
& 1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^{2+1/4}*I*b^3/c^2*e^2*arctanh(c*x)^2*Pi*c \\
& sgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x \\
& +1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+1/4*a*b^2*c/e*\ln(c*x+1)^2*d^3 \\
& +3/4*a*b^2/c*e*\ln(c*x+1)^2*d+3/4*a*b^2/c*e*\ln(c*x-1)^2*d+3*a*b^2/c*e*\ln(c*x \\
& -1)*d+3*a*b^2/c*e*\ln(c*x+1)*d+a*b^2*c/e*arctanh(c*x)^2*d^3-1/2*a*b^2/c^2*e^ \\
& 2*\ln(c*x-1)*\ln(1/2*c*x+1/2)+1/2*a*b^2/c^2*e^2*\ln(c*x+1)*\ln(-1/2*c*x+1/2)-1/ \\
& 2*a*b^2/c^2*e^2*\ln(-1/2*c*x+1/2)*\ln(1/2*c*x+1/2)+a*b^2/c^2*e^2*arctanh(c*x) \\
& *\ln(c*x-1)+a*b^2/c^2*e^2*arctanh(c*x)*\ln(c*x+1)+1/4*a*b^2*c/e*\ln(c*x-1)^2*d \\
& ^3+3*b^3/c*e*\ln((c*x+1)/(-c^2*x^2+1))^{(1/2))*d*arctanh(c*x)^2-6*b^3/c*e*d*ar \\
& ctanh(c*x)*\ln(1+I*(c*x+1)/(-c^2*x^2+1))^{(1/2))-1/2*b^3*c/e*arctanh(c*x)^2*\ln \\
& (c*x+1)*d^3+1/2*b^3*c/e*arctanh(c*x)^2*\ln(c*x-1)*d^3+b^3*c/e*\ln((c*x+1)/(-c \\
& ^2*x^2+1))^{(1/2))*d^3*arctanh(c*x)^2+3/2*b^3/c*e*arctanh(c*x)^2*\ln(c*x-1)*d- \\
& 3/2*b^3/c*e*arctanh(c*x)^2*\ln(c*x+1)*d-1/2*I*b^3/c^2*e^2*arctanh(c*x)^2*Pi- \\
& 3/2*I*b^3*arctanh(c*x)^2*Pi*d^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^{3+3/2}*I* \\
& b^3*arctanh(c*x)^2*Pi*d^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^{2-3/4}*I*b^3*ar \\
& ctanh(c*x)^2*Pi*d^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^{3-3/4}*I*b^3*arctanh(c*x)^ \\
& 2*Pi*d^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^{3-6*b^3/c \\
& *e*d*arctanh(c*x)*\ln(1-I*(c*x+1)/(-c^2*x^2+1))^{(1/2))+a^2*b*c/e*arctanh(c*x) \\
& *d^3+1/2*a^2*b*c/e*\ln(c*x-1)*d^3+3/2*a^2*b*c/e*...
\end{aligned}$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^3,x, algorithm="maxima")

[Out] $\frac{1}{3}a^3x^3e^2 + a^3d^2x^2e + a^3d^2x + \frac{3}{2}(2x^2\operatorname{arctanh}(cx) + c(2x/c^2 - \log(cx+1)/c^3 + \log(cx-1)/c^3))a^2bde + \frac{3}{2}(2cx\operatorname{arctanh}(cx) + \log(-c^2x^2+1))a^2bd^2/c + \frac{1}{2}(2x^3\operatorname{arctanh}(cx) + c(x^2/c^2 + \log(c^2x^2-1)/c^4))a^2be^2 - \frac{1}{24}((b^3c^3x^3e^2 + 3b^3c^3dx^2e + 3b^3c^3d^2x - 3b^3c^2d^2 - 3b^3cde - b^3e^2)\log(-cx+1)^3 - 3(2ab^2c^3x^3e^2 + (6ab^2c^3de + b^3c^2e^2)x^2 + 6(ab^2c^3d^2 + b^3c^2de)x + (b^3c^3x^3e^2 + 3b^3c^3dx^2e + 3b^3c^3d^2x + 3b^3c^2d^2 - 3b^3cde + b^3e^2)\log(cx+1))\log(-cx+1)^2)/c^3 - \int (-1/8((b^3c^3x^3e^2 - b^3c^2d^2 + (2b^3c^3de - b^3c^2e^2)x^2 + (b^3c^3d^2 - 2b^3c^2de)x)\log(cx+1)^3 + 6(ab^2c^3x^3e^2 - ab^2c^2d^2 + (2ab^2c^3de - ab^2c^2e^2)x^2 + (ab^2c^3d^2 - 2ab^2c^2de)x)\log(cx+1)^2 - (4ab^2c^3x^3e^2 + 2(6ab^2c^3de + b^3c^2e^2)x^2 + 3(b^3c^3x^3e^2 - b^3c^2d^2 + (2b^3c^3de - b^3c^2e^2)x^2 + (b^3c^3d^2 - 2b^3c^2de)x)\log(cx+1)^2 + 12(ab^2c^3d^2 + b^3c^2de)x - 2(6ab^2c^2d^2 - 3b^3c^2d^2 + 3b^3cde - (6ab^2c^3 + b^3c^3)x^3e^2 - b^3e^2 + 3(2ab^2c^2e^2 - (4ab^2c^3d + b^3c^3d)e)x^2 - 3(2ab^2c^3d^2 + b^3c^3d^2 - 4ab^2c^2de)x)\log(cx+1))\log(-cx+1))/(c^3x - c^2), x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^3,x, algorithm="fricas")

[Out] $\int (a^3x^2e^2 + 2a^3dx^2e + a^3d^2 + (b^3x^2e^2 + 2b^3dx^2e + b^3d^2)\operatorname{arctanh}(cx))^3 + 3(a^2bx^2e^2 + 2a^2bd^2e + a^2bd^2)\operatorname{arctanh}(cx)^2 + 3(a^2bx^2e^2 + 2a^2bd^2e + a^2bd^2)\operatorname{arctanh}(cx), x)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^3 (d + ex)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*(a+b*atanh(c*x))**3,x)

[Out] Integral((a + b*atanh(c*x))**3*(d + e*x)**2, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^3,x, algorithm="giac")

[Out] integrate((e*x + d)^2*(b*arctanh(c*x) + a)^3, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{atanh}(cx))^3 (d + ex)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^3*(d + e*x)^2,x)

[Out] int((a + b*atanh(c*x))^3*(d + e*x)^2, x)

3.17 $\int (d + ex) (a + b \tanh^{-1}(cx))^3 dx$

Optimal. Leaf size=244

$$\frac{3be(a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{3bex(a + b \tanh^{-1}(cx))^2}{2c} + \frac{d(a + b \tanh^{-1}(cx))^3}{c} - \frac{\left(d^2 + \frac{e^2}{c^2}\right)(a + b \tanh^{-1}(cx))^3}{2e}$$

[Out] $3/2*b*e*(a+b*\operatorname{arctanh}(c*x))^2/c^2+3/2*b*e*x*(a+b*\operatorname{arctanh}(c*x))^2/c+d*(a+b*\operatorname{arctanh}(c*x))^3/c-1/2*(d^2+e^2/c^2)*(a+b*\operatorname{arctanh}(c*x))^3/e+1/2*(e*x+d)^2*(a+b*\operatorname{arctanh}(c*x))^3/e-3*b^2*e*(a+b*\operatorname{arctanh}(c*x))*\ln(2/(-c*x+1))/c^2-3*b*d*(a+b*\operatorname{arctanh}(c*x))^2*\ln(2/(-c*x+1))/c-3/2*b^3*e*\operatorname{polylog}(2,1-2/(-c*x+1))/c^2-3*b^2*d*(a+b*\operatorname{arctanh}(c*x))*\operatorname{polylog}(2,1-2/(-c*x+1))/c+3/2*b^3*d*\operatorname{polylog}(3,1-2/(-c*x+1))/c$

Rubi [A]

time = 0.43, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6065, 6021, 6131, 6055, 2449, 2352, 6195, 6095, 6205, 6745}

$$\frac{3b^2e \log\left(\frac{d}{1-cx}\right)(a+b \tanh^{-1}(cx))}{c^2} - \frac{3b^2d \operatorname{Li}_2\left(1-\frac{d}{1-cx}\right)(a+b \tanh^{-1}(cx))}{c} - \frac{\left(\frac{d}{c} + \frac{d^2}{2c}\right)(a+b \tanh^{-1}(cx))^3}{2c} + \frac{3be(a+b \tanh^{-1}(cx))^2}{2c^2} + \frac{(d+ex)^2(a+b \tanh^{-1}(cx))^3}{2c} + \frac{d(a+b \tanh^{-1}(cx))^3}{c} - \frac{3d \log\left(\frac{d}{1-cx}\right)(a+b \tanh^{-1}(cx))^2}{c} + \frac{3ber(a+b \tanh^{-1}(cx))^2}{2c} - \frac{3b^2e \operatorname{Li}_2\left(1-\frac{d}{1-cx}\right)}{2c^2} + \frac{3b^2d \operatorname{Li}_2\left(1-\frac{d}{1-cx}\right)}{2c}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(d + e*x)*(a + b*\operatorname{ArcTanh}[c*x])^3, x]$

[Out] $(3*b*e*(a + b*\operatorname{ArcTanh}[c*x])^2)/(2*c^2) + (3*b*e*x*(a + b*\operatorname{ArcTanh}[c*x])^2)/(2*c) + (d*(a + b*\operatorname{ArcTanh}[c*x])^3)/c - ((d^2 + e^2/c^2)*(a + b*\operatorname{ArcTanh}[c*x])^3)/(2*e) + ((d + e*x)^2*(a + b*\operatorname{ArcTanh}[c*x])^3)/(2*e) - (3*b^2*e*(a + b*\operatorname{ArcTanh}[c*x])*Log[2/(1 - c*x)])/c^2 - (3*b*d*(a + b*\operatorname{ArcTanh}[c*x])^2*Log[2/(1 - c*x)])/c - (3*b^3*e*PolyLog[2, 1 - 2/(1 - c*x)])/(2*c^2) - (3*b^2*d*(a + b*\operatorname{ArcTanh}[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/c + (3*b^3*d*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c)$

Rule 2352

$\operatorname{Int}[\operatorname{Log}[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] \rightarrow \operatorname{Simp}[(-e^(-1))*\operatorname{PolyLog}[2, 1 - c*x], x] /;$ $\operatorname{FreeQ}\{c, d, e, x\} \ \&\& \ \operatorname{EqQ}[e + c*d, 0]$

Rule 2449

$\operatorname{Int}[\operatorname{Log}[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] \rightarrow \operatorname{Dist}[-e/g, \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /;$ $\operatorname{FreeQ}\{c, d, e, f, g, x\} \ \&\& \ \operatorname{EqQ}[c, 2*d] \ \&\& \ \operatorname{EqQ}[e^2*f + d^2*g, 0]$

Rule 6021

$\operatorname{Int}[(a_. + \operatorname{ArcTanh}[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] \rightarrow \operatorname{Simp}[x*(a + b*\operatorname{ArcTanh}[c*x^n])^p, x] - \operatorname{Dist}[b*c*n*p, \operatorname{Int}[x^n*(a + b*\operatorname{ArcTanh}[c*x^n])^p, x]]$

$(p - 1)/(1 - c^2 x^{2n})$, x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && (EqQ[n, 1] || EqQ[p, 1])

Rule 6055

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 6065

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol] := Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] - Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 6095

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 6131

Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6195

Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_.))^(m_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]

Rule 6205

Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol] := Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int (d + ex) (a + b \tanh^{-1}(cx))^3 dx &= \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))^3}{2e} - \frac{(3bc) \int \left(-\frac{e^2 (a + b \tanh^{-1}(cx))^2}{c^2} + \frac{(c^2 d^2 + e^2)}{2e} \right) dx}{2e} \\
&= \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))^3}{2e} - \frac{(3b) \int \frac{(c^2 d^2 + e^2 + 2c^2 dex) (a + b \tanh^{-1}(cx))^2}{1 - c^2 x^2} dx}{2ce} \\
&= \frac{3bex (a + b \tanh^{-1}(cx))^2}{2c} + \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))^3}{2e} - \frac{(3b) \int \left(\frac{e^2 (a + b \tanh^{-1}(cx))^2}{c^2} - \frac{(c^2 d^2 + e^2 + 2c^2 dex) (a + b \tanh^{-1}(cx))^2}{1 - c^2 x^2} \right) dx}{2ce} \\
&= \frac{3be (a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{3bex (a + b \tanh^{-1}(cx))^2}{2c} + \frac{(d + ex)^2 (a + b \tanh^{-1}(cx))^3}{2e} \\
&= \frac{3be (a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{3bex (a + b \tanh^{-1}(cx))^2}{2c} + \frac{d(a + b \tanh^{-1}(cx))^3}{c} \\
&= \frac{3be (a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{3bex (a + b \tanh^{-1}(cx))^2}{2c} + \frac{d(a + b \tanh^{-1}(cx))^3}{c} \\
&= \frac{3be (a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{3bex (a + b \tanh^{-1}(cx))^2}{2c} + \frac{d(a + b \tanh^{-1}(cx))^3}{c} \\
&= \frac{3be (a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{3bex (a + b \tanh^{-1}(cx))^2}{2c} + \frac{d(a + b \tanh^{-1}(cx))^3}{c}
\end{aligned}$$

Mathematica [A]

time = 0.44, size = 331, normalized size = 1.36

Integrate[(d + e*x)*(a + b*ArcTanh[c*x])^3, x] /; FreeQ[{a, b, c, d, e}, x] && !FreeQ[c, x] && !FreeQ[d + e*x, x] && !FreeQ[1 - c*x, x] && !FreeQ[1 + c*x, x] && !FreeQ[1 - c^2*x^2, x] && !FreeQ[1 + c^2*x^2, x] && !FreeQ[1 - c^2*x^2, x] && !FreeQ[1 + c^2*x^2, x]

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x])^3, x]

[Out] (2*a^2*c*(2*a*c*d + 3*b*e)*x + 2*a^3*c^2*e*x^2 + 6*a^2*b*c^2*x*(2*d + e*x)*ArcTanh[c*x] + 3*a^2*b*(2*c*d + e)*Log[1 - c*x] + 3*a^2*b*(2*c*d - e)*Log[1 + c*x] + 6*a*b^2*e*(2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + Log[1 - c^2*x^2]) - 2*b^3*e*(ArcTanh[c*x]*((3 - 3*c*x)*ArcTanh[c*x] + (1 - c

$$\begin{aligned} & ^2x^2) \operatorname{ArcTanh}[c*x]^2 + 6 \operatorname{Log}[1 + E^{(-2 \operatorname{ArcTanh}[c*x])}] - 3 \operatorname{PolyLog}[2, -E^{(-2 \operatorname{ArcTanh}[c*x])}] \\ & + 12*a*b^2*c*d*(\operatorname{ArcTanh}[c*x]*((-1 + c*x)*\operatorname{ArcTanh}[c*x] - 2*\operatorname{Log}[1 + E^{(-2 \operatorname{ArcTanh}[c*x])}]) \\ & + \operatorname{PolyLog}[2, -E^{(-2 \operatorname{ArcTanh}[c*x])}]) + 4*b^3*c*d*(\operatorname{ArcTanh}[c*x]^2*((-1 + c*x)*\operatorname{ArcTanh}[c*x] - 3*\operatorname{Log}[1 + E^{(-2 \operatorname{ArcTanh}[c*x])}]) \\ & + 3*\operatorname{ArcTanh}[c*x]*\operatorname{PolyLog}[2, -E^{(-2 \operatorname{ArcTanh}[c*x])}] + (3*\operatorname{PolyLog}[3, -E^{(-2 \operatorname{ArcTanh}[c*x])}])/2)) / (4*c^2) \end{aligned}$$

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.66, size = 12134, normalized size = 49.73

method	result	size
derivativedivides	Expression too large to display	12134
default	Expression too large to display	12134

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)`

[Out] result too large to display

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*arctanh(c*x))^3,x, algorithm="maxima")`

[Out]
$$\begin{aligned} & 1/2*a^3*x^2*e + a^3*d*x + 3/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3))*a^2*b*e + 3/2*(2*c*x*arctanh(c*x) + \log(-c^2*x^2 + 1))*a^2*b*d/c - 1/16*((b^3*c^2*x^2*e + 2*b^3*c^2*d*x - 2*b^3*c*d - b^3*e)*\log(-c*x + 1)^3 - 3*(2*a*b^2*c^2*x^2*e + 2*(2*a*b^2*c^2*d + b^3*c*e)*x + (b^3*c^2*x^2*e + 2*b^3*c^2*d*x + 2*b^3*c*d - b^3*e)*\log(c*x + 1))*\log(-c*x + 1)^2)/c^2 - \operatorname{integrate}(-1/8*((b^3*c^2*x^2*e - b^3*c*d + (b^3*c^2*d - b^3*c*e)*x)*\log(c*x + 1)^3 + 6*(a*b^2*c^2*x^2*e - a*b^2*c*d + (a*b^2*c^2*d - a*b^2*c*e)*x)*\log(c*x + 1)^2 - 3*(2*a*b^2*c^2*x^2*e + (b^3*c^2*x^2*e - b^3*c*d + (b^3*c^2*d - b^3*c*e)*x)*\log(c*x + 1)^2 + 2*(2*a*b^2*c^2*d + b^3*c*e)*x - (4*a*b^2*c*d - 2*b^3*c*d + b^3*e - (4*a*b^2*c^2 + b^3*c^2)*x^2*e - 2*(2*a*b^2*c^2*d + b^3*c^2*d - 2*a*b^2*c*e)*x)*\log(c*x + 1))*\log(-c*x + 1))/c^2 * x - c), x) \end{aligned}$$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arctanh(c*x))^3,x, algorithm="fricas")

[Out] integral(a^3*x*e + a^3*d + (b^3*x*e + b^3*d)*arctanh(c*x)^3 + 3*(a*b^2*x*e + a*b^2*d)*arctanh(c*x)^2 + 3*(a^2*b*x*e + a^2*b*d)*arctanh(c*x), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^3 (d + ex) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*atanh(c*x))**3,x)

[Out] Integral((a + b*atanh(c*x))**3*(d + e*x), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arctanh(c*x))^3,x, algorithm="giac")

[Out] integrate((e*x + d)*(b*arctanh(c*x) + a)^3, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{atanh}(cx))^3 (d + ex) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^3*(d + e*x),x)

[Out] int((a + b*atanh(c*x))^3*(d + e*x), x)

$$3.18 \quad \int \frac{(a+b \tanh^{-1}(cx))^3}{d+ex} dx$$

Optimal. Leaf size=272

$$-\frac{(a+b \tanh^{-1}(cx))^3 \log\left(\frac{2}{1+cx}\right)}{e} + \frac{(a+b \tanh^{-1}(cx))^3 \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{3b(a+b \tanh^{-1}(cx))^2 \text{PolyLog}(2, \frac{2c(d+ex)}{(cd+e)(1+cx)})}{2e}$$

[Out] $-(a+b*\text{arctanh}(c*x))^3*\ln(2/(c*x+1))/e+(a+b*\text{arctanh}(c*x))^3*\ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+3/2*b*(a+b*\text{arctanh}(c*x))^2*\text{polylog}(2,1-2/(c*x+1))/e-3/2*b*(a+b*\text{arctanh}(c*x))^2*\text{polylog}(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+3/2*b^2*(a+b*\text{arctanh}(c*x))*\text{polylog}(3,1-2/(c*x+1))/e-3/2*b^2*(a+b*\text{arctanh}(c*x))*\text{polylog}(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+3/4*b^3*\text{polylog}(4,1-2/(c*x+1))/e-3/4*b^3*\text{polylog}(4,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e$

Rubi [A]

time = 0.04, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6061}

$$\frac{3b^2(a+b \tanh^{-1}(cx)) \text{Li}_2\left(1-\frac{2cd+ex}{(cd+e)(1+cx)}\right)}{2e} + \frac{3b^2 \text{Li}_2\left(1-\frac{2}{1+cx}\right) (a+b \tanh^{-1}(cx))}{2e} - \frac{3b(a+b \tanh^{-1}(cx))^2 \text{Li}_2\left(1-\frac{2cd+ex}{(cd+e)(1+cx)}\right)}{2e} + \frac{(a+b \tanh^{-1}(cx))^3 \log\left(\frac{2cd+ex}{(cd+e)(1+cx)}\right)}{e} + \frac{3b \text{Li}_2\left(1-\frac{2}{1+cx}\right) (a+b \tanh^{-1}(cx))^2}{2e} - \frac{\log\left(\frac{2}{1+cx}\right) (a+b \tanh^{-1}(cx))^3}{e} - \frac{3b^2 \text{Li}_2\left(1-\frac{2cd+ex}{(cd+e)(1+cx)}\right)}{4e} + \frac{3b^2 \text{Li}_2\left(1-\frac{2}{1+cx}\right)}{4e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])^3/(d + e*x), x]

[Out] $-(((a + b*\text{ArcTanh}[c*x])^3*\text{Log}[2/(1 + c*x)]))/e) + ((a + b*\text{ArcTanh}[c*x])^3*\text{Log}[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (3*b*(a + b*\text{ArcTanh}[c*x])^2*\text{PolyLog}[2, 1 - 2/(1 + c*x)])/(2*e) - (3*b*(a + b*\text{ArcTanh}[c*x])^2*\text{PolyLog}[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/ (2*e) + (3*b^2*(a + b*\text{ArcTanh}[c*x])* \text{PolyLog}[3, 1 - 2/(1 + c*x)])/(2*e) - (3*b^2*(a + b*\text{ArcTanh}[c*x])* \text{PolyLog}[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/ (2*e) + (3*b^3*\text{PolyLog}[4, 1 - 2/(1 + c*x)])/(4*e) - (3*b^3*\text{PolyLog}[4, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/ (4*e)$

Rule 6061

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^3/((d_.) + (e_.)*(x_.)), x_Symbol] :=
 Simp[(-(a + b*ArcTanh[c*x])^3)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcTanh[c*x])^3*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x))])/e), x] + Simp[3*b*(a + b*ArcTanh[c*x])^2*(PolyLog[2, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[3*b*(a + b*ArcTanh[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x))])/ (2*e)), x] + Simp[3*b^2*(a + b*ArcTanh[c*x])* (PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[3*b^2*(a + b*ArcTanh[c*x])* (PolyLog[3, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x))])/ (2*e)), x] + Simp[3*b^3*(PolyLog[4, 1 - 2/(1 + c*x)]/(4*e)), x] - Simp[3*b^3*(PolyLog[4, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x))])/ (4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rubi steps

$$\int \frac{(a + b \tanh^{-1}(cx))^3}{d + ex} dx = -\frac{(a + b \tanh^{-1}(cx))^3 \log\left(\frac{2}{1+cx}\right)}{e} + \frac{(a + b \tanh^{-1}(cx))^3 \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + 3$$

Mathematica [C] Result contains complex when optimal does not.
time = 31.92, size = 2179, normalized size = 8.01

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c*x])^3/(d + e*x), x]

[Out] (a^3*Log[d + e*x])/e - ((3*I)*a^2*b*(I*ArcTanh[c*x]*(-Log[1/Sqrt[1 - c^2*x^2]] + Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]])) + ((-I)*(I*ArcTanh[(c*d)/e] + I*ArcTanh[c*x])^2 - (I/4)*(Pi - (2*I)*ArcTanh[c*x])^2 + 2*(I*ArcTanh[(c*d)/e] + I*ArcTanh[c*x])*Log[1 - E^((2*I)*(I*ArcTanh[(c*d)/e] + I*ArcTanh[c*x]))] + (Pi - (2*I)*ArcTanh[c*x])*Log[1 - E^(I*(Pi - (2*I)*ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c*x])*Log[2*Sin[(Pi - (2*I)*ArcTanh[c*x])/2]] - 2*(I*ArcTanh[(c*d)/e] + I*ArcTanh[c*x])*Log[(2*I)*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - I*PolyLog[2, E^((2*I)*(I*ArcTanh[(c*d)/e] + I*ArcTanh[c*x]))] - I*PolyLog[2, E^(I*(Pi - (2*I)*ArcTanh[c*x]))]/2)/e + (a*b^2*(-8*c*d*ArcTanh[c*x]^3 + 4*e*ArcTanh[c*x]^3 - (4*sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] - 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])]) - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(1 + E^(2*ArcTanh[c*x]))/(2*E^ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e)] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*c*d*ArcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x])))/(2*E^ArcTanh[c*x])] + (6*I)*c*d*Pi*ArcTanh[c*x]*Log[1/Sqrt[1 - c^2*x^2]] - 6*c*d*ArcTanh[c*x]^2*Log[(c*d)/sqrt[1 - c^2*x^2] + (c*e*x)/sqrt[1 - c^2*x^2]] - 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + 6*c*d*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]*PolyLog[2, -((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e)] + 12*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c*d*ArcTanh[c*x]*PolyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 3*c*d*PolyLog[3, -E^(-2*ArcTanh[c*x])] + 3*c*d*PolyLog[3, -((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e)] - 12*c*d*PolyLog[3, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^

$$\begin{aligned}
& (\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x]) - 3*c*d*\text{PolyLog}[3, E^{(2*(\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x])} - 3*c*d*\text{PolyLog}[3, E^{(2*(\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x])} \\
&] + \text{ArcTanh}[c*x])))]/(2*c*d*e) + (b^3*(-5*c*d*\text{ArcTanh}[c*x]^4 + 3*e*\text{ArcTanh}[c*x]^4 - (3*\text{Sqrt}[1 - (c^2*d^2)/e^2]*e*\text{ArcTanh}[c*x]^4)/E^{\text{ArcTanh}[(c*d)/e] - \\
& 4*c*d*\text{ArcTanh}[c*x]^3*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c*x])}] - (6*I)*c*d*\text{Pi}*\text{ArcTanh}[c*x]^2*\text{Log}[1 - I*E^{\text{ArcTanh}[c*x]}] - (6*I)*c*d*\text{Pi}*\text{ArcTanh}[c*x]^2*\text{Log}[1 + I*E^{\text{ArcTanh}[c*x]}] \\
& + 4*c*d*\text{ArcTanh}[c*x]^3*\text{Log}[1 - (\text{Sqrt}[c*d + e]*E^{\text{ArcTanh}[c*x]})/\text{Sqrt}[-(c*d) + e]] + 4*c*d*\text{ArcTanh}[c*x]^3*\text{Log}[1 + (\text{Sqrt}[c*d + e]*E^{\text{ArcTanh}[c*x]})/\text{Sqrt}[-(c*d) + e]] + (6*I)*c*d*\text{Pi}*\text{ArcTanh}[c*x]^2*\text{Log}[1 + E^{(2*\text{ArcTanh}[c*x])}] - (6*I)*c*d*\text{Pi}*\text{ArcTanh}[c*x]^2*\text{Log}[(1 + E^{(2*\text{ArcTanh}[c*x])})/(2*E^{\text{ArcTanh}[c*x]})] - 12*c*d*\text{ArcTanh}[c*x]^3*\text{Log}[1 + ((c*d + e)*E^{(2*\text{ArcTanh}[c*x])})/(c*d - e)] + 12*c*d*\text{ArcTanh}[c*x]^3*\text{Log}[1 - E^{(2*(\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x])}] + 12*c*d*\text{ArcTanh}[(c*d)/e]*\text{ArcTanh}[c*x]^2*\text{Log}[(I/2)*E^{(-\text{ArcTanh}[(c*d)/e] - \text{ArcTanh}[c*x])}*(-1 + E^{(2*(\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x])})] + 8*c*d*\text{ArcTanh}[c*x]^3*\text{Log}[(e*(-1 + E^{(2*\text{ArcTanh}[c*x])}) + c*d*(1 + E^{(2*\text{ArcTanh}[c*x])}))]/(2*E^{\text{ArcTanh}[c*x]})] + (6*I)*c*d*\text{Pi}*\text{ArcTanh}[c*x]^2*\text{Log}[1/\text{Sqrt}[1 - c^2*x^2]] - 8*c*d*\text{ArcTanh}[c*x]^3*\text{Log}[(c*d)/\text{Sqrt}[1 - c^2*x^2] + (c*e*x)/\text{Sqrt}[1 - c^2*x^2]] - 12*c*d*\text{ArcTanh}[(c*d)/e]*\text{ArcTanh}[c*x]^2*\text{Log}[I*\text{Sinh}[\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x]]] + 6*c*d*\text{ArcTanh}[c*x]^2*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c*x])}] - (12*I)*c*d*\text{Pi}*\text{ArcTanh}[c*x]*\text{PolyLog}[2, (-I)*E^{\text{ArcTanh}[c*x]}] - (12*I)*c*d*\text{Pi}*\text{ArcTanh}[c*x]*\text{PolyLog}[2, I*E^{\text{ArcTanh}[c*x]}] + 12*c*d*\text{ArcTanh}[c*x]^2*\text{PolyLog}[2, -((\text{Sqrt}[c*d + e]*E^{\text{ArcTanh}[c*x]})/\text{Sqrt}[-(c*d) + e])] + 12*c*d*\text{ArcTanh}[c*x]^2*\text{PolyLog}[2, (\text{Sqrt}[c*d + e]*E^{\text{ArcTanh}[c*x]})/\text{Sqrt}[-(c*d) + e]] + (6*I)*c*d*\text{Pi}*\text{ArcTanh}[c*x]*\text{PolyLog}[2, -E^{(2*\text{ArcTanh}[c*x])}] - 18*c*d*\text{ArcTanh}[c*x]^2*\text{PolyLog}[2, -(((c*d + e)*E^{(2*\text{ArcTanh}[c*x])})/(c*d - e))] + 18*c*d*\text{ArcTanh}[c*x]^2*\text{PolyLog}[2, E^{(2*(\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x])}] + 6*c*d*\text{ArcTanh}[c*x]*\text{PolyLog}[3, -E^{(-2*\text{ArcTanh}[c*x])}] + (12*I)*c*d*\text{Pi}*\text{PolyLog}[3, (-I)*E^{\text{ArcTanh}[c*x]}] + (12*I)*c*d*\text{Pi}*\text{PolyLog}[3, I*E^{\text{ArcTanh}[c*x]}] - 24*c*d*\text{ArcTanh}[c*x]*\text{PolyLog}[3, -((\text{Sqrt}[c*d + e]*E^{\text{ArcTanh}[c*x]})/\text{Sqrt}[-(c*d) + e])] - 24*c*d*\text{ArcTanh}[c*x]*\text{PolyLog}[3, (\text{Sqrt}[c*d + e]*E^{\text{ArcTanh}[c*x]})/\text{Sqrt}[-(c*d) + e]] - (3*I)*c*d*\text{Pi}*\text{PolyLog}[3, -E^{(2*\text{ArcTanh}[c*x])}] + 18*c*d*\text{ArcTanh}[c*x]*\text{PolyLog}[3, -(((c*d + e)*E^{(2*\text{ArcTanh}[c*x])})/(c*d - e))] - 18*c*d*\text{ArcTanh}[c*x]*\text{PolyLog}[3, E^{(2*(\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x])}] + 3*c*d*\text{PolyLog}[4, -E^{(-2*\text{ArcTanh}[c*x])}] + 24*c*d*\text{PolyLog}[4, -((\text{Sqrt}[c*d + e]*E^{\text{ArcTanh}[c*x]})/\text{Sqrt}[-(c*d) + e])] + 24*c*d*\text{PolyLog}[4, (\text{Sqrt}[c*d + e]*E^{\text{ArcTanh}[c*x]})/\text{Sqrt}[-(c*d) + e]] - 9*c*d*\text{PolyLog}[4, -(((c*d + e)*E^{(2*\text{ArcTanh}[c*x])})/(c*d - e))] + 9*c*d*\text{PolyLog}[4, E^{(2*(\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x])}))/ (4*c*d*e)
\end{aligned}$$

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.47, size = 2415, normalized size = 8.88

method	result	size
derivativedivides	Expression too large to display	2415
default	Expression too large to display	2415

Verification of antiderivative is not currently implemented for this CAS.

$x^2+1)/(-c*d+e))-3/4*b^3*c/e*polylog(4,-(c*x+1)^2/(-c^2*x^2+1))+3/4*b^3*c/(c*d+e)*polylog(4,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+a^3*c*ln(c*e*x+c*d)/e)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^3/(e*x+d),x, algorithm="maxima")

[Out] $a^3*e^{-1}*\log(x*e + d) + \int \frac{1}{8}b^3*(\log(c*x + 1) - \log(-c*x + 1))^3/(x*e + d) + \frac{3}{4}a*b^2*(\log(c*x + 1) - \log(-c*x + 1))^2/(x*e + d) + \frac{3}{2}a^2*b*(\log(c*x + 1) - \log(-c*x + 1))/(x*e + d), x$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^3/(e*x+d),x, algorithm="fricas")

[Out] $\int \frac{(b^3*\arctanh(c*x)^3 + 3*a*b^2*\arctanh(c*x)^2 + 3*a^2*b*\arctanh(c*x) + a^3)/(x*e + d), x$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))**3/(e*x+d),x)

[Out] $\int (a + b*\operatorname{atanh}(c*x))**3/(d + e*x), x$

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^3/(e*x+d),x, algorithm="giac")

[Out] $\int (b*\operatorname{arctanh}(c*x) + a)^3/(e*x + d), x$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^3/(d + e*x), x)

[Out] int((a + b*atanh(c*x))^3/(d + e*x), x)

$$3.19 \quad \int \frac{(a+b \tanh^{-1}(cx))^3}{(d+ex)^2} dx$$

Optimal. Leaf size=517

$$-\frac{(a+b \tanh^{-1}(cx))^3}{e(d+ex)} + \frac{3bc(a+b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1-cx}\right)}{2e(cd+e)} - \frac{3bc(a+b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1+cx}\right)}{2(cd-e)e} + \frac{3bc(a+b \tanh^{-1}(cx))}{e^2(d+ex)^2}$$

```
[Out] -(a+b*arctanh(c*x))^3/e/(e*x+d)+3/2*b*c*(a+b*arctanh(c*x))^2*ln(2/(-c*x+1))
/e/(c*d+e)-3/2*b*c*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/(c*d-e)/e+3*b*c*(a+b*
arctanh(c*x))^2*ln(2/(c*x+1))/(c^2*d^2-e^2)-3*b*c*(a+b*arctanh(c*x))^2*ln(2
*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)+3/2*b^2*c*(a+b*arctanh(c*x))*poly
log(2,1-2/(-c*x+1))/e/(c*d+e)+3/2*b^2*c*(a+b*arctanh(c*x))*polylog(2,1-2/(c
*x+1))/(c*d-e)/e-3*b^2*c*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/(c^2*d^2
-e^2)+3*b^2*c*(a+b*arctanh(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(
c^2*d^2-e^2)-3/4*b^3*c*polylog(3,1-2/(-c*x+1))/e/(c*d+e)+3/4*b^3*c*polylog(
3,1-2/(c*x+1))/(c*d-e)/e-3/2*b^3*c*polylog(3,1-2/(c*x+1))/(c^2*d^2-e^2)+3/2
*b^3*c*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)
```

Rubi [A]

time = 0.38, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6065, 6055, 6095, 6205, 6745, 6203, 6059}

$\frac{3b^2d^2c^2 \operatorname{arctanh}\left(\frac{a+bx}{c}\right)}{2e^2(d+ex)^2} + \frac{3b^2cd^2 \operatorname{arctanh}\left(\frac{a+bx}{c}\right) \log\left(\frac{2}{1-cx}\right)}{2e^2(d+ex)^2} - \frac{3b^2cd^2 \operatorname{arctanh}\left(\frac{a+bx}{c}\right) \log\left(\frac{2}{1+cx}\right)}{2e^2(d+ex)^2} + \frac{3b^2cd^2}{e^2(d+ex)^2}$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])^3/(d + e*x)^2,x]

```
[Out] -((a + b*ArcTanh[c*x])^3/(e*(d + e*x))) + (3*b*c*(a + b*ArcTanh[c*x])^2*Log
[2/(1 - c*x)]/(2*e*(c*d + e)) - (3*b*c*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c
*x)]/(2*(c*d - e)*e) + (3*b*c*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)]/(c^
2*d^2 - e^2) - (3*b*c*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e
*(1 + c*x)))]/(c^2*d^2 - e^2) + (3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1
- 2/(1 - c*x)]/(2*e*(c*d + e)) + (3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog[2,
1 - 2/(1 + c*x)]/(2*(c*d - e)*e) - (3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog[2
, 1 - 2/(1 + c*x)]/(c^2*d^2 - e^2) + (3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog
[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/(c^2*d^2 - e^2) - (3*b^3*c*
PolyLog[3, 1 - 2/(1 - c*x)]/(4*e*(c*d + e)) + (3*b^3*c*PolyLog[3, 1 - 2/(1
+ c*x)]/(4*(c*d - e)*e) - (3*b^3*c*PolyLog[3, 1 - 2/(1 + c*x)]/(2*(c^2*d
^2 - e^2)) + (3*b^3*c*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))
]/(2*(c^2*d^2 - e^2))
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^p]/((d_.) + (e_.)*(x_.)), x_Symbol
] := Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
```

(p/e) , $\text{Int}[(a + b \cdot \text{ArcTanh}[c \cdot x])^{(p-1)} \cdot (\text{Log}[2/(1 + e \cdot (x/d))]) / (1 - c^2 \cdot x^2)$, $x]$, $x]$ /; $\text{FreeQ}\{a, b, c, d, e\}, x\}$ && $\text{IGtQ}[p, 0]$ && $\text{EqQ}[c^2 \cdot d^2 - e^2, 0]$

Rule 6059

$\text{Int}[(a + b \cdot \text{ArcTanh}[c \cdot x])^{(p-1)} \cdot (\text{Log}[2/(1 + e \cdot (x/d))]) / (1 - c^2 \cdot x^2)$, $x]$, $x]$ /; $\text{FreeQ}\{a, b, c, d, e\}, x\}$ && $\text{IGtQ}[p, 0]$ && $\text{EqQ}[c^2 \cdot d^2 - e^2, 0]$

$\text{Int}[(a + b \cdot \text{ArcTanh}[c \cdot x])^{(p-1)} \cdot (\text{Log}[2/(1 + e \cdot (x/d))]) / (1 - c^2 \cdot x^2)$, $x]$, $x]$ /; $\text{FreeQ}\{a, b, c, d, e\}, x\}$ && $\text{IGtQ}[p, 0]$ && $\text{EqQ}[c^2 \cdot d^2 - e^2, 0]$

Rule 6065

$\text{Int}[(a + b \cdot \text{ArcTanh}[c \cdot x])^{(p-1)} \cdot (\text{Log}[2/(1 + e \cdot (x/d))]) / (1 - c^2 \cdot x^2)$, $x]$, $x]$ /; $\text{FreeQ}\{a, b, c, d, e\}, x\}$ && $\text{IGtQ}[p, 1]$ && $\text{IntegerQ}[q]$ && $\text{NeQ}[q, -1]$

Rule 6095

$\text{Int}[(a + b \cdot \text{ArcTanh}[c \cdot x])^{(p-1)} \cdot (\text{Log}[2/(1 + e \cdot (x/d))]) / (1 - c^2 \cdot x^2)$, $x]$, $x]$ /; $\text{FreeQ}\{a, b, c, d, e, p\}, x\}$ && $\text{EqQ}[c^2 \cdot d + e, 0]$ && $\text{NeQ}[p, -1]$

Rule 6203

$\text{Int}[(a + b \cdot \text{ArcTanh}[c \cdot x])^{(p-1)} \cdot (\text{Log}[2/(1 + e \cdot (x/d))]) / (1 - c^2 \cdot x^2)$, $x]$, $x]$ /; $\text{FreeQ}\{a, b, c, d, e\}, x\}$ && $\text{IGtQ}[p, 0]$ && $\text{EqQ}[c^2 \cdot d + e, 0]$ && $\text{EqQ}[(1 - u)^2 - (1 - 2/(1 + c \cdot x))^2, 0]$

Rule 6205

$\text{Int}[(a + b \cdot \text{ArcTanh}[c \cdot x])^{(p-1)} \cdot (\text{Log}[2/(1 + e \cdot (x/d))]) / (1 - c^2 \cdot x^2)$, $x]$, $x]$ /; $\text{FreeQ}\{a, b, c, d, e\}, x\}$ && $\text{IGtQ}[p, 0]$ && $\text{EqQ}[c^2 \cdot d + e, 0]$ && $\text{EqQ}[(1 - u)^2 - (1 - 2/(1 - c \cdot x))^2, 0]$

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + b \tanh^{-1}(cx))^3}{(d + ex)^2} dx &= -\frac{(a + b \tanh^{-1}(cx))^3}{e(d + ex)} + \frac{(3bc) \int \left(-\frac{c(a+b \tanh^{-1}(cx))^2}{2(cd+e)(-1+cx)} + \frac{c(a+b \tanh^{-1}(cx))^2}{2(cd-e)(1+cx)} + \frac{e^2(a+b \tanh^{-1}(cx))^2}{(-cd+e)} \right) dx}{e} \\ &= -\frac{(a + b \tanh^{-1}(cx))^3}{e(d + ex)} + \frac{(3bc^2) \int \frac{(a+b \tanh^{-1}(cx))^2}{1+cx} dx}{2(cd - e)e} - \frac{(3bc^2) \int \frac{(a+b \tanh^{-1}(cx))^2}{-1+cx} dx}{2e(cd + e)} \\ &= -\frac{(a + b \tanh^{-1}(cx))^3}{e(d + ex)} + \frac{3bc(a + b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1-cx}\right)}{2e(cd + e)} - \frac{3bc(a + b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1+cx}\right)}{2e(cd + e)} \\ &= -\frac{(a + b \tanh^{-1}(cx))^3}{e(d + ex)} + \frac{3bc(a + b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1-cx}\right)}{2e(cd + e)} - \frac{3bc(a + b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1+cx}\right)}{2e(cd + e)} \\ &= -\frac{(a + b \tanh^{-1}(cx))^3}{e(d + ex)} + \frac{3bc(a + b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1-cx}\right)}{2e(cd + e)} - \frac{3bc(a + b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1+cx}\right)}{2e(cd + e)} \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 11.58, size = 990, normalized size = 1.91

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(d + e*x)^2, x]
```

```
[Out] -(a^3/(e*(d + e*x))) - (3*a^2*b*ArcTanh[c*x])/(e*(d + e*x)) - (3*a^2*b*c*Log[1 - c*x])/(2*e*(c*d + e)) + (3*a^2*b*c*Log[1 + c*x])/(2*c*d*e - 2*e^2) - (3*a^2*b*c*Log[d + e*x])/(c^2*d^2 - e^2) + (3*a*b^2*(-(ArcTanh[c*x]^2/(Sqrt[1 - (c^2*d^2)/e^2]*e*E^ArcTanh[(c*d)/e])) + (x*ArcTanh[c*x]^2)/(d + e*x) + (c*d*(I*Pi*Log[1 + E^(2*ArcTanh[c*x])]) - 2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - I*Pi*(ArcTanh[c*x] - Log[1 - c^2*x^2]/2) - 2*ArcTanh[(c*d)/e]*(ArcTanh[c*x] + Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]]) + PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]/(c^2*d^2 - e^2))/d + (b^3*((x*ArcTanh[c*x]^3)/(d + e*x) + (3*(6*c*d*ArcTanh[c*x]^3 - 2*e*ArcTanh[c*x]^3 + (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] + (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2] + 6*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e)] - 6*c*d*ArcTanh[c*x]^2*Log[1 - E^ArcTanh[(c*d)/e] + ArcTanh[c*x]]) - 6*c*d*ArcTanh[c*x]^2*L
```

$$\begin{aligned} & \log[1 + E^{\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x]}] - 6*c*d*\text{ArcTanh}[c*x]^2*\log[1 - \\ & E^{2*(\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x])}] - 12*c*d*\text{ArcTanh}[(c*d)/e]*\text{ArcTanh}[\\ & c*x]*\log[(I/2)*E^{(-\text{ArcTanh}[(c*d)/e] - \text{ArcTanh}[c*x])}*(-1 + E^{2*(\text{ArcTanh}[(c* \\ & d)/e] + \text{ArcTanh}[c*x])})] - 6*c*d*\text{ArcTanh}[c*x]^2*\log[(e*(-1 + E^{2*\text{ArcTanh}[c \\ & *x]) + c*d*(1 + E^{2*\text{ArcTanh}[c*x])})/(2*E^{\text{ArcTanh}[c*x]})] + 6*c*d*\text{ArcTanh}[c \\ & *x]^2*\log[(c*(d + e*x))/\text{Sqrt}[1 - c^2*x^2]] + (3*I)*c*d*\text{Pi}*\text{ArcTanh}[c*x]*\log[\\ & 1 - c^2*x^2] + 12*c*d*\text{ArcTanh}[(c*d)/e]*\text{ArcTanh}[c*x]*\log[I*\text{Sinh}[\text{ArcTanh}[(c*d \\ &)/e] + \text{ArcTanh}[c*x]]] + 6*c*d*\text{ArcTanh}[c*x]*\text{PolyLog}[2, -((c*d + e)*E^{2*\text{Arc} \\ & \text{Tanh}[c*x]})/(c*d - e)] - 12*c*d*\text{ArcTanh}[c*x]*\text{PolyLog}[2, -E^{\text{ArcTanh}[(c*d)/ \\ & e] + \text{ArcTanh}[c*x]}] - 12*c*d*\text{ArcTanh}[c*x]*\text{PolyLog}[2, E^{\text{ArcTanh}[(c*d)/e] + \\ & \text{ArcTanh}[c*x]}] - 6*c*d*\text{ArcTanh}[c*x]*\text{PolyLog}[2, E^{2*(\text{ArcTanh}[(c*d)/e] + \text{Arc} \\ & \text{Tanh}[c*x])}] - 3*c*d*\text{PolyLog}[3, -((c*d + e)*E^{2*\text{ArcTanh}[c*x]})/(c*d - e) \\ &] + 12*c*d*\text{PolyLog}[3, -E^{\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x]}] + 12*c*d*\text{PolyLo} \\ & \text{g}[3, E^{\text{ArcTanh}[(c*d)/e] + \text{ArcTanh}[c*x]}] + 3*c*d*\text{PolyLog}[3, E^{2*(\text{ArcTanh}[\\ & (c*d)/e] + \text{ArcTanh}[c*x])})]/(6*c^2*d^2 - 6*e^2))/d \end{aligned}$$

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.04, size = 3591, normalized size = 6.95

method	result	size
derivativedivides	Expression too large to display	3591
default	Expression too large to display	3591

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x))^3/(e*x+d)^2,x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & 1/c*(-3*a^2*b*c^2/(c*d+e)/(c*d-e)*\ln(c*e*x+c*d)-3/4*I*b^3*c^2/(c*d+e)/(c*d- \\ & e)*\arctanh(c*x)^2*\text{Picsgn}(I*(c*x+1)^2/(c^2*x^2-1))^3-3/2*I*b^3*c^2/(c*d+e)/ \\ & (c*d-e)*\arctanh(c*x)^2*\text{Picsgn}(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1) \\ & ^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/2*I*b^3*c^2/(c*d+e)/(c* \\ & d-e)*\arctanh(c*x)^2*\text{Picsgn}(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+3/2*I*b^3*c^2/(\\ & c*d+e)/(c*d-e)*\arctanh(c*x)^2*\text{Picsgn}(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/4*I \\ & *b^3*c^2/(c*d+e)/(c*d-e)*\arctanh(c*x)^2*\text{Picsgn}(I*(c*x+1)^2/(c^2*x^2-1)/(1+ \\ & (c*x+1)^2/(-c^2*x^2+1)))^3-3*a^2*b*c^2/(c*e*x+c*d)/e*\arctanh(c*x)-3*a^2*b*c \\ & ^2/e/(2*c*d+2*e)*\ln(c*x-1)+3*a^2*b*c^2/e/(2*c*d-2*e)*\ln(c*x+1)+3/2*b^3*c^3/ \\ & (c*d+e)^2/(c*d-e)*d*\text{polylog}(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3*b^ \\ & 3*c^2*\arctanh(c*x)^2/(c*d+e)/(c*d-e)*\ln(c*e*x+c*d)+3*b^3*c^2*\arctanh(c*x)^2 \\ & /(c*d+e)/(c*d-e)*\ln(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1) \\ &)-1))-3*b^3*c^2/(c*d+e)/(c*d-e)*\arctanh(c*x)^2*\ln(2)+3/2*b^3*c^2*e/(c*d+e)^ \\ & 2/(c*d-e)*\text{polylog}(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+3*b^3*c^2/e*\ar \\ & \text{ctanh}(c*x)^2/(2*c*d-2*e)*\ln(c*x+1)-3*b^3*c^2/e*\arctanh(c*x)^2/(2*c*d+2*e)*\ln \\ & (c*x-1)-3*b^3*c^2/e*\arctanh(c*x)^2/(c*d-e)*\ln((c*x+1)/(-c^2*x^2+1)^{(1/2)})- \\ & 3*a*b^2*c^2/(c*e*x+c*d)/e*\arctanh(c*x)^2-3*a*b^2*c^2/(c*d+e)/(c*d-e)*\text{dilog} \\ & (c*e*x-e)/(-c*d-e)+3*a*b^2*c^2/(c*d+e)/(c*d-e)*\text{dilog}((c*e*x+e)/(-c*d+e))-3 \\ & /2*a*b^2*c^2/e/(c*d-e)*\text{dilog}(1/2*c*x+1/2)-3/4*a*b^2*c^2/e/(c*d-e)*\ln(c*x+1) \end{aligned}$$

$$\begin{aligned}
& -2-3/4*a*b^2*c^2/e/(c*d+e)*\ln(c*x-1)^2+3/2*a*b^2*c^2/e/(c*d+e)*\operatorname{dilog}(1/2*c*x+1/2)+3/4*I*b^3*c^2/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*c\operatorname{sgn}(I/(1+(c*x+1)^2/(-c^2*x^2+1))) *c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1))*c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))) -3/4*I*b^3*c^3/e/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/2*I*b^3*c^2/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*c\operatorname{sgn}(I/(1+(c*x+1)^2/(-c^2*x^2+1))) *c\operatorname{sgn}(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))) *c\operatorname{sgn}(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1))) +3/2*I*b^3*c^3/e/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*d*c\operatorname{sgn}(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/4*I*b^3*c^3/e/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*d*c\operatorname{sgn}(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+3/2*I*b^3*c^2/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*c\operatorname{sgn}(I/(1+(c*x+1)^2/(-c^2*x^2+1))) *c\operatorname{sgn}(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+3/4*I*b^3*c^2/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1))*c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/2*I*b^3*c^2/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*c\operatorname{sgn}(I*(c*x+1)/(-c^2*x^2+1)^(1/2)) *c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1))^2-3/4*I*b^3*c^2/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*c\operatorname{sgn}(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1))-3/4*I*b^3*c^2/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*c\operatorname{sgn}(I/(1+(c*x+1)^2/(-c^2*x^2+1))) *c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+3/2*I*b^3*c^2/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*c\operatorname{sgn}(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))) *c\operatorname{sgn}(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+3/2*I*b^3*c^3/e/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*d*6*a*b^2*c^2/e*\operatorname{arctanh}(c*x)/(2*c*d+2*e)*\ln(c*x-1)+6*a*b^2*c^2/e*\operatorname{arctanh}(c*x)/(2*c*d-2*e)*\ln(c*x+1)-6*a*b^2*c^2*\operatorname{arctanh}(c*x)/(c*d+e)/(c*d-e)*\ln(c*e*x+c*d)-3*a*b^2*c^2/(c*d+e)/(c*d-e)*\ln(c*e*x+c*d)*\ln((c*e*x-e)/(-c*d-e))+3*a*b^2*c^2/(c*d+e)/(c*d-e)*\ln(c*e*x+c*d)*\ln((c*e*x+e)/(-c*d+e))+3/2*a*b^2*c^2/e/(c*d-e)*\ln(-1/2*c*x+1/2)*\ln(c*x+1)-3/2*a*b^2*c^2/e/(c*d-e)*\ln(-1/2*c*x+1/2)*\ln(1/2*c*x+1/2)+3/2*a*b^2*c^2/e/(c*d+e)*\ln(c*x-1)*\ln(1/2*c*x+1/2)-3*b^3*c^2*e/(c*d+e)^2/(c*d-e)*\operatorname{arctanh}(c*x)*\operatorname{polylog}(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3*b^3*c^2*e/(c*d+e)^2/(c*d-e)*\operatorname{arctanh}(c*x)^2*\ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3*b^3*c^3/(c*d+e)^2/(c*d-e)*d*\operatorname{arctanh}(c*x)^2*\ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3*b^3*c^3/(c*d+e)^2/(c*d-e)*d*\operatorname{arctanh}(c*x)*\operatorname{polylog}(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3/2*I*b^3*c^2/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}-3/2*I*b^3*c^3/e/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*d*c\operatorname{sgn}(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1))^2-3/4*I*b^3*c^3/e/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*d*c\operatorname{sgn}(I/(1+(c*x+1)^2/(-c^2*x^2+1))) *c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/4*I*b^3*c^3/e/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*d*c\operatorname{sgn}(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1))+3/4*I*b^3*c^3/e/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*d*c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1))*c\operatorname{sgn}(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-b^3*c^2/(c*e*x+c*d)/e*\operatorname{arctanh}(c*x)^3+b^3*c^2/e*\operatorname{arctanh}(c*x)^3/(c*d-e)-a^3*c^2/(c*e*x+c*d)/e+3/4*I*b^3*c^3/e/(c*d+e)/(c*d-e)*\operatorname{arctanh}(c*x)^2*\operatorname{Pi}*d*c\operatorname{sgn}(I/(1+(c
\end{aligned}$$

$*x+1)^2/(-c^2*x^2+1))) * csgn(I*(c*x+1)^2/(c^2*x^2-1)) * csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^2,x, algorithm="maxima")

[Out] $3/2*(c*(\log(c*x + 1)/(c*d*e - e^2) - \log(c*x - 1)/(c*d*e + e^2) - 2*\log(x*e + d)/(c^2*d^2 - e^2)) - 2*\arctanh(c*x)/(x*e^2 + d*e))*a^2*b - a^3/(x*e^2 + d*e) + 1/8*((b^3*c*d*e - b^3*e^2 - (b^3*c^2*d*e - b^3*c*e^2)*x)*\log(-c*x + 1)^3 - 3*(2*a*b^2*c^2*d^2 - 2*a*b^2*e^2 - (b^3*c*d*e + b^3*e^2 + (b^3*c^2*d*e + b^3*c*e^2)*x)*\log(c*x + 1))*\log(-c*x + 1)^2)/(c^2*d^3*e + (c^2*d^2*e^2 - e^4)*x - d*e^3) - \text{integrate}(-1/8*((b^3*c*d*e - b^3*e^2 - (b^3*c^2*d*e - b^3*c*e^2)*x)*\log(c*x + 1)^3 + 6*(a*b^2*c*d*e - a*b^2*e^2 - (a*b^2*c^2*d*e - a*b^2*c*e^2)*x)*\log(c*x + 1)^2 - 3*(4*a*b^2*c^2*d^2 - 4*a*b^2*c*d*e + (b^3*c*d*e - b^3*e^2 - (b^3*c^2*d*e - b^3*c*e^2)*x)*\log(c*x + 1)^2 + 4*(a*b^2*c^2*d*e - a*b^2*c*e^2)*x - 2*(b^3*c^2*x^2*e^2 + 2*a*b^2*e^2 - ((2*a*b^2*c - b^3*c)*e^2 - (2*a*b^2*c^2*d + b^3*c^2*d)*e)*x - (2*a*b^2*c*d - b^3*c*d)*e)*\log(c*x + 1))*\log(-c*x + 1))/(c*d^3*e - (c^2*d*e^3 - c*e^4)*x^3 - (2*c^2*d^2*e^2 - 3*c*d*e^3 + e^4)*x^2 - d^2*e^2 - (c^2*d^3*e - 3*c*d^2*e^2 + 2*d*e^3)*x), x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^2,x, algorithm="fricas")

[Out] $\text{integral}((b^3*\arctanh(c*x))^3 + 3*a*b^2*\arctanh(c*x)^2 + 3*a^2*b*\arctanh(c*x) + a^3)/(x^2*e^2 + 2*d*x*e + d^2), x)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))**3/(e*x+d)**2,x)

[Out] Integral((a + b*atanh(c*x))**3/(d + e*x)**2, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^2,x, algorithm="giac")

[Out] integrate((b*arctanh(c*x) + a)^3/(e*x + d)^2, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^3/(d + e*x)^2,x)

[Out] int((a + b*atanh(c*x))^3/(d + e*x)^2, x)

$$3.20 \quad \int \frac{(a+b \tanh^{-1}(cx))^3}{(d+ex)^3} dx$$

Optimal. Leaf size=953

$$\frac{3bc(a+b \tanh^{-1}(cx))^2}{2(c^2d^2 - e^2)(d+ex)} - \frac{(a+b \tanh^{-1}(cx))^3}{2e(d+ex)^2} - \frac{3b^2c^2(a+b \tanh^{-1}(cx)) \log\left(\frac{2}{1-cx}\right)}{2(cd-e)(cd+e)^2} + \frac{3bc^2(a+b \tanh^{-1}(cx))}{4e(cd+e)^2}$$

[Out] $3/2*b*c*(a+b*\operatorname{arctanh}(c*x))^2/(c^2*d^2-e^2)/(e*x+d)-1/2*(a+b*\operatorname{arctanh}(c*x))^3/e/(e*x+d)^2-3/2*b^2*c^2*(a+b*\operatorname{arctanh}(c*x))*\ln(2/(-c*x+1))/(c*d-e)/(c*d+e)^2+3/4*b*c^2*(a+b*\operatorname{arctanh}(c*x))^2*\ln(2/(-c*x+1))/e/(c*d+e)^2+3/2*b^2*c^2*(a+b*\operatorname{arctanh}(c*x))*\ln(2/(c*x+1))/(c*d-e)^2/(c*d+e)-3*b^2*c^2*e*(a+b*\operatorname{arctanh}(c*x))*\ln(2/(c*x+1))/(c^2*d^2-e^2)^2-3/4*b*c^2*(a+b*\operatorname{arctanh}(c*x))^2*\ln(2/(c*x+1))/(c*d-e)^2/e+3*b*c^3*d*(a+b*\operatorname{arctanh}(c*x))^2*\ln(2/(c*x+1))/(c^2*d^2-e^2)^2+3*b^2*c^2*e*(a+b*\operatorname{arctanh}(c*x))*\ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)^2-3*b*c^3*d*(a+b*\operatorname{arctanh}(c*x))^2*\ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)^2-3/4*b^3*c^2*\operatorname{polylog}(2,1-2/(-c*x+1))/(c*d-e)/(c*d+e)^2+3/4*b^2*c^2*(a+b*\operatorname{arctanh}(c*x))*\operatorname{polylog}(2,1-2/(-c*x+1))/e/(c*d+e)^2-3/4*b^3*c^2*\operatorname{polylog}(2,1-2/(c*x+1))/(c*d-e)^2/(c*d+e)+3/2*b^3*c^2*e*\operatorname{polylog}(2,1-2/(c*x+1))/(c^2*d^2-e^2)^2+3/4*b^2*c^2*(a+b*\operatorname{arctanh}(c*x))*\operatorname{polylog}(2,1-2/(c*x+1))/(c*d-e)^2/e-3*b^2*c^3*d*(a+b*\operatorname{arctanh}(c*x))*\operatorname{polylog}(2,1-2/(c*x+1))/(c^2*d^2-e^2)^2-3/2*b^3*c^2*e*\operatorname{polylog}(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)^2+3*b^2*c^3*d*(a+b*\operatorname{arctanh}(c*x))*\operatorname{polylog}(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)^2-3/8*b^3*c^2*\operatorname{polylog}(3,1-2/(-c*x+1))/e/(c*d+e)^2+3/8*b^3*c^2*\operatorname{polylog}(3,1-2/(c*x+1))/(c*d-e)^2/e-3/2*b^3*c^3*d*\operatorname{polylog}(3,1-2/(c*x+1))/(c^2*d^2-e^2)^2+3/2*b^3*c^3*d*\operatorname{polylog}(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)^2$

Rubi [A]

time = 0.76, antiderivative size = 953, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6065, 6055, 6095, 6205, 6745, 6203, 2449, 2352, 6057, 2497, 6059}

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])^3/(d + e*x)^3, x]

[Out] $(3*b*c*(a+b*\operatorname{ArcTanh}[c*x])^2)/(2*(c^2*d^2 - e^2)*(d+e*x)) - (a+b*\operatorname{ArcTanh}[c*x])^3/(2*e*(d+e*x)^2) - (3*b^2*c^2*(a+b*\operatorname{ArcTanh}[c*x])*Log[2/(1-c*x)])/(2*(c*d-e)*(c*d+e)^2) + (3*b*c^2*(a+b*\operatorname{ArcTanh}[c*x])^2*Log[2/(1-c*x)])/(4*e*(c*d+e)^2) - (3*b^2*c^2*e*(a+b*\operatorname{ArcTanh}[c*x])*Log[2/(1+c*x)])/((c*d-e)^2*(c*d+e)^2) + (3*b^2*c^2*(a+b*\operatorname{ArcTanh}[c*x])*Log[2/(1+c*x)])/(2*(c*d-e)^2*(c*d+e)) - (3*b*c^2*(a+b*\operatorname{ArcTanh}[c*x])^2*Log[2/(1+c*x)])/(4*(c*d-e)^2*e) + (3*b*c^3*d*(a+b*\operatorname{ArcTanh}[c*x])^2*Log[2/(1+c*x)])/(4*(c*d-e)^2*e) + (3*b*c^3*d*(a+b*\operatorname{ArcTanh}[c*x])^2*Log[2/(1+c*x)])/(4*(c*d-e)^2*e)$

```

+ c*x)]/((c*d - e)^2*(c*d + e)^2) + (3*b^2*c^2*e*(a + b*ArcTanh[c*x])*Log[
(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/((c*d - e)^2*(c*d + e)^2) - (3*b*c^
3*d*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/((c*
d - e)^2*(c*d + e)^2) - (3*b^3*c^2*PolyLog[2, 1 - 2/(1 - c*x)])/(4*(c*d - e
)*(c*d + e)^2) + (3*b^2*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x
)]/(4*e*(c*d + e)^2) + (3*b^3*c^2*e*PolyLog[2, 1 - 2/(1 + c*x)])/(2*(c*d -
e)^2*(c*d + e)^2) - (3*b^3*c^2*PolyLog[2, 1 - 2/(1 + c*x)])/(4*(c*d - e)^2*
(c*d + e)) + (3*b^2*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)]/(
4*(c*d - e)^2*e) - (3*b^2*c^3*d*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 +
c*x)]/((c*d - e)^2*(c*d + e)^2) - (3*b^3*c^2*e*PolyLog[2, 1 - (2*c*(d + e*
x))/((c*d + e)*(1 + c*x))]/(2*(c*d - e)^2*(c*d + e)^2) + (3*b^2*c^3*d*(a +
b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/((c*
d - e)^2*(c*d + e)^2) - (3*b^3*c^2*PolyLog[3, 1 - 2/(1 - c*x)])/(8*e*(c*d
+ e)^2) + (3*b^3*c^2*PolyLog[3, 1 - 2/(1 + c*x)])/(8*(c*d - e)^2*e) - (3*b^
3*c^3*d*PolyLog[3, 1 - 2/(1 + c*x)])/(2*(c*d - e)^2*(c*d + e)^2) + (3*b^3*c
^3*d*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/(2*(c*d - e)^2*
(c*d + e)^2)

```

Rule 2352

```

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

```

Rule 2449

```

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

```

Rule 2497

```

Int[Log[u]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]

```

Rule 6055

```

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0]

```

Rule 6057

```

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := S
imp[(-(a + b*ArcTanh[c*x])*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L

```

og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])*(Log[2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/e), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 6059

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^2/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-a + b*ArcTanh[c*x])^2*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcTanh[c*x])^2*(Log[2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/e), x] + Simp[b*(a + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/e), x] + Simp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 - 2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 6065

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^(p_)*((d_) + (e_.)*(x_))^(q_), x_Symbol] := Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] - Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 6095

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 6203

Int[(Log[u]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^(p_))/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]

Rule 6205

Int[(Log[u]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^(p_))/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(-a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx))^3}{(d + ex)^3} dx &= -\frac{(a + b \tanh^{-1}(cx))^3}{2e(d + ex)^2} + \frac{(3bc) \int \left(-\frac{c^2(a + b \tanh^{-1}(cx))^2}{2(cd+e)^2(-1+cx)} + \frac{c^2(a + b \tanh^{-1}(cx))^2}{2(cd-e)^2(1+cx)} + \frac{e^2(a + b \tanh^{-1}(cx))^2}{(-cd+e)^2} \right) dx}{2e} \\
&= -\frac{(a + b \tanh^{-1}(cx))^3}{2e(d + ex)^2} + \frac{(3bc^3) \int \frac{(a + b \tanh^{-1}(cx))^2}{1+cx} dx}{4(cd - e)^2 e} - \frac{(3bc^3) \int \frac{(a + b \tanh^{-1}(cx))^2}{-1+cx} dx}{4e(cd + e)^2} \\
&= \frac{3bc(a + b \tanh^{-1}(cx))^2}{2(cd - e)(cd + e)(d + ex)} - \frac{(a + b \tanh^{-1}(cx))^3}{2e(d + ex)^2} + \frac{3bc^2(a + b \tanh^{-1}(cx))^2 \log\left(\frac{d + ex}{cd + e}\right)}{4e(cd + e)^2} \\
&= \frac{3bc(a + b \tanh^{-1}(cx))^2}{2(cd - e)(cd + e)(d + ex)} - \frac{(a + b \tanh^{-1}(cx))^3}{2e(d + ex)^2} + \frac{3bc^2(a + b \tanh^{-1}(cx))^2 \log\left(\frac{d + ex}{cd + e}\right)}{4e(cd + e)^2} \\
&= \frac{3bc(a + b \tanh^{-1}(cx))^2}{2(cd - e)(cd + e)(d + ex)} - \frac{(a + b \tanh^{-1}(cx))^3}{2e(d + ex)^2} - \frac{3b^2c^2(a + b \tanh^{-1}(cx)) \log\left(\frac{d + ex}{cd + e}\right)}{2(cd - e)(cd + e)^2} \\
&= \frac{3bc(a + b \tanh^{-1}(cx))^2}{2(cd - e)(cd + e)(d + ex)} - \frac{(a + b \tanh^{-1}(cx))^3}{2e(d + ex)^2} - \frac{3b^2c^2(a + b \tanh^{-1}(cx)) \log\left(\frac{d + ex}{cd + e}\right)}{2(cd - e)(cd + e)^2} \\
&= \frac{3bc(a + b \tanh^{-1}(cx))^2}{2(cd - e)(cd + e)(d + ex)} - \frac{(a + b \tanh^{-1}(cx))^3}{2e(d + ex)^2} - \frac{3b^2c^2(a + b \tanh^{-1}(cx)) \log\left(\frac{d + ex}{cd + e}\right)}{2(cd - e)(cd + e)^2}
\end{aligned}$$

Mathematica [F]

time = 62.53, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tanh^{-1}(cx))^3}{(d + ex)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(d + e*x)^3, x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x])^3/(d + e*x)^3, x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 15.39, size = 53542, normalized size = 56.18

method	result	size
derivativedivides	Expression too large to display	53542
default	Expression too large to display	53542

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -3/4*((4*c^2*d*log(x*e + d)/(c^4*d^4 - 2*c^2*d^2*e^2 + e^4) - c*log(c*x + 1)
)/(c^2*d^2*e - 2*c*d*e^2 + e^3) + c*log(c*x - 1)/(c^2*d^2*e + 2*c*d*e^2 + e
^3) - 2/(c^2*d^3 + (c^2*d^2*e - e^3)*x - d*e^2))*c + 2*arctanh(c*x)/(x^2*e
^3 + 2*d*x*e^2 + d^2*e))*a^2*b - 1/2*a^3/(x^2*e^3 + 2*d*x*e^2 + d^2*e) + 1/1
6*((2*b^3*c^3*d^3*e - 3*b^3*c^2*d^2*e^2 + b^3*e^4 - (b^3*c^4*d^2*e^2 - 2*b
^3*c^3*d*e^3 + b^3*c^2*e^4)*x^2 - 2*(b^3*c^4*d^3*e - 2*b^3*c^3*d^2*e^2 + b^3
*c^2*d*e^3)*x)*log(-c*x + 1)^3 - 3*(2*a*b^2*c^4*d^4 - 2*b^3*c^3*d^3*e - 4*a
*b^2*c^2*d^2*e^2 + 2*b^3*c*d*e^3 + 2*a*b^2*e^4 - 2*(b^3*c^3*d^2*e^2 - b^3*c
*e^4)*x - (2*b^3*c^3*d^3*e + 3*b^3*c^2*d^2*e^2 - b^3*e^4 + (b^3*c^4*d^2*e^2
+ 2*b^3*c^3*d*e^3 + b^3*c^2*e^4)*x^2 + 2*(b^3*c^4*d^3*e + 2*b^3*c^3*d^2*e
^2 + b^3*c^2*d*e^3)*x)*log(c*x + 1))*log(-c*x + 1)^2)/(c^4*d^6*e - 2*c^2*d^4
*e^3 + (c^4*d^4*e^3 - 2*c^2*d^2*e^5 + e^7)*x^2 + d^2*e^5 + 2*(c^4*d^5*e^2 -
2*c^2*d^3*e^4 + d*e^6)*x) - integrate(-1/8*((b^3*c^3*d^3*e - b^3*c^2*d^2*e
^2 - b^3*c*d*e^3 + b^3*e^4 - (b^3*c^4*d^3*e - b^3*c^3*d^2*e^2 - b^3*c^2*d*e
^3 + b^3*c*e^4)*x)*log(c*x + 1)^3 + 6*(a*b^2*c^3*d^3*e - a*b^2*c^2*d^2*e^2
- a*b^2*c*d*e^3 + a*b^2*e^4 - (a*b^2*c^4*d^3*e - a*b^2*c^3*d^2*e^2 - a*b^2*
c^2*d*e^3 + a*b^2*c*e^4)*x)*log(c*x + 1)^2 - 3*(2*a*b^2*c^4*d^4 + 2*a*b^2*c
*d*e^3 - 2*(b^3*c^3*d*e^3 - b^3*c^2*e^4)*x^2 + (b^3*c^3*d^3*e - b^3*c^2*d^2
*e^2 - b^3*c*d*e^3 + b^3*e^4 - (b^3*c^4*d^3*e - b^3*c^3*d^2*e^2 - b^3*c^2*d
*e^3 + b^3*c*e^4)*x)*log(c*x + 1)^2 + 2*(a*b^2*c^4*d^3*e + a*b^2*c*e^4 - (a
*b^2*c^2*d - 2*b^3*c^2*d)*e^3 - (a*b^2*c^3*d^2 + 2*b^3*c^3*d^2)*e^2)*x - 2*
(a*b^2*c^2*d^2 - b^3*c^2*d^2)*e^2 - 2*(a*b^2*c^3*d^3 + b^3*c^3*d^3)*e - ((b
^3*c^4*d^3*e + b^3*c^3*e^4)*x^3 - 4*a*b^2*e^4 + 3*(b^3*c^4*d^2*e^2 + b^3*c^
3*d*e^3)*x^2 + ((4*a*b^2*c - b^3*c)*e^4 - (4*a*b^2*c^2*d - b^3*c^2*d)*e^3 -
4*(a*b^2*c^3*d^2 - b^3*c^3*d^2)*e^2 + 2*(2*a*b^2*c^4*d^3 + b^3*c^4*d^3)*e)
*x + (4*a*b^2*c*d - b^3*c*d)*e^3 + (4*a*b^2*c^2*d^2 + b^3*c^2*d^2)*e^2 - 2*
(2*a*b^2*c^3*d^3 - b^3*c^3*d^3)*e)*log(c*x + 1))*log(-c*x + 1))/(c^3*d^6*e
```

- c^2*d^5*e^2 - c*d^4*e^3 - (c^4*d^3*e^4 - c^3*d^2*e^5 - c^2*d*e^6 + c*e^7)
 *x^4 - (3*c^4*d^4*e^3 - 4*c^3*d^3*e^4 - 2*c^2*d^2*e^5 + 4*c*d*e^6 - e^7)*x^3
 + d^3*e^4 - 3*(c^4*d^5*e^2 - 2*c^3*d^4*e^3 + 2*c^2*d^2*e^5 - d*e^6)*x^2 - (c^4*d^6*e
 - 4*c^3*d^5*e^2 + 2*c^2*d^4*e^3 + 4*c*d^3*e^4 - 3*d^2*e^5)*x), x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x)
) + a^3)/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))**3/(e*x+d)**3,x)

[Out] Integral((a + b*atanh(c*x))**3/(d + e*x)**3, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^3,x, algorithm="giac")

[Out] integrate((b*arctanh(c*x) + a)^3/(e*x + d)^3, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^3/(d + e*x)^3,x)

[Out] int((a + b*atanh(c*x))^3/(d + e*x)^3, x)

$$3.21 \quad \int \frac{a+b \tanh^{-1}(cx)}{1+2cx} dx$$

Optimal. Leaf size=67

$$\frac{(a - b \tanh^{-1}(\frac{1}{2})) \log(-\frac{1+2cx}{2d})}{2c} - \frac{b \text{PolyLog}(2, -1 - 2cx)}{4c} + \frac{b \text{PolyLog}(2, \frac{1}{3}(1 + 2cx))}{4c}$$

[Out] 1/2*(a-b*arctanh(1/2))*ln(1/2*(-2*c*x-1)/d)/c-1/4*b*polylog(2,-2*c*x-1)/c+1/4*b*polylog(2,2/3*c*x+1/3)/c

Rubi [A]

time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.63, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6057, 2449, 2352, 2497}

$$-\frac{\log(\frac{2}{cx+1})(a+b \tanh^{-1}(cx))}{2c} + \frac{\log(\frac{2(2cx+1)}{3(cx+1)})(a+b \tanh^{-1}(cx))}{2c} + \frac{b \text{Li}_2(1 - \frac{2}{cx+1})}{4c} - \frac{b \text{Li}_2(1 - \frac{2(2cx+1)}{3(cx+1)})}{4c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])/(1 + 2*c*x), x]

[Out] -1/2*((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/c + ((a + b*ArcTanh[c*x])*Log[(2*(1 + 2*c*x))/(3*(1 + c*x))])/(2*c) + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(4*c) - (b*PolyLog[2, 1 - (2*(1 + 2*c*x))/(3*(1 + c*x))])/(4*c)

Rule 2352

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2449

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 2497

Int[Log[u]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 6057

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-a + b*ArcTanh[c*x])*Log[2/(1 + c*x)]/e, x] + (Dist[b*(c/e), Int[L

og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rubi steps

$$\int \frac{a + b \tanh^{-1}(cx)}{1 + 2cx} dx = -\frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{2c} + \frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2(1+2cx)}{3(1+cx)}\right)}{2c} + \frac{1}{2}b \int \frac{\log\left(\frac{2}{1+cx}\right)}{1+cx} dx$$

$$= -\frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{2c} + \frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2(1+2cx)}{3(1+cx)}\right)}{2c} - \frac{b \operatorname{Li}_2\left(1 - \frac{2}{1+cx}\right)}{4c}$$

$$= -\frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1+cx}\right)}{2c} + \frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2(1+2cx)}{3(1+cx)}\right)}{2c} + \frac{b \operatorname{Li}_2\left(1 - \frac{2}{1+cx}\right)}{4c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 240, normalized size = 3.58

$a \log(1+2cx) + b \operatorname{arctanh}(cx) \log(1-c^2x^2) + \log(\operatorname{tanh}(\frac{1}{2} + \operatorname{arctanh}(cx))) - \frac{1}{2} \left((1-2 \operatorname{arctanh}(cx))^2 + (\operatorname{tanh}^{-1}(cx))^2 + (1-2 \operatorname{arctanh}(cx)) \log(1+c^{2 \operatorname{arctanh}(cx)}) + 2 \operatorname{arctanh}(cx) \log(1-c^{2 \operatorname{arctanh}(cx)}) \right) - (1-2 \operatorname{arctanh}(cx)) \log\left(\frac{2(1+2cx)}{3(1+cx)}\right) - 2(\operatorname{tanh}^{-1}(cx) \log(\operatorname{tanh}(\frac{1}{2} + \operatorname{arctanh}(cx))) - \operatorname{PolyLog}(2, -e^{2 \operatorname{arctanh}(cx)}) - \operatorname{PolyLog}(2, e^{-2 \operatorname{arctanh}(cx)})) \right)$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c*x])/(1 + 2*c*x), x]
[Out] (a*Log[1 + 2*c*x] + b*ArcTanh[c*x]*(Log[1 - c^2*x^2]/2 + Log[I*Sinh[ArcTanh[1/2] + ArcTanh[c*x]]]) - (I/2)*b*((-1/4*I)*(Pi - (2*I)*ArcTanh[c*x])^2 + I*(ArcTanh[1/2] + ArcTanh[c*x])^2 + (Pi - (2*I)*ArcTanh[c*x])*Log[1 + E^(2*ArcTanh[c*x])]) + (2*I)*(ArcTanh[1/2] + ArcTanh[c*x])*Log[1 - E^(-2*(ArcTanh[1/2] + ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c*x])*Log[2/Sqrt[1 - c^2*x^2]] - (2*I)*(ArcTanh[1/2] + ArcTanh[c*x])*Log[(2*I)*Sinh[ArcTanh[1/2] + ArcTanh[c*x]]) - I*PolyLog[2, -E^(2*ArcTanh[c*x])] - I*PolyLog[2, E^(-2*(ArcTanh[1/2] + ArcTanh[c*x]))])/(2*c)

Maple [A]
time = 0.26, size = 101, normalized size = 1.51

method	result
derivativedivides	$\frac{a \ln(2cx+1)}{2} + \frac{b \ln(2cx+1) \operatorname{arctanh}(cx)}{2} + \frac{b \ln\left(\frac{2}{3} - \frac{2cx}{3}\right) \ln(2cx+1)}{4} - \frac{b \ln\left(\frac{2}{3} - \frac{2cx}{3}\right) \ln\left(\frac{2cx}{3} + \frac{1}{3}\right)}{4} - \frac{b \operatorname{dilog}\left(\frac{2cx}{3} + \frac{1}{3}\right)}{4} - \frac{b \operatorname{dilog}(2cx+2)}{4} - \frac{b \operatorname{dilog}(2cx+2)}{4}$
default	$\frac{a \ln(2cx+1)}{2} + \frac{b \ln(2cx+1) \operatorname{arctanh}(cx)}{2} + \frac{b \ln\left(\frac{2}{3} - \frac{2cx}{3}\right) \ln(2cx+1)}{4} - \frac{b \ln\left(\frac{2}{3} - \frac{2cx}{3}\right) \ln\left(\frac{2cx}{3} + \frac{1}{3}\right)}{4} - \frac{b \operatorname{dilog}\left(\frac{2cx}{3} + \frac{1}{3}\right)}{4} - \frac{b \operatorname{dilog}(2cx+2)}{4} - \frac{b \operatorname{dilog}(2cx+2)}{4}$

risch	$\frac{b \ln\left(\frac{2cx}{3} + \frac{1}{3}\right) \ln\left(\frac{2}{3} - \frac{2cx}{3}\right)}{4c} - \frac{b \ln\left(\frac{2cx}{3} + \frac{1}{3}\right) \ln(-cx+1)}{4c} + \frac{b \operatorname{dilog}\left(\frac{2}{3} - \frac{2cx}{3}\right)}{4c} + \frac{a \ln(-2cx-1)}{2c} - \frac{b \ln(-2cx-1) \ln(2cx+1)}{4c}$
-------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x))/(2*c*x+1),x,method=_RETURNVERBOSE)`

[Out] $1/c*(1/2*a*\ln(2*c*x+1)+1/2*b*\ln(2*c*x+1)*\operatorname{arctanh}(c*x)+1/4*b*\ln(2/3-2/3*c*x)*\ln(2*c*x+1)-1/4*b*\ln(2/3-2/3*c*x)*\ln(2/3*c*x+1/3)-1/4*b*\operatorname{dilog}(2/3*c*x+1/3)-1/4*b*\operatorname{dilog}(2*c*x+2)-1/4*b*\ln(2*c*x+1)*\ln(2*c*x+2))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))/(2*c*x+1),x, algorithm="maxima")`

[Out] $1/2*b*\operatorname{integrate}((\log(cx + 1) - \log(-cx + 1))/(2*c*x + 1), x) + 1/2*a*\log(2*c*x + 1)/c$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))/(2*c*x+1),x, algorithm="fricas")`

[Out] `integral((b*arctanh(c*x) + a)/(2*c*x + 1), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}(cx)}{2cx + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x))/(2*c*x+1),x)`

[Out] `Integral((a + b*atanh(c*x))/(2*c*x + 1), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(2*c*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/(2*c*x + 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{atanh}(cx)}{2cx + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(2*c*x + 1),x)
```

```
[Out] int((a + b*atanh(c*x))/(2*c*x + 1), x)
```

$$3.22 \quad \int \frac{\tanh^{-1}(x)}{1 - \sqrt{2}x} dx$$

Optimal. Leaf size=88

$$-\frac{\tanh^{-1}\left(\frac{1}{\sqrt{2}}\right) \log\left(1 - \sqrt{2}x\right)}{\sqrt{2}} - \frac{\text{PolyLog}\left(2, -\frac{\sqrt{2}-2x}{2-\sqrt{2}}\right)}{2\sqrt{2}} + \frac{\text{PolyLog}\left(2, \frac{\sqrt{2}-2x}{2+\sqrt{2}}\right)}{2\sqrt{2}}$$

[Out] $-1/2*\text{arctanh}(1/2*2^{(1/2)})*\ln(1-x*2^{(1/2)})*2^{(1/2)}-1/4*\text{polylog}(2, (2*x-2^{(1/2)})/(2-2^{(1/2)}))*2^{(1/2)}+1/4*\text{polylog}(2, (-2*x+2^{(1/2)})/(2+2^{(1/2)}))*2^{(1/2)}$

Rubi [A]

time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6057, 2449, 2352, 2497}

$$-\frac{\text{Li}_2\left(1 - \frac{2}{x+1}\right)}{2\sqrt{2}} + \frac{\text{Li}_2\left(\frac{2^{(1+\sqrt{2})}(1-\sqrt{2}x)}{x+1} + 1\right)}{2\sqrt{2}} + \frac{\log\left(\frac{2}{x+1}\right) \tanh^{-1}(x)}{\sqrt{2}} - \frac{\log\left(-\frac{2^{(1+\sqrt{2})}(1-\sqrt{2}x)}{x+1}\right) \tanh^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] `Int[ArcTanh[x]/(1 - Sqrt[2]*x), x]`

[Out] $(\text{ArcTanh}[x]*\text{Log}[2/(1+x)])/\text{Sqrt}[2] - (\text{ArcTanh}[x]*\text{Log}[(-2*(1+\text{Sqrt}[2]))*(1-\text{Sqrt}[2]*x)/(1+x)])/\text{Sqrt}[2] - \text{PolyLog}[2, 1 - 2/(1+x)]/(2*\text{Sqrt}[2]) + \text{PolyLog}[2, 1 + (2*(1+\text{Sqrt}[2]))*(1-\text{Sqrt}[2]*x)/(1+x)]/(2*\text{Sqrt}[2])$

Rule 2352

`Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]`

Rule 2449

`Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]`

Rule 2497

`Int[Log[u]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[Pq^m*((1-u)/D[u, x])]}, Simp[C*PolyLog[2, 1-u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]`

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/((d_.) + (e_.)*(x_.)), x_Symbol] := S
imp[(- (a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(x)}{1 - \sqrt{2}x} dx &= \frac{\tanh^{-1}(x) \log\left(\frac{2}{1+x}\right)}{\sqrt{2}} - \frac{\tanh^{-1}(x) \log\left(-\frac{2(1+\sqrt{2})(1-\sqrt{2}x)}{1+x}\right)}{\sqrt{2}} - \frac{\int \frac{\log\left(\frac{2}{1+x}\right) dx}{1-x^2}}{\sqrt{2}} + \frac{\int \frac{\log\left(-\frac{2(1+\sqrt{2})(1-\sqrt{2}x)}{1+x}\right) dx}{1-x^2}}{\sqrt{2}} \\ &= \frac{\tanh^{-1}(x) \log\left(\frac{2}{1+x}\right)}{\sqrt{2}} - \frac{\tanh^{-1}(x) \log\left(-\frac{2(1+\sqrt{2})(1-\sqrt{2}x)}{1+x}\right)}{\sqrt{2}} + \frac{\text{Li}_2\left(1 + \frac{2(1+\sqrt{2})(1-\sqrt{2}x)}{1+x}\right)}{2\sqrt{2}} \\ &= \frac{\tanh^{-1}(x) \log\left(\frac{2}{1+x}\right)}{\sqrt{2}} - \frac{\tanh^{-1}(x) \log\left(-\frac{2(1+\sqrt{2})(1-\sqrt{2}x)}{1+x}\right)}{\sqrt{2}} - \frac{\text{Li}_2\left(1 - \frac{2}{1+x}\right)}{2\sqrt{2}} + \frac{\text{Li}_2\left(1 + \frac{2(1+\sqrt{2})(1-\sqrt{2}x)}{1+x}\right)}{2\sqrt{2}} \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 272, normalized size = 3.09

Integrate[ArcTanh[x]/(1 - Sqrt[2]*x), x]

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTanh[x]/(1 - Sqrt[2]*x), x]

[Out] (Pi^2 - 4*ArcTanh[1/Sqrt[2]]^2 - (4*I)*Pi*ArcTanh[x] + 8*ArcTanh[1/Sqrt[2]]*ArcTanh[x] - 8*ArcTanh[x]^2 + 8*ArcTanh[1/Sqrt[2]]*Log[1 - E^(2*ArcTanh[1/Sqrt[2]] - 2*ArcTanh[x])] - 8*ArcTanh[x]*Log[1 - E^(2*ArcTanh[1/Sqrt[2]] - 2*ArcTanh[x])] + (4*I)*Pi*Log[1 + E^(2*ArcTanh[x])] + 8*ArcTanh[x]*Log[1 + E^(2*ArcTanh[x])] - (4*I)*Pi*Log[2/Sqrt[1 - x^2]] - 8*ArcTanh[x]*Log[2/Sqrt[1 - x^2]] - 4*ArcTanh[x]*Log[1 - x^2] - 8*ArcTanh[x]*Log[(-I)*Sinh[ArcTanh[1/Sqrt[2]] - ArcTanh[x]]] - 8*ArcTanh[1/Sqrt[2]]*Log[(-2*I)*Sinh[ArcTanh[1/Sqrt[2]] - ArcTanh[x]]] + 8*ArcTanh[x]*Log[(-2*I)*Sinh[ArcTanh[1/Sqrt[2]] - ArcTanh[x]]] + 4*PolyLog[2, E^(2*ArcTanh[1/Sqrt[2]] - 2*ArcTanh[x])] + 4*PolyLog[2, -E^(2*ArcTanh[x])])/(8*Sqrt[2])

Maple [A]

time = 0.18, size = 127, normalized size = 1.44

method	result
default	$-\frac{\ln(x\sqrt{2}-1)\sqrt{2}\operatorname{arctanh}(x)}{2} - \frac{\sqrt{2}\ln(x\sqrt{2}-1)\ln\left(\frac{\sqrt{2}-x\sqrt{2}}{\sqrt{2}-1}\right)}{4} + \frac{\sqrt{2}\ln(x\sqrt{2}-1)\ln\left(\frac{\sqrt{2}+x\sqrt{2}}{1+\sqrt{2}}\right)}{4}$
risch	$\frac{\sqrt{2}\ln\left(\frac{-2x+\sqrt{2}}{2+\sqrt{2}}\right)\ln\left(\frac{2+2x}{2+\sqrt{2}}\right)}{4} - \frac{\sqrt{2}\ln\left(\frac{-2x+\sqrt{2}}{2+\sqrt{2}}\right)\ln(1+x)}{4} + \frac{\sqrt{2}\operatorname{dilog}\left(\frac{-2+2x}{2+\sqrt{2}}\right)}{4} + \frac{\sqrt{2}\ln\left(\frac{2x-\sqrt{2}}{2-\sqrt{2}}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctanh(x)/(1-x*2^(1/2)),x,method=_RETURNVERBOSE)`

[Out]
$$-1/2*\ln(x*2^{(1/2)}-1)*2^{(1/2)}*\operatorname{arctanh}(x)-1/4*2^{(1/2)}*\ln(x*2^{(1/2)}-1)*\ln((2^{(1/2)}-x*2^{(1/2)})/(2^{(1/2)}-1))+1/4*2^{(1/2)}*\ln(x*2^{(1/2)}-1)*\ln((2^{(1/2)}+x*2^{(1/2)})/(1+2^{(1/2)}))-1/4*2^{(1/2)}*\operatorname{dilog}((2^{(1/2)}-x*2^{(1/2)})/(2^{(1/2)}-1))+1/4*2^{(1/2)}*\operatorname{dilog}((2^{(1/2)}+x*2^{(1/2)})/(1+2^{(1/2)}))$$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(69) = 138.

time = 0.46, size = 144, normalized size = 1.64

$$\frac{1}{4}\sqrt{2}(\log(x+1)-\log(x-1))\log(\sqrt{2}x-1)-\frac{1}{2}\sqrt{2}\operatorname{arctanh}(x)\log(\sqrt{2}x-1)-\frac{1}{4}\sqrt{2}\left(\log(x+1)\log\left(-\frac{\sqrt{2}x+\sqrt{2}}{\sqrt{2}+1}+1\right)+\operatorname{Li}_2\left(\frac{\sqrt{2}x+\sqrt{2}}{\sqrt{2}+1}\right)\right)+\frac{1}{4}\sqrt{2}\left(\log(x-1)\log\left(\frac{\sqrt{2}x-\sqrt{2}}{\sqrt{2}-1}+1\right)+\operatorname{Li}_2\left(-\frac{\sqrt{2}x-\sqrt{2}}{\sqrt{2}-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctanh(x)/(1-x*2^(1/2)),x, algorithm="maxima")`

[Out]
$$1/4*\sqrt{2}*(\log(x+1)-\log(x-1))*\log(\sqrt{2}*x-1)-1/2*\sqrt{2}*\operatorname{arctanh}(x)*\log(\sqrt{2}*x-1)-1/4*\sqrt{2}*(\log(x+1)*\log(-(\sqrt{2}*x+\sqrt{2})/(\sqrt{2}+1)+1)+\operatorname{dilog}((\sqrt{2}*x+\sqrt{2})/(\sqrt{2}+1)))+1/4*\sqrt{2}*(\log(x-1)*\log((\sqrt{2}*x-\sqrt{2})/(\sqrt{2}-1)+1)+\operatorname{dilog}(-(\sqrt{2}*x-\sqrt{2})/(\sqrt{2}-1)))$$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctanh(x)/(1-x*2^(1/2)),x, algorithm="fricas")`**[Out]** `integral(-(\sqrt{2}*x+1)*arctanh(x)/(2*x^2-1), x)`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}(x)}{\sqrt{2}x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atanh(x)/(1-x*2**(1/2)),x)`

[Out] `-Integral(atanh(x)/(sqrt(2)*x - 1), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctanh(x)/(1-x*2^(1/2)),x, algorithm="giac")`

[Out] `integrate(-arctanh(x)/(sqrt(2)*x - 1), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$- \int \frac{\operatorname{atanh}(x)}{\sqrt{2} x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-atanh(x)/(2^(1/2)*x - 1),x)`

[Out] `-int(atanh(x)/(2^(1/2)*x - 1), x)`

3.23 $\int (d + ex)^3 (a + b \tanh^{-1}(cx^2)) dx$

Optimal. Leaf size=182

$$\frac{2bde^2x}{c} + \frac{be^3x^2}{4c} + \frac{bd(cd^2 - e^2) \operatorname{ArcTan}(\sqrt{c}x)}{c^{3/2}} - \frac{bd(cd^2 + e^2) \tanh^{-1}(\sqrt{c}x)}{c^{3/2}} + \frac{(d + ex)^4 (a + b \tanh^{-1}(cx^2))}{4e}$$

```
[Out] 2*b*d*e^2*x/c+1/4*b*e^3*x^2/c+b*d*(c*d^2-e^2)*arctan(x*c^(1/2))/c^(3/2)+1/4
*(e*x+d)^4*(a+b*arctanh(c*x^2))/e-b*d*(c*d^2+e^2)*arctanh(x*c^(1/2))/c^(3/2
)+1/8*b*(c^2*d^4+6*c*d^2*e^2+e^4)*ln(-c*x^2+1)/c^2/e-1/8*b*(c^2*d^4-6*c*d^2
*e^2+e^4)*ln(c*x^2+1)/c^2/e
```

Rubi [A]

time = 0.20, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6071, 1847, 1294, 1181, 211, 214, 1833, 1824, 647, 31}

$$\frac{(d + ex)^4 (a + b \tanh^{-1}(cx^2))}{4e} + \frac{bd \operatorname{ArcTan}(\sqrt{c}x) (cd^2 - e^2)}{c^{3/2}} - \frac{bd (cd^2 + e^2) \tanh^{-1}(\sqrt{c}x)}{c^{3/2}} + \frac{b(e^2d^4 + 6cd^2e^2 + e^4) \log(1 - cx^2)}{8c^2e} - \frac{b(e^2d^4 - 6cd^2e^2 + e^4) \log(cx^2 + 1)}{8c^2e} + \frac{2bde^2x}{c} + \frac{be^3x^2}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcTanh[c*x^2]), x]
```

```
[Out] (2*b*d*e^2*x)/c + (b*e^3*x^2)/(4*c) + (b*d*(c*d^2 - e^2)*ArcTan[Sqrt[c]*x])/
/c^(3/2) - (b*d*(c*d^2 + e^2)*ArcTanh[Sqrt[c]*x])/c^(3/2) + ((d + e*x)^4*(a
+ b*ArcTanh[c*x^2]))/(4*e) + (b*(c^2*d^4 + 6*c*d^2*e^2 + e^4)*Log[1 - c*x^
2])/(8*c^2*e) - (b*(c^2*d^4 - 6*c*d^2*e^2 + e^4)*Log[1 + c*x^2])/(8*c^2*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt
[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

$(-a)*c]$

Rule 1181

$\text{Int}[(d_ + (e_)*(x_)^2)/((a_ + (c_)*(x_)^4), x_Symbol] := \text{With}[\{q = \text{Rt}[-a]*c, 2]\}, \text{Dist}[e/2 + c*(d/(2*q)), \text{Int}[1/(-q + c*x^2), x], x] + \text{Dist}[e/2 - c*(d/(2*q)), \text{Int}[1/(q + c*x^2), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[(-a)*c]$

Rule 1294

$\text{Int}[(f_)*(x_)^{(m_)}*((d_ + (e_)*(x_)^2)*((a_ + (c_)*(x_)^4)^{(p_)}), x_Symbol] := \text{Simp}[e*f*(f*x)^{(m-1)}*((a + c*x^4)^{(p+1)})/(c*(m + 4*p + 3)), x] - \text{Dist}[f^2/(c*(m + 4*p + 3)), \text{Int}[(f*x)^{(m-2)}*(a + c*x^4)^p*(a*e*(m-1) - c*d*(m + 4*p + 3)*x^2), x], x] /; \text{FreeQ}[\{a, c, d, e, f, p\}, x] \&\& \text{GtQ}[m, 1] \&\& \text{NeQ}[m + 4*p + 3, 0] \&\& \text{IntegerQ}[2*p] \&\& (\text{IntegerQ}[p] \ || \ \text{IntegerQ}[m])]$

Rule 1824

$\text{Int}[(Pq_)*((a_ + (b_)*(x_)^2)^{(p_)}), x_Symbol] := \text{Int}[\text{ExpandIntegrand}[Pq*(a + b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[p, -2]$

Rule 1833

$\text{Int}[(Pq_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)}))^{(p_)}), x_Symbol] := \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^{(m+1)}, Pq, x]*(a + b*x^{\text{Simplify}[n/(m+1)])}]^p, x], x, x^{(m+1)}], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{NeQ}[m, -1] \&\& \text{IGtQ}[\text{Simplify}[n/(m+1)], 0] \&\& \text{PolyQ}[Pq, x^{(m+1)}]$

Rule 1847

$\text{Int}[(Pq_)*((c_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)}))^{(p_)}), x_Symbol] := \text{Module}[\{q = \text{Expon}[Pq, x], j, k\}, \text{Int}[\text{Sum}[(c*x)^{(m+j)}/c^j]*\text{Sum}[\text{Coeff}[Pq, x, j + k*(n/2)]*x^{(k*(n/2))}, \{k, 0, 2*((q-j)/n) + 1\}*(a + b*x^n)^p, \{j, 0, n/2 - 1\}], x]] /; \text{FreeQ}[\{a, b, c, m, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{!PolyQ}[Pq, x^{(n/2)}]$

Rule 6071

$\text{Int}[(a_ + \text{ArcTanh}[c_)*(x_)^{(n_)}]*(b_)*((d_ + (e_)*(x_)^{(m_)}), x_Symbol] := \text{Simp}[(d + e*x)^{(m+1)}*((a + b*\text{ArcTanh}[c*x^n])/(e*(m+1))), x] - \text{Dist}[b*c*(n/(e*(m+1))), \text{Int}[x^{(n-1)}*((d + e*x)^{(m+1)})/(1 - c^2*x^{(2*n)})], x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, n\}, x] \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int (d + ex)^3 (a + b \tanh^{-1}(cx^2)) dx &= \int (a(d + ex)^3 + b(d + ex)^3 \tanh^{-1}(cx^2)) dx \\
&= \frac{a(d + ex)^4}{4e} + b \int (d + ex)^3 \tanh^{-1}(cx^2) dx \\
&= \frac{a(d + ex)^4}{4e} + b \int (d^3 \tanh^{-1}(cx^2) + 3d^2 ex \tanh^{-1}(cx^2) + 3de^2 x^2 \tanh^{-1}(cx^2) + bde^2 x^3 \tanh^{-1}(cx^2)) dx \\
&= \frac{a(d + ex)^4}{4e} + (bd^3) \int \tanh^{-1}(cx^2) dx + (3bd^2 e) \int x \tanh^{-1}(cx^2) dx \\
&= \frac{a(d + ex)^4}{4e} + bd^3 x \tanh^{-1}(cx^2) + \frac{3}{2} bd^2 ex^2 \tanh^{-1}(cx^2) + bde^2 x^3 \tanh^{-1}(cx^2) + \frac{2bde^2 x}{c} \\
&= \frac{2bde^2 x}{c} + \frac{a(d + ex)^4}{4e} + bd^3 x \tanh^{-1}(cx^2) + \frac{3}{2} bd^2 ex^2 \tanh^{-1}(cx^2) + \frac{bde^2 x^3 \tanh^{-1}(cx^2)}{e} \\
&= \frac{2bde^2 x}{c} + \frac{be^3 x^2}{4c} + \frac{a(d + ex)^4}{4e} + \frac{bd^3 \tan^{-1}(\sqrt{c} x)}{\sqrt{c}} - \frac{bd^3 \tan^{-1}(\sqrt{c} x)}{\sqrt{c}} \\
&= \frac{2bde^2 x}{c} + \frac{be^3 x^2}{4c} + \frac{a(d + ex)^4}{4e} + \frac{bd^3 \tan^{-1}(\sqrt{c} x)}{\sqrt{c}} - \frac{bde^2 \tan^{-1}(\sqrt{c} x)}{c^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.34, size = 254, normalized size = 1.40

$$\frac{1}{8} \left(\frac{8d(acd^2 + 2be^2)x}{c} + \frac{2(6acd^2 + be^2)x^2}{c} + 8ade^2x^3 + 2ae^3x^4 + \frac{8bd(cd^2 - e^2)\text{ArcTan}(\sqrt{c}x)}{c^{3/2}} + 2bx(4d^3 + 6d^2ex + 4de^2x^2 + e^3x^3) \tanh^{-1}(cx^2) + \frac{b(4c^{3/2}d^3 + 4\sqrt{c}de^2 + e^3) \log(1 - \sqrt{c}x)}{c^2} + \frac{b(-4c^{3/2}d^3 - 4de^2 + \sqrt{c}e^3) \log(1 + \sqrt{c}x)}{c^2} - \frac{be^3 \log(1 + cx^2)}{c^2} + \frac{6bde^2 \log(1 - cx^2)}{c} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(d + e*x)^3*(a + b*ArcTanh[c*x^2]), x]`

```

[Out] ((8*d*(a*c*d^2 + 2*b*e^2)*x)/c + (2*e*(6*a*c*d^2 + b*e^2)*x^2)/c + 8*a*d*e^2*x^3 + 2*a*e^3*x^4 + (8*b*d*(c*d^2 - e^2)*ArcTan[Sqrt[c]*x])/c^(3/2) + 2*b*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcTanh[c*x^2] + (b*(4*c^(3/2)*d^3 + 4*Sqrt[c]*d*e^2 + e^3)*Log[1 - Sqrt[c]*x])/c^2 + (b*(-4*c^(3/2)*d^3 - 4*c*d*e^2 + Sqrt[c]*e^3)*Log[1 + Sqrt[c]*x])/c^(5/2) - (b*e^3*Log[1 + c*x^2])/c^2 + (6*b*d^2*e*Log[1 - c^2*x^4])/c)/8

```

Maple [A]

time = 0.35, size = 276, normalized size = 1.52

method	result
default	$\frac{(ex+d)^4 a}{4e} + \frac{be^3 \operatorname{arctanh}(cx^2)x^4}{4} + be^2 \operatorname{arctanh}(cx^2)x^3 d + \frac{3be \operatorname{arctanh}(cx^2)x^2 d^2}{2} + b \operatorname{arctanh}(cx^2)x d^3 + \dots$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4}(e*x+d)^4*a/e + \frac{1}{4}*b*e^3*arctanh(c*x^2)*x^4 + b*e^2*arctanh(c*x^2)*x^3*d + \frac{3}{2}*b*e*arctanh(c*x^2)*x^2*d^2 + b*arctanh(c*x^2)*x*d^3 + \frac{1}{4}*b/e*arctanh(c*x^2)*d^4 + \frac{1}{4}*b*e^3*x^2/c + 2*b*d*e^2*x/c + \frac{1}{8}*b/e*\ln(c*x^2-1)*d^4 + \frac{3}{4}*b*e/c*\ln(c*x^2-1)*d^2 + \frac{1}{8}*b*e^3/c^2*\ln(c*x^2-1) - b/c^{(1/2)}*arctanh(x*c^{(1/2)})*d^3 - b*e^2/c^{(3/2)}*arctanh(x*c^{(1/2)})*d - \frac{1}{8}*b/e*\ln(c*x^2+1)*d^4 + \frac{3}{4}*b*e/c*\ln(c*x^2+1)*d^2 - \frac{1}{8}*b*e^3/c^2*\ln(c*x^2+1) + b/c^{(1/2)}*arctan(x*c^{(1/2)})*d^3 - b*e^2/c^{(3/2)}*arctan(x*c^{(1/2)})*d$

Maxima [A]

time = 0.47, size = 238, normalized size = 1.31

$$\frac{1}{4}ax^4e^3 + ad^3x^3e^2 + \frac{3}{2}a^2d^2x^2e + \frac{1}{2}c \left(\frac{2 \arctan(\sqrt{c}x)}{c^3} + \frac{\log\left(\frac{a-\sqrt{c}}{a+\sqrt{c}}\right)}{c^3} \right) + 2x \operatorname{arctanh}(cx^2) b d^3 + a d^2 x + \frac{1}{2} \left(2x^2 \operatorname{arctanh}(cx^2) + \left(\frac{4x}{c^2} - \frac{2 \arctan(\sqrt{c}x)}{c^3} + \frac{\log\left(\frac{a-\sqrt{c}}{a+\sqrt{c}}\right)}{c^3} \right) \right) b d e^2 + \frac{3(2c^2 \operatorname{arctanh}(cx^2) + \log(-c^2x^4 + 1)) b d^2 e}{4c} + \frac{1}{8} \left(2x^2 \operatorname{arctanh}(cx^2) + c \left(\frac{2x^2}{c^2} - \frac{\log(cx^2 + 1)}{c^3} + \frac{\log(cx^2 - 1)}{c^3} \right) \right) b e^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3*(a+b*arctanh(c*x^2)),x, algorithm="maxima")`

[Out] $\frac{1}{4}a^2x^4e^3 + a*d*x^3e^2 + \frac{3}{2}a*d^2*x^2*e + \frac{1}{2}*(c*(2*\arctan(\sqrt{c}*x)/c^{(3/2)} + \log((c*x - \sqrt{c})/(c*x + \sqrt{c}))/c^{(3/2)})) + 2*x*arctanh(c*x^2)*b*d^3 + a*d^3*x + \frac{1}{2}*(2*x^3*arctanh(c*x^2) + c*(4*x/c^2 - 2*\arctan(\sqrt{c}*x)/c^{(5/2)} + \log((c*x - \sqrt{c})/(c*x + \sqrt{c}))/c^{(5/2)}))*b*d*e^2 + \frac{3}{4}*(2*c*x^2*arctanh(c*x^2) + \log(-c^2*x^4 + 1))*b*d^2*e/c + \frac{1}{8}*(2*x^4*arctanh(c*x^2) + c*(2*x^2/c^2 - \log(c*x^2 + 1)/c^3 + \log(c*x^2 - 1)/c^3))*b*e^3$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(156) = 312.

time = 0.40, size = 1137, normalized size = 6.25

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3*(a+b*arctanh(c*x^2)),x, algorithm="fricas")`

[Out] $\frac{1}{8}*(12*a*c^2*d^2*x^2*\cosh(1) + 8*a*c^2*d^3*x + 2*(a*c^2*x^4 + b*c*x^2)*\cosh(1)^3 + 2*(a*c^2*x^4 + b*c*x^2)*\sinh(1)^3 + 8*(a*c^2*d*x^3 + 2*b*c*d*x)*\cosh(1)^2 + 2*(4*a*c^2*d*x^3 + 8*b*c*d*x + 3*(a*c^2*x^4 + b*c*x^2)*\cosh(1))*\sinh(1)^2 + 8*(b*c*d^3 - b*d*\cosh(1)^2 - 2*b*d*\cosh(1)*\sinh(1) - b*d*\sinh(1)^2)*\sqrt{c}*arctan(\sqrt{c}*x) + 4*(b*c*d^3 + b*d*\cosh(1)^2 + 2*b*d*\cosh(1)*\sinh(1) + b*d*\sinh(1)^2)*\sqrt{c}*\log((c*x^2 - 2*\sqrt{c}*x + 1)/(c*x^2 - 1)) + (6*b*c*d^2*\cosh(1) - b*\cosh(1)^3 - 3*b*\cosh(1)*\sinh(1)^2 - b*\sinh(1)^3$

```

+ 3*(2*b*c*d^2 - b*cosh(1)^2)*sinh(1))*log(c*x^2 + 1) + (6*b*c*d^2*cosh(1)
+ b*cosh(1)^3 + 3*b*cosh(1)*sinh(1)^2 + b*sinh(1)^3 + 3*(2*b*c*d^2 + b*cosh
(1)^2)*sinh(1))*log(c*x^2 - 1) + (b*c^2*x^4*cosh(1)^3 + b*c^2*x^4*sinh(1)^3
+ 4*b*c^2*d*x^3*cosh(1)^2 + 6*b*c^2*d^2*x^2*cosh(1) + 4*b*c^2*d^3*x + (3*b
*c^2*x^4*cosh(1) + 4*b*c^2*d*x^3)*sinh(1)^2 + (3*b*c^2*x^4*cosh(1)^2 + 8*b*
c^2*d*x^3*cosh(1) + 6*b*c^2*d^2*x^2)*sinh(1))*log(-(c*x^2 + 1)/(c*x^2 - 1))
+ 2*(6*a*c^2*d^2*x^2 + 3*(a*c^2*x^4 + b*c*x^2)*cosh(1)^2 + 8*(a*c^2*d*x^3
+ 2*b*c*d*x)*cosh(1))*sinh(1))/c^2, 1/8*(12*a*c^2*d^2*x^2*cosh(1) + 8*a*c^2
*d^3*x + 2*(a*c^2*x^4 + b*c*x^2)*cosh(1)^3 + 2*(a*c^2*x^4 + b*c*x^2)*sinh(1)
)^3 + 8*(a*c^2*d*x^3 + 2*b*c*d*x)*cosh(1)^2 + 2*(4*a*c^2*d*x^3 + 8*b*c*d*x
+ 3*(a*c^2*x^4 + b*c*x^2)*cosh(1))*sinh(1)^2 + 8*(b*c*d^3 + b*d*cosh(1)^2 +
2*b*d*cosh(1)*sinh(1) + b*d*sinh(1)^2)*sqrt(-c)*arctan(sqrt(-c)*x) + 4*(b*
c*d^3 - b*d*cosh(1)^2 - 2*b*d*cosh(1)*sinh(1) - b*d*sinh(1)^2)*sqrt(-c)*log
((c*x^2 + 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + (6*b*c*d^2*cosh(1) - b*cosh(1)^3
- 3*b*cosh(1)*sinh(1)^2 - b*sinh(1)^3 + 3*(2*b*c*d^2 - b*cosh(1)^2)*sinh(1)
))*log(c*x^2 + 1) + (6*b*c*d^2*cosh(1) + b*cosh(1)^3 + 3*b*cosh(1)*sinh(1)^
2 + b*sinh(1)^3 + 3*(2*b*c*d^2 + b*cosh(1)^2)*sinh(1))*log(c*x^2 - 1) + (b*
c^2*x^4*cosh(1)^3 + b*c^2*x^4*sinh(1)^3 + 4*b*c^2*d*x^3*cosh(1)^2 + 6*b*c^2
*d^2*x^2*cosh(1) + 4*b*c^2*d^3*x + (3*b*c^2*x^4*cosh(1) + 4*b*c^2*d*x^3)*si
nh(1)^2 + (3*b*c^2*x^4*cosh(1)^2 + 8*b*c^2*d*x^3*cosh(1) + 6*b*c^2*d^2*x^2)
*sinh(1))*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*(6*a*c^2*d^2*x^2 + 3*(a*c^2*x^4
+ b*c*x^2)*cosh(1)^2 + 8*(a*c^2*d*x^3 + 2*b*c*d*x)*cosh(1))*sinh(1))/c^2]

```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2786 vs. $2(168) = 336$.

time = 8.67, size = 2786, normalized size = 15.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**3*(a+b*atanh(c*x**2)),x)

```

[Out] Piecewise((8*a*c**2*d**3*x*sqrt(-1/c)*sqrt(1/c)/(2*c**3*(-1/c)**(3/2)*sqrt(
1/c) - 2*c**3*sqrt(-1/c)*(1/c)**(3/2) + 12*c**2*sqrt(-1/c)*sqrt(1/c)) + 12*
a*c**2*d**2*e*x**2*sqrt(-1/c)*sqrt(1/c)/(2*c**3*(-1/c)**(3/2)*sqrt(1/c) - 2
*c**3*sqrt(-1/c)*(1/c)**(3/2) + 12*c**2*sqrt(-1/c)*sqrt(1/c)) + 8*a*c**2*d*
e**2*x**3*sqrt(-1/c)*sqrt(1/c)/(2*c**3*(-1/c)**(3/2)*sqrt(1/c) - 2*c**3*sqrt
(-1/c)*(1/c)**(3/2) + 12*c**2*sqrt(-1/c)*sqrt(1/c)) + 2*a*c**2*e**3*x**4*s
qrt(-1/c)*sqrt(1/c)/(2*c**3*(-1/c)**(3/2)*sqrt(1/c) - 2*c**3*sqrt(-1/c)*(1/
c)**(3/2) + 12*c**2*sqrt(-1/c)*sqrt(1/c)) + 8*b*c**2*d**3*x*sqrt(-1/c)*sqrt
(1/c)*atanh(c*x**2)/(2*c**3*(-1/c)**(3/2)*sqrt(1/c) - 2*c**3*sqrt(-1/c)*(1/
c)**(3/2) + 12*c**2*sqrt(-1/c)*sqrt(1/c)) - 2*b*c**2*d**3*(-1/c)**(3/2)*log
(x + sqrt(-1/c))/(2*c**3*(-1/c)**(3/2)*sqrt(1/c) - 2*c**3*sqrt(-1/c)*(1/c)*
*(3/2) + 12*c**2*sqrt(-1/c)*sqrt(1/c)) + 2*b*c**2*d**3*(1/c)**(3/2)*log(x +
sqrt(-1/c))/(2*c**3*(-1/c)**(3/2)*sqrt(1/c) - 2*c**3*sqrt(-1/c)*(1/c)**(3/

```


$a*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4), True))$

Giac [A]

time = 7.83, size = 227, normalized size = 1.25

$$\frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{(6acd^2e + be^3)x^2}{4c} + \frac{1}{8}(be^3x^4 + 4bde^2x^3 + 6bd^2ex^2 + 4bd^3x) \log\left(\frac{-cx^2+1}{-cx^2-1}\right) + \frac{(acd^3 + 2bde^2)x}{c} + \frac{(bcd^3 - bde^2)\arctan(\sqrt{c}x)}{c^3} + \frac{(bcd^3 + bde^2)\arctan\left(\frac{-x}{\sqrt{-c}}\right)}{\sqrt{-c}c} + \frac{(6bcd^2e - be^3)\log(cx^2+1)}{8c^2} + \frac{(6bcd^2e + be^3)\log(cx^2-1)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x^2)),x, algorithm="giac")

[Out] $\frac{1}{4}a*e^3*x^4 + a*d*e^2*x^3 + \frac{1}{4}*(6*a*c*d^2*e + b*e^3)*x^2/c + \frac{1}{8}*(b*e^3*x^4 + 4*b*d*e^2*x^3 + 6*b*d^2*e*x^2 + 4*b*d^3*x)*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + (a*c*d^3 + 2*b*d*e^2)*x/c + (b*c*d^3 - b*d*e^2)*\arctan(\sqrt{c}*x)/c^{3/2} + (b*c*d^3 + b*d*e^2)*\arctan(c*x/\sqrt{-c})/(\sqrt{-c}*c) + \frac{1}{8}*(6*b*c*d^2*e - b*e^3)*\log(c*x^2 + 1)/c^2 + \frac{1}{8}*(6*b*c*d^2*e + b*e^3)*\log(c*x^2 - 1)/c^2$

Mupad [B]

time = 2.04, size = 823, normalized size = 4.52

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^2))*(d + e*x)^3,x)

[Out] $\log(cx^2 + 1)*((b*e^3*x^4)/8 + (b*d^3*x)/2 + (3*b*d^2*e*x^2)/4 + (b*d*e^2*x^3)/2) - \log(1 - cx^2)*((b*e^3*x^4)/8 + (b*d^3*x)/2 + (3*b*d^2*e*x^2)/4 + (b*d*e^2*x^3)/2) + (a*e^3*x^4)/4 - (\log(8*c^5*d^6 - c^2*e^6 - 4*d*e^5*(-c^5)^{1/2}) + e^6*x*(-c^5)^{1/2} + 8*c^3*d^2*e^4 + 4*c^4*d^3*e^3*x - 4*c^3*d*e^5*x + 4*c*d^3*e^3*(-c^5)^{1/2} - 8*c^3*d^6*x*(-c^5)^{1/2} - 8*c*d^2*e^4*x*(-c^5)^{1/2} + 4*b*c^2*e^3 - 4*b*c*d^3*(-c^5)^{1/2} + 4*b*d*e^2*(-c^5)^{1/2} - 6*b*c^3*d^2*e)/(8*c^4) - (\log(8*c^5*d^6 - c^2*e^6 + 4*d*e^5*(-c^5)^{1/2}) - e^6*x*(-c^5)^{1/2} + 8*c^3*d^2*e^4 + 4*c^4*d^3*e^3*x - 4*c^3*d*e^5*x - 4*c*d^3*e^3*(-c^5)^{1/2} + 8*c^3*d^6*x*(-c^5)^{1/2} + 8*c*d^2*e^4*x*(-c^5)^{1/2})*(b*c^2*e^3 + 4*b*c*d^3*(-c^5)^{1/2} - 4*b*d*e^2*(-c^5)^{1/2} - 6*b*c^3*d^2*e)/(8*c^4) + (x*(2*a*c^2*d^3 + 4*b*c*d*e^2))/(2*c^2) + (\log(64*c^2*d^12*(c^5)^{7/2} + 128*d^8*e^4*(c^5)^{7/2} - 64*c^20*d^12*x - c^14*e^12*x + c*e^12*(c^5)^{5/2} - 32*c^16*d^4*e^8*x - 128*c^18*d^8*e^4*x + 32*c^3*d^4*e^8*(c^5)^{5/2})*(b*c^2*e^3 + 4*b*c*d^3*(c^5)^{1/2} + 4*b*d*e^2*(c^5)^{1/2} + 6*b*c^3*d^2*e))/(8*c^4) + (\log(8*c^10*d^6 + c^7*e^6 + 8*c^8*d^2*e^4 - 4*d*e^5*(c^5)^{3/2} + e^6*x*(c^5)^{3/2} - 4*c^9*d^3*e^3*x - 4*c*d^3*e^3*(c^5)^{3/2} + 8*c^3*d^6*x*(c^5)^{3/2} - 4*c^8*d*e^5*x + 8*c*d^2*e^4*x*(c^5)^{3/2})*(b*c^2*e^3 - 4*b*c*d^3*(c^5)^{1/2} - 4*b*d*e^2*(c^5)^{1/2} + 6*b*c^3*d^2*e))/(8*c^4) + (x^2*(b*c*e^3 + 6*a*c^2*d^2*e))/(4*c^2) + a*d*e^2*x^3$

3.24 $\int (d + ex)^2 (a + b \tanh^{-1}(cx^2)) dx$

Optimal. Leaf size=158

$$\frac{2be^2x}{3c} + \frac{b(3cd^2 - e^2) \operatorname{ArcTan}(\sqrt{c}x)}{3c^{3/2}} - \frac{b(3cd^2 + e^2) \tanh^{-1}(\sqrt{c}x)}{3c^{3/2}} + \frac{(d + ex)^3 (a + b \tanh^{-1}(cx^2))}{3e} + \frac{bd(cd^2 + e^2)}{3c}$$

[Out] $2/3*b*e^2*x/c + 1/3*b*(3*c*d^2 - e^2)*\arctan(x*c^{(1/2)})/c^{(3/2)} + 1/3*(e*x+d)^3*(a + b*\arctanh(c*x^2))/e - 1/3*b*(3*c*d^2 + e^2)*\arctanh(x*c^{(1/2)})/c^{(3/2)} + 1/6*b*d*(c*d^2 + 3*e^2)*\ln(-c*x^2 + 1)/c - 1/6*b*d*(c*d^2 - 3*e^2)*\ln(c*x^2 + 1)/c/e$

Rubi [A]

time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6071, 1845, 1262, 647, 31, 1294, 1181, 211, 214}

$$\frac{(d + ex)^3 (a + b \tanh^{-1}(cx^2))}{3e} + \frac{b \operatorname{ArcTan}(\sqrt{c}x) (3cd^2 - e^2)}{3c^{3/2}} - \frac{b(3cd^2 + e^2) \tanh^{-1}(\sqrt{c}x)}{3c^{3/2}} + \frac{bd(cd^2 + 3e^2) \log(1 - cx^2)}{6ce} - \frac{bd(cd^2 - 3e^2) \log(cx^2 + 1)}{6ce} + \frac{2be^2x}{3c}$$

Antiderivative was successfully verified.

[In] `Int[(d + e*x)^2*(a + b*ArcTanh[c*x^2]),x]`

[Out] $(2*b*e^2*x)/(3*c) + (b*(3*c*d^2 - e^2)*\operatorname{ArcTan}[\operatorname{Sqrt}[c]*x])/(3*c^{(3/2)}) - (b*(3*c*d^2 + e^2)*\operatorname{ArcTanh}[\operatorname{Sqrt}[c]*x])/(3*c^{(3/2)}) + ((d + e*x)^3*(a + b*\operatorname{ArcTanh}[c*x^2]))/(3*e) + (b*d*(c*d^2 + 3*e^2)*\operatorname{Log}[1 - c*x^2])/(6*c*e) - (b*d*(c*d^2 - 3*e^2)*\operatorname{Log}[1 + c*x^2])/(6*c*e)$

Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 211

`Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

Rule 214

`Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 647

`Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[`

$(-a)*c]$

Rule 1181

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-a]*c, 2\}, \text{Dist}[e/2 + c*(d/(2*q)), \text{Int}[1/(-q + c*x^2), x], x] + \text{Dist}[e/2 - c*(d/(2*q)), \text{Int}[1/(q + c*x^2), x], x]] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[(-a)*c]$

Rule 1262

$\text{Int}[(x_.)*((d_.) + (e_.)*(x_.)^2)^{(q_.)}*((a_.) + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x]$

Rule 1294

$\text{Int}[(f_.)*(x_.)^{(m_.)}*((d_.) + (e_.)*(x_.)^2)*((a_.) + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[e*f*(f*x)^{(m-1)}*((a + c*x^4)^{(p+1)}/(c*(m + 4*p + 3))), x] - \text{Dist}[f^2/(c*(m + 4*p + 3)), \text{Int}[(f*x)^{(m-2)}*(a + c*x^4)^p*(a*e*(m-1) - c*d*(m + 4*p + 3)*x^2), x], x] /; \text{FreeQ}\{a, c, d, e, f, p\}, x] \&\& \text{GtQ}[m, 1] \&\& \text{NeQ}[m + 4*p + 3, 0] \&\& \text{IntegerQ}[2*p] \&\& (\text{IntegerQ}[p] \mid \mid \text{IntegerQ}[m])]$

Rule 1845

$\text{Int}[(Pq_.)*((c_.)*(x_.)^{(m_.)})/((a_.) + (b_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow \text{With}[\{v = \text{Sum}[(c*x)^{(m+ii)}*((\text{Coeff}[Pq, x, ii] + \text{Coeff}[Pq, x, n/2 + ii])*x^{(n/2)})/(c^{ii}*(a + b*x^n))\}, \{ii, 0, n/2 - 1\}\}, \text{Int}[v, x] /; \text{SumQ}[v] /; \text{FreeQ}\{a, b, c, m\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{Expon}[Pq, x] < n]$

Rule 6071

$\text{Int}[(a_.) + \text{ArcTanh}[(c_.)*(x_.)^{(n_.)}]* (b_.)]*((d_.) + (e_.)*(x_.)^{(m_.)}), x_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(m+1)}*((a + b*\text{ArcTanh}[c*x^n])/(e*(m+1))), x] - \text{Dist}[b*c*(n/(e*(m+1))), \text{Int}[x^{(n-1)}*((d + e*x)^{(m+1)}/(1 - c^2*x^{(2*n)})), x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int (d+ex)^2 (a+b \tanh^{-1}(cx^2)) dx &= \frac{(d+ex)^3 (a+b \tanh^{-1}(cx^2))}{3e} - \frac{b \int \frac{2cx(d+ex)^3}{1-c^2x^4} dx}{3e} \\
&= \frac{(d+ex)^3 (a+b \tanh^{-1}(cx^2))}{3e} - \frac{(2bc) \int \frac{x(d+ex)^3}{1-c^2x^4} dx}{3e} \\
&= \frac{(d+ex)^3 (a+b \tanh^{-1}(cx^2))}{3e} - \frac{(2bc) \int \left(\frac{x(d^3+3de^2x^2)}{1-c^2x^4} + \frac{x^2(3d^2e+e^3x^2)}{1-c^2x^4} \right) dx}{3e} \\
&= \frac{(d+ex)^3 (a+b \tanh^{-1}(cx^2))}{3e} - \frac{(2bc) \int \frac{x(d^3+3de^2x^2)}{1-c^2x^4} dx}{3e} - \frac{(2bc) \int \frac{x^2(3d^2e+e^3x^2)}{1-c^2x^4} dx}{3e} \\
&= \frac{2be^2x}{3c} + \frac{(d+ex)^3 (a+b \tanh^{-1}(cx^2))}{3e} - \frac{(2b) \int \frac{e^3+3c^2d^2ex^2}{1-c^2x^4} dx}{3ce} - \frac{(bc) \int \frac{x^2(3d^2e+e^3x^2)}{1-c^2x^4} dx}{6e} \\
&= \frac{2be^2x}{3c} + \frac{(d+ex)^3 (a+b \tanh^{-1}(cx^2))}{3e} + \frac{(bcd(cd^2-3e^2)) \text{Subst}\left(\int \frac{x^2(3d^2e+e^3x^2)}{1-c^2x^4} dx, \sqrt{c}x\right)}{6e} \\
&= \frac{2be^2x}{3c} + \frac{b(3cd^2-e^2) \tan^{-1}(\sqrt{c}x)}{3c^{3/2}} - \frac{b(3cd^2+e^2) \tanh^{-1}(\sqrt{c}x)}{3c^{3/2}} + \dots
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 170, normalized size = 1.08

$$\frac{1}{6} \left(6ad^2x + \frac{4be^2x}{c} + 6adex^2 + 2ae^2x^3 + \frac{2b(3cd^2-e^2) \text{ArcTan}(\sqrt{c}x)}{c^{3/2}} + 2bx(3d^2+3dex+e^2x^2) \tanh^{-1}(cx^2) + \frac{b(3cd^2+e^2) \log(1-\sqrt{c}x)}{c^{3/2}} - \frac{b(3cd^2+e^2) \log(1+\sqrt{c}x)}{c^{3/2}} + \frac{3bde \log(1-c^2x^4)}{c} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(d + e*x)^2*(a + b*ArcTanh[c*x^2]), x]`

```
[Out] (6*a*d^2*x + (4*b*e^2*x)/c + 6*a*d*e*x^2 + 2*a*e^2*x^3 + (2*b*(3*c*d^2 - e^2)*ArcTan[Sqrt[c]*x])/c^(3/2) + 2*b*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcTanh[c*x^2] + (b*(3*c*d^2 + e^2)*Log[1 - Sqrt[c]*x])/c^(3/2) - (b*(3*c*d^2 + e^2)*Log[1 + Sqrt[c]*x])/c^(3/2) + (3*b*d*e*Log[1 - c^2*x^4])/c)/6
```

Maple [A]

time = 0.28, size = 205, normalized size = 1.30

method	result
default	$\frac{(ex+d)^3 a}{3e} + \frac{be^2 \operatorname{arctanh}(cx^2)x^3}{3} + be \operatorname{arctanh}(cx^2)x^2d + b \operatorname{arctanh}(cx^2)x d^2 + \frac{b \operatorname{arctanh}(cx^2)d^3}{3e} + \frac{2be^2x}{3c} + \dots$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((e*x+d)^2*(a+b*arctanh(c*x^2)), x, method=_RETURNVERBOSE)`

[Out] $\frac{1}{3}(e*x+d)^3*a/e+1/3*b*e^2*\operatorname{arctanh}(c*x^2)*x^3+b*e*\operatorname{arctanh}(c*x^2)*x^2*d+b*a*\operatorname{rctanh}(c*x^2)*x*d^2+1/3*b/e*\operatorname{arctanh}(c*x^2)*d^3+2/3*b*e^2*x/c+1/6*b/e*\ln(c*x^2-1)*d^3+1/2*b*e/c*\ln(c*x^2-1)*d-b/c^{1/2}*\operatorname{arctanh}(x*c^{1/2})*d^2-1/3*b*e^2/c^{3/2}*\operatorname{arctanh}(x*c^{1/2})-1/6*b/e*\ln(c*x^2+1)*d^3+1/2*b*e/c*\ln(c*x^2+1)*d+b/c^{1/2}*\operatorname{arctan}(x*c^{1/2})*d^2-1/3*b*e^2/c^{3/2}*\operatorname{arctan}(x*c^{1/2})$

Maxima [A]

time = 0.47, size = 171, normalized size = 1.08

$$\frac{1}{3}ax^3e^2 + adx^2e + \frac{1}{2} \left(c \left(\frac{2 \arctan(\sqrt{c}x)}{c^3} + \frac{\log\left(\frac{c-x\sqrt{c}}{c+\sqrt{c}}\right)}{c^3} \right) + 2x \operatorname{arctanh}(cx^2) \right) bd^2 + ad^2x + \frac{1}{6} \left(2x^3 \operatorname{arctanh}(cx^2) + c \left(\frac{4x}{c^2} - \frac{2 \arctan(\sqrt{c}x)}{c^3} + \frac{\log\left(\frac{c-x\sqrt{c}}{c+\sqrt{c}}\right)}{c^3} \right) \right) be^2 + \frac{(2cx^2 \operatorname{arctanh}(cx^2) + \log(-c^2x^4 + 1))bde}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*(a+b*arctanh(c*x^2)),x, algorithm="maxima")`

[Out] $\frac{1}{3}a*x^3*e^2 + a*d*x^2*e + \frac{1}{2}*(c*(2*\operatorname{arctan}(\operatorname{sqrt}(c)*x)/c^{3/2} + \log((c*x - \operatorname{sqrt}(c))/(c*x + \operatorname{sqrt}(c))))/c^{3/2}) + 2*x*\operatorname{arctanh}(c*x^2)*b*d^2 + a*d^2*x + \frac{1}{6}*(2*x^3*\operatorname{arctanh}(c*x^2) + c*(4*x/c^2 - 2*\operatorname{arctan}(\operatorname{sqrt}(c)*x)/c^{5/2} + \log((c*x - \operatorname{sqrt}(c))/(c*x + \operatorname{sqrt}(c))))/c^{5/2}))*b*e^2 + \frac{1}{2}*(2*c*x^2*\operatorname{arctanh}(c*x^2) + \log(-c^2*x^4 + 1))*b*d*e/c$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(131) = 262.

time = 0.46, size = 677, normalized size = 4.28

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*(a+b*arctanh(c*x^2)),x, algorithm="fricas")`

[Out] $\frac{1}{6}*(6*a*c^2*d*x^2*\cosh(1) + 6*a*c^2*d^2*x + 2*(a*c^2*x^3 + 2*b*c*x))*\cosh(1)^2 + 2*(a*c^2*x^3 + 2*b*c*x)*\sinh(1)^2 + 2*(3*b*c*d^2 - b*\cosh(1)^2 - 2*b*\cosh(1)*\sinh(1) - b*\sinh(1)^2)*\operatorname{sqrt}(c)*\operatorname{arctan}(\operatorname{sqrt}(c)*x) + (3*b*c*d^2 + b*\cosh(1)^2 + 2*b*\cosh(1)*\sinh(1) + b*\sinh(1)^2)*\operatorname{sqrt}(c)*\log((c*x^2 - 2*\operatorname{sqrt}(c)*x + 1)/(c*x^2 - 1)) + 3*(b*c*d*\cosh(1) + b*c*d*\sinh(1))*\log(c*x^2 + 1) + 3*(b*c*d*\cosh(1) + b*c*d*\sinh(1))*\log(c*x^2 - 1) + (b*c^2*x^3*\cosh(1)^2 + b*c^2*x^3*\sinh(1)^2 + 3*b*c^2*d*x^2*\cosh(1) + 3*b*c^2*d^2*x + (2*b*c^2*x^3*\cosh(1) + 3*b*c^2*d*x^2)*\sinh(1))*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*(3*a*c^2*d*x^2 + 2*(a*c^2*x^3 + 2*b*c*x))*\cosh(1))*\sinh(1))/c^2, \frac{1}{6}*(6*a*c^2*d*x^2*\cosh(1) + 6*a*c^2*d^2*x + 2*(a*c^2*x^3 + 2*b*c*x))*\cosh(1)^2 + 2*(a*c^2*x^3 + 2*b*c*x)*\sinh(1)^2 + 2*(3*b*c*d^2 + b*\cosh(1)^2 + 2*b*\cosh(1)*\sinh(1) + b*\sinh(1)^2)*\operatorname{sqrt}(-c)*\operatorname{arctan}(\operatorname{sqrt}(-c)*x) + (3*b*c*d^2 - b*\cosh(1)^2 - 2*b*\cosh(1)*\sinh(1) - b*\sinh(1)^2)*\operatorname{sqrt}(-c)*\log((c*x^2 + 2*\operatorname{sqrt}(-c)*x - 1)/(c*x^2 + 1)) + 3*(b*c*d*\cosh(1) + b*c*d*\sinh(1))*\log(c*x^2 + 1) + 3*(b*c*d*\cosh(1) + b*c*d*\sinh(1))*\log(c*x^2 - 1) + (b*c^2*x^3*\cosh(1)^2 + b*c^2*x^3*\sinh(1)^2 + 3*b*c^2*d*x^2*\cosh(1) + 3*b*c^2*d^2*x + (2*b*c^2*x^3*\cosh(1) + 3*b*c$

$$\begin{aligned}
& c*x**2)/(12*c**3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2) + 24*c**2*sqrt(-1/c) \\
& + 24*c**2*sqrt(1/c)) + 3*b*c**2*d*e*(-1/c)**(3/2)*log(x - sqrt(-1/c))/(12*c \\
& **3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2) + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt \\
& t(1/c)) + 12*b*c**2*d*e*(-1/c)**(3/2)*log(x + sqrt(-1/c))/(12*c**3*(-1/c)** \\
& (3/2) - 12*c**3*(1/c)**(3/2) + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt(1/c)) - 3* \\
& b*c**2*d*e*(1/c)**(3/2)*log(x - sqrt(-1/c))/(12*c**3*(-1/c)**(3/2) - 12*c** \\
& 3*(1/c)**(3/2) + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt(1/c)) - 12*b*c**2*d*e*(1 \\
& /c)**(3/2)*log(x + sqrt(-1/c))/(12*c**3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2 \\
&) + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt(1/c)) + 4*b*c**2*e**2*x**3*sqrt(-1/c) \\
& *atanh(c*x**2)/(12*c**3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2) + 24*c**2*sqrt \\
& (-1/c) + 24*c**2*sqrt(1/c)) + 4*b*c**2*e**2*x**3*sqrt(1/c)*atanh(c*x**2)/(1 \\
& 2*c**3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2) + 24*c**2*sqrt(-1/c) + 24*c**2* \\
& sqrt(1/c)) + b*c**2*e**2*(-1/c)**(3/2)*sqrt(1/c)*log(x - sqrt(1/c))/(12*c** \\
& 3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2) + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt(\\
& 1/c)) - b*c**2*e**2*sqrt(-1/c)*(1/c)**(3/2)*log(x - sqrt(1/c))/(12*c**3*(-1 \\
& /c)**(3/2) - 12*c**3*(1/c)**(3/2) + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt(1/c)) \\
& - 12*b*c*d**2*log(x + sqrt(-1/c))/(12*c**3*(-1/c)**(3/2) - 12*c**3*(1/c)** \\
& (3/2) + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt(1/c)) + 12*b*c*d**2*log(x - sqrt(\\
& 1/c))/(12*c**3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2) + 24*c**2*sqrt(-1/c) + \\
& 24*c**2*sqrt(1/c)) + 12*b*c*d**2*atanh(c*x**2)/(12*c**3*(-1/c)**(3/2) - 12* \\
& c**3*(1/c)**(3/2) + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt(1/c)) + 15*b*c*d*e*sq \\
& rt(-1/c)*log(x - sqrt(-1/c))/(12*c**3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2) \\
& + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt(1/c)) + 24*b*c*d*e*sqrt(-1/c)*log(x + s \\
& qrt(-1/c))/(12*c**3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2) + 24*c**2*sqrt(-1/ \\
& c) + 24*c**2*sqrt(1/c)) - 12*b*c*d*e*sqrt(-1/c)*atanh(c*x**2)/(12*c**3*(-1/ \\
& c)**(3/2) - 12*c**3*(1/c)**(3/2) + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt(1/c)) \\
& + 15*b*c*d*e*sqrt(1/c)*log(x - sqrt(-1/c))/(12*c**3*(-1/c)**(3/2) - 12*c**3 \\
& *(1/c)**(3/2) + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt(1/c)) + 24*b*c*d*e*sqrt(1 \\
& /c)*log(x + sqrt(-1/c))/(12*c**3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2) + 24* \\
& c**2*sqrt(-1/c) + 24*c**2*sqrt(1/c)) - 12*b*c*d*e*sqrt(1/c)*atanh(c*x**2)/(\\
& 12*c**3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2) + 24*c**2*sqrt(-1/c) + 24*c**2 \\
& *sqrt(1/c)) + 8*b*c*e**2*x*sqrt(-1/c)/(12*c**3*(-1/c)**(3/2) - 12*c**3*(1/c \\
&)** (3/2) + 24*c**2*sqrt(-1/c) + 24*c**2*sqrt(1/c)) + 8*b*c*e**2*x*sqrt(1/c) \\
& / (12*c**3*(-1/c)**(3/2) - 12*c**3*(1/c)**(3/2) ...
\end{aligned}$$

Giac [A]

time = 1.31, size = 171, normalized size = 1.08

$$\frac{1}{3}ae^2x^3 + adex^2 + \frac{bde \log(cx^2 + 1)}{2c} + \frac{bde \log(cx^2 - 1)}{2c} + \frac{1}{6}(be^2x^3 + 3bdex^2 + 3bd^2x) \log\left(\frac{-cx^2 + 1}{cx^2 - 1}\right) + \frac{(3acd^2 + 2be^2)x}{3c} + \frac{(3bcd^2 - be^2) \arctan(\sqrt{c}x)}{3c^{\frac{3}{2}}} + \frac{(3bcd^2 + be^2) \arctan\left(\frac{cx}{\sqrt{-c}}\right)}{3\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x^2)),x, algorithm="giac")

[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + 1/2*b*d*e*log(c*x^2 + 1)/c + 1/2*b*d*e*log(c*x^2 - 1)/c + 1/6*(b*e^2*x^3 + 3*b*d*e*x^2 + 3*b*d^2*x)*log(-(c*x^2 + 1)/(c*x^2 - 1))

2 - 1)) + 1/3*(3*a*c*d^2 + 2*b*e^2)*x/c + 1/3*(3*b*c*d^2 - b*e^2)*arctan(sqrt(c)*x)/c^(3/2) + 1/3*(3*b*c*d^2 + b*e^2)*arctan(c*x/sqrt(-c))/(sqrt(-c)*c)

Mupad [B]

time = 1.39, size = 309, normalized size = 1.96

$\ln(x^2 + 1) \left(\frac{bdx}{2} + \frac{bdx^2}{2} + \frac{bd^2x^3}{6} \right) - \ln(1 - cx^2) \left(\frac{bdx}{2} + \frac{bdx^2}{2} + \frac{bd^2x^3}{6} \right) + \frac{x(3bd^2d + 2bd^2d^2)}{3cd} + \frac{ad^2x^2}{3} - \frac{\ln(c + x\sqrt{cd}) (bd^2\sqrt{cd} + 3bdcd\sqrt{cd} - 3bd^2d)}{6cd} + \frac{\ln(c - x\sqrt{cd}) (bd^2\sqrt{cd} + 3bdcd\sqrt{cd} + 3bd^2d)}{6cd} + \frac{\ln(c + x\sqrt{-cd}) (bd^2\sqrt{-cd} + 3bdcd\sqrt{-cd} - 3bd^2d)}{6cd} + \frac{\ln(c - x\sqrt{-cd}) (3bd^2d - bd^2\sqrt{-cd} + 3bdcd\sqrt{-cd})}{6cd} + adex^2$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^2))*(d + e*x)^2,x)

[Out] log(c*x^2 + 1)*((b*e^2*x^3)/6 + (b*d^2*x)/2 + (b*d*e*x^2)/2) - log(1 - c*x^2)*((b*e^2*x^3)/6 + (b*d^2*x)/2 + (b*d*e*x^2)/2) + (x*(3*a*c^2*d^2 + 2*b*c*e^2))/(3*c^2) + (a*e^2*x^3)/3 - (log(c + x*(c^3)^(1/2))*(b*e^2*(c^3)^(1/2) + 3*b*c*d^2*(c^3)^(1/2) - 3*b*c^2*d*e))/(6*c^3) + (log(c - x*(c^3)^(1/2))*(b*e^2*(c^3)^(1/2) + 3*b*c*d^2*(c^3)^(1/2) + 3*b*c^2*d*e))/(6*c^3) + (log(c + x*(-c^3)^(1/2))*(b*e^2*(-c^3)^(1/2) + 3*b*c^2*d*e - 3*b*c*d^2*(-c^3)^(1/2)))/(6*c^3) + (log(c - x*(-c^3)^(1/2))*(3*b*c^2*d*e - b*e^2*(-c^3)^(1/2) + 3*b*c*d^2*(-c^3)^(1/2)))/(6*c^3) + a*d*e*x^2

3.25 $\int (d + ex) (a + b \tanh^{-1}(cx^2)) dx$

Optimal. Leaf size=117

$$\frac{bd\text{ArcTan}(\sqrt{c}x)}{\sqrt{c}} - \frac{bd \tanh^{-1}(\sqrt{c}x)}{\sqrt{c}} + \frac{(d+ex)^2(a+b \tanh^{-1}(cx^2))}{2e} + \frac{b(cd^2+e^2)\log(1-cx^2)}{4ce} - \frac{b(cd^2-e^2)\log(cx^2+1)}{4ce}$$

[Out] 1/2*(e*x+d)^2*(a+b*arctanh(c*x^2))/e+1/4*b*(c*d^2+e^2)*ln(-c*x^2+1)/c/e-1/4*b*(c*d^2-e^2)*ln(c*x^2+1)/c/e+b*d*arctan(x*c^(1/2))/c^(1/2)-b*d*arctanh(x*c^(1/2))/c^(1/2)

Rubi [A]

time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6071, 1845, 304, 209, 212, 1262, 647, 31}

$$\frac{(d+ex)^2(a+b \tanh^{-1}(cx^2))}{2e} + \frac{bd\text{ArcTan}(\sqrt{c}x)}{\sqrt{c}} + \frac{b(cd^2+e^2)\log(1-cx^2)}{4ce} - \frac{b(cd^2-e^2)\log(cx^2+1)}{4ce} - \frac{bd \tanh^{-1}(\sqrt{c}x)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)*(a + b*ArcTanh[c*x^2]),x]

[Out] (b*d*ArcTan[Sqrt[c]*x])/Sqrt[c] - (b*d*ArcTanh[Sqrt[c]*x])/Sqrt[c] + ((d + e*x)^2*(a + b*ArcTanh[c*x^2]))/(2*e) + (b*(c*d^2 + e^2)*Log[1 - c*x^2])/(4*c*e) - (b*(c*d^2 - e^2)*Log[1 + c*x^2])/(4*c*e)

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 304

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x]

] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 647

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[(-a)*c]

Rule 1262

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 1845

Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 6071

Int[((a_) + ArcTanh[(c_)*(x_)^(n_)]*(b_))*((d_) + (e_)*(x_)^(m_)), x_Symbol] := Simp[(d + e*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(e*(m + 1))), x] - Dist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int (d+ex)(a+b \tanh^{-1}(cx^2)) dx &= \int (a(d+ex) + b(d+ex) \tanh^{-1}(cx^2)) dx \\
&= \frac{a(d+ex)^2}{2e} + b \int (d+ex) \tanh^{-1}(cx^2) dx \\
&= \frac{a(d+ex)^2}{2e} + b \int (d \tanh^{-1}(cx^2) + ex \tanh^{-1}(cx^2)) dx \\
&= \frac{a(d+ex)^2}{2e} + (bd) \int \tanh^{-1}(cx^2) dx + (be) \int x \tanh^{-1}(cx^2) dx \\
&= \frac{a(d+ex)^2}{2e} + bdx \tanh^{-1}(cx^2) + \frac{1}{2} bex^2 \tanh^{-1}(cx^2) - (2bcd) \int \frac{1}{1-cx^2} dx \\
&= \frac{a(d+ex)^2}{2e} + bdx \tanh^{-1}(cx^2) + \frac{1}{2} bex^2 \tanh^{-1}(cx^2) + \frac{be \log(1-cx^2)}{4c} \\
&= \frac{a(d+ex)^2}{2e} + \frac{bd \tan^{-1}(\sqrt{c}x)}{\sqrt{c}} - \frac{bd \tanh^{-1}(\sqrt{c}x)}{\sqrt{c}} + bdx \tanh^{-1}(cx^2)
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 104, normalized size = 0.89

$$adx + \frac{1}{2} aex^2 + bdx \tanh^{-1}(cx^2) + \frac{1}{2} bex^2 \tanh^{-1}(cx^2) + \frac{bd(2\text{ArcTan}(\sqrt{c}x) + \log(1-\sqrt{c}x) - \log(1+\sqrt{c}x))}{2\sqrt{c}} + \frac{be \log(1-c^2x^4)}{4c}$$

Antiderivative was successfully verified.

`[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x^2]), x]`

```
[Out] a*d*x + (a*e*x^2)/2 + b*d*x*ArcTanh[c*x^2] + (b*e*x^2*ArcTanh[c*x^2])/2 + (
b*d*(2*ArcTan[Sqrt[c]*x] + Log[1 - Sqrt[c]*x] - Log[1 + Sqrt[c]*x]))/(2*Sqr
t[c]) + (b*e*Log[1 - c^2*x^4])/(4*c)
```

Maple [A]

time = 0.15, size = 92, normalized size = 0.79

method	result
default	$ a\left(\frac{1}{2}ex^2 + dx\right) + \frac{b \operatorname{arctanh}(cx^2)x^2e}{2} + b \operatorname{arctanh}(cx^2) dx + \frac{be \ln(cx^2-1)}{4c} - \frac{bd \operatorname{arctanh}(x\sqrt{c})}{\sqrt{c}} + \frac{be \ln(cx^2+1)}{4c} $
risch	$ \frac{bx(ex+2d) \ln(cx^2+1)}{4} - \frac{be x^2 \ln(-cx^2+1)}{4} - \frac{bdx \ln(-cx^2+1)}{2} + \frac{aex^2}{2} + \frac{b \ln(1-x\sqrt{c})d}{2\sqrt{c}} - \frac{b \ln(1+x\sqrt{c})d}{2\sqrt{c}} + \frac{b \ln(1-cx^2)}{2\sqrt{c}} $

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((e*x+d)*(a+b*arctanh(c*x^2)), x, method=_RETURNVERBOSE)`

$1/c)*\sqrt{1/c}) - c*(-1/c)**(3/2)*\log(x + \sqrt{-1/c))/(c**2*(-1/c)**(3/2)*\sqrt{1/c} - c**2*\sqrt{-1/c}*(1/c)**(3/2) + 6*c*\sqrt{-1/c}*\sqrt{1/c}) + c*(1/c)**(3/2)*\log(x + \sqrt{-1/c))/(c**2*(-1/c)**(3/2)*\sqrt{1/c} - c**2*\sqrt{-1/c}*(1/c)**(3/2) + 6*c*\sqrt{-1/c}*\sqrt{1/c}) - 2*\sqrt{-1/c}*\log(x - \sqrt{-1/c))/(c**2*(-1/c)**(3/2)*\sqrt{1/c} - c**2*\sqrt{-1/c}*(1/c)**(3/2) + 6*c*\sqrt{-1/c}*\sqrt{1/c}) - 3*\sqrt{-1/c}*\log(x + \sqrt{-1/c))/(c**2*(-1/c)**(3/2)*\sqrt{1/c} - c**2*\sqrt{-1/c}*(1/c)**(3/2) + 6*c*\sqrt{-1/c}*\sqrt{1/c}) + 4*\sqrt{-1/c}*\log(x - \sqrt{1/c))/(c**2*(-1/c)**(3/2)*\sqrt{1/c} - c**2*\sqrt{-1/c}*(1/c)**(3/2) + 6*c*\sqrt{-1/c}*\sqrt{1/c}) + 4*\sqrt{-1/c}*\operatorname{atanh}(c*x**2)/(c**2*(-1/c)**(3/2)*\sqrt{1/c} - c**2*\sqrt{-1/c}*(1/c)**(3/2) + 6*c*\sqrt{-1/c}*\sqrt{1/c}) + 2*\sqrt{1/c}*\log(x - \sqrt{-1/c))/(c**2*(-1/c)**(3/2)*\sqrt{1/c} - c**2*\sqrt{-1/c}*(1/c)**(3/2) + 6*c*\sqrt{-1/c}*\sqrt{1/c}) - 3*\sqrt{1/c}*\log(x + \sqrt{-1/c))/(c**2*(-1/c)**(3/2)*\sqrt{1/c} - c**2*\sqrt{-1/c}*(1/c)**(3/2) + 6*c*\sqrt{-1/c}*\sqrt{1/c}), \operatorname{Ne}(c, 0)), (0, \operatorname{True})) + b*e*\operatorname{Piecewise}((x**2*\operatorname{tanh}(c*x**2)/2 + \log(c*x**2 + 1)/(2*c) - \operatorname{atanh}(c*x**2)/(2*c), \operatorname{Ne}(c, 0)), (0, \operatorname{True}))$

Giac [A]

time = 0.55, size = 135, normalized size = 1.15

$$\frac{1}{2} a e x^2 + a d x + \frac{b d \sqrt{|c|} \arctan\left(x \sqrt{|c|}\right)}{c} + \frac{b e \log\left(x^2 + \frac{1}{|c|}\right)}{4 c} + \frac{1}{4} (b e x^2 + 2 b d x) \log\left(-\frac{c x^2 + 1}{c x^2 - 1}\right) - \frac{(2 b c d \sqrt{|c|} - b c e) \log\left(x + \frac{1}{\sqrt{|c|}}\right)}{4 c^2} + \frac{(2 b c d \sqrt{|c|} + b c e) \log\left(x - \frac{1}{\sqrt{|c|}}\right)}{4 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*arctanh(c*x^2)),x, algorithm="giac")`

[Out] $1/2*a*e*x^2 + a*d*x + b*d*\sqrt{\operatorname{abs}(c)}*\arctan(x*\sqrt{\operatorname{abs}(c)})/c + 1/4*b*e*\log(x^2 + 1/\operatorname{abs}(c))/c + 1/4*(b*e*x^2 + 2*b*d*x)*\log(-(c*x^2 + 1)/(c*x^2 - 1)) - 1/4*(2*b*c*d*\sqrt{\operatorname{abs}(c)} - b*c*e)*\log(\operatorname{abs}(x + 1/\sqrt{\operatorname{abs}(c)}))/c^2 + 1/4*(2*b*c*d*\sqrt{\operatorname{abs}(c)} + b*c*e)*\log(\operatorname{abs}(x - 1/\sqrt{\operatorname{abs}(c)}))/c^2$

Mupad [B]

time = 1.31, size = 242, normalized size = 2.07

$$a d x + \frac{a e x^2}{2} + \frac{b d x \ln(c x^2 + 1)}{2} - \frac{b d x \ln(1 - c x^2)}{2} + \frac{b e \ln(c + x \sqrt{c^3})}{4 c} + \frac{b e \ln(c - x \sqrt{c^3})}{4 c} + \frac{b e x^2 \ln(c x^2 + 1)}{4} - \frac{b e x^2 \ln(1 - c x^2)}{4} + \frac{b e \ln(c + x \sqrt{-c^3})}{4 c} + \frac{b e \ln(c - x \sqrt{-c^3})}{4 c} - \frac{b d \ln(c + x \sqrt{c^3}) \sqrt{c^3}}{2 c^2} + \frac{b d \ln(c - x \sqrt{c^3}) \sqrt{c^3}}{2 c^2} - \frac{b d \ln(c + x \sqrt{-c^3}) \sqrt{-c^3}}{2 c^2} + \frac{b d \ln(c - x \sqrt{-c^3}) \sqrt{-c^3}}{2 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c*x^2))*(d + e*x),x)`

[Out] $a*d*x + (a*e*x^2)/2 + (b*d*x*\log(c*x^2 + 1))/2 - (b*d*x*\log(1 - c*x^2))/2 + (b*e*\log(c + x*(c^3)^(1/2)))/(4*c) + (b*e*\log(c - x*(c^3)^(1/2)))/(4*c) + (b*e*x^2*\log(c*x^2 + 1))/4 - (b*e*x^2*\log(1 - c*x^2))/4 + (b*e*\log(c + x*(-c^3)^(1/2)))/(4*c) + (b*e*\log(c - x*(-c^3)^(1/2)))/(4*c) - (b*d*\log(c + x*(c^3)^(1/2))*(c^3)^(1/2))/(2*c^2) + (b*d*\log(c - x*(c^3)^(1/2))*(c^3)^(1/2))/(2*c^2) - (b*d*\log(c + x*(-c^3)^(1/2))*(-c^3)^(1/2))/(2*c^2) + (b*d*\log(c - x*(-c^3)^(1/2))*(-c^3)^(1/2))/(2*c^2)$

$$3.26 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{d+ex} dx$$

Optimal. Leaf size=325

$$\frac{(a + b \tanh^{-1}(cx^2)) \log(d + ex)}{e} - \frac{b \log\left(\frac{e(1 - \sqrt{-c}x)}{\sqrt{-c}d + e}\right) \log(d + ex)}{2e} - \frac{b \log\left(-\frac{e(1 + \sqrt{-c}x)}{\sqrt{-c}d - e}\right) \log(d + ex)}{2e} +$$

[Out] (a+b*arctanh(c*x^2))*ln(e*x+d)/e-1/2*b*ln(e*x+d)*ln(e*(1-x*(-c)^(1/2))/(e+d*(-c)^(1/2)))/e-1/2*b*ln(e*x+d)*ln(-e*(1+x*(-c)^(1/2))/(-e+d*(-c)^(1/2)))/e+1/2*b*ln(e*x+d)*ln(e*(1-x*c^(1/2))/(e+d*c^(1/2)))/e+1/2*b*ln(e*x+d)*ln(-e*(1+x*c^(1/2))/(-e+d*c^(1/2)))/e-1/2*b*polylog(2,(e*x+d)*(-c)^(1/2)/(-e+d*(-c)^(1/2)))/e-1/2*b*polylog(2,(e*x+d)*(-c)^(1/2)/(e+d*(-c)^(1/2)))/e+1/2*b*polylog(2,(e*x+d)*c^(1/2)/(-e+d*c^(1/2)))/e+1/2*b*polylog(2,(e*x+d)*c^(1/2)/(e+d*c^(1/2)))/e

Rubi [A]

time = 0.41, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6067, 281, 212, 2463, 266, 2441, 2440, 2438}

$$\frac{\log(d+ex)(a+b \tanh^{-1}(cx^2))}{e} - \frac{b \operatorname{Li}_2\left(\frac{\sqrt{-c}(d+ex)}{\sqrt{-c}d+e}\right)}{2e} + \frac{b \operatorname{Li}_2\left(\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+e}\right)}{2e} - \frac{b \operatorname{Li}_2\left(\frac{\sqrt{-c}(d+ex)}{\sqrt{-c}d-e}\right)}{2e} + \frac{b \operatorname{Li}_2\left(\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-e}\right)}{2e} - \frac{b \log(d+ex) \log\left(\frac{e(1-\sqrt{-c}x)}{\sqrt{-c}d+e}\right)}{2e} - \frac{b \log(d+ex) \log\left(-\frac{e(1-\sqrt{-c}x)}{\sqrt{-c}d-e}\right)}{2e} + \frac{b \log(d+ex) \log\left(\frac{e(1+\sqrt{c}x)}{\sqrt{c}d+e}\right)}{2e} + \frac{b \log(d+ex) \log\left(-\frac{e(1+\sqrt{c}x)}{\sqrt{c}d-e}\right)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x^2])/(d + e*x),x]

[Out] ((a + b*ArcTanh[c*x^2])*Log[d + e*x])/e - (b*Log[(e*(1 - Sqrt[-c]*x))/(Sqrt[-c]*d + e)]*Log[d + e*x])/(2*e) - (b*Log[-((e*(1 + Sqrt[-c]*x))/(Sqrt[-c]*d - e))]*Log[d + e*x])/(2*e) + (b*Log[(e*(1 - Sqrt[c]*x))/(Sqrt[c]*d + e)]*Log[d + e*x])/(2*e) + (b*Log[-((e*(1 + Sqrt[c]*x))/(Sqrt[c]*d - e))]*Log[d + e*x])/(2*e) - (b*PolyLog[2, (Sqrt[-c]*(d + e*x))/(Sqrt[-c]*d - e)])/(2*e) + (b*PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - e)])/(2*e) - (b*PolyLog[2, (Sqrt[-c]*(d + e*x))/(Sqrt[-c]*d + e)])/(2*e) + (b*PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + e)])/(2*e)

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 281

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 2438

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 2440

Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))])*(b_)/((f_) + (g_)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]

Rule 2441

Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))/((f_) + (g_)*(x_)), x_Symbol] := Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

Rule 2463

Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))^(p_)*((h_)*(x_))^(m_)*((f_) + (g_)*(x_))^(r_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 6067

Int[((a_) + ArcTanh[(c_)*(x_)^(n_)]*(b_))/((d_) + (e_)*(x_)), x_Symbol] := Simp[Log[d + e*x]*((a + b*ArcTanh[c*x^n])/e), x] - Dist[b*c*(n/e), Int[x^(n - 1)*(Log[d + e*x]/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[n]

Rubi steps

$$\int \frac{a + b \tanh^{-1}(cx^2)}{d + ex} dx = \int \left(\frac{a}{d + ex} + \frac{b \tanh^{-1}(cx^2)}{d + ex} \right) dx$$

$$= \frac{a \log(d + ex)}{e} + b \int \frac{\tanh^{-1}(cx^2)}{d + ex} dx$$

Mathematica [C] Result contains complex when optimal does not.

time = 13.63, size = 285, normalized size = 0.88

$$\frac{a \log(d+ex) + b \left(2 \tanh^{-1}(cx^2) \log(d+ex) - \log\left(\frac{e(-\sqrt{C}x)}{\sqrt{C}d+ex}\right) \log(d+ex) - \log\left(\frac{e(+\sqrt{C}x)}{\sqrt{C}d-ex}\right) \log(d+ex) + \log\left(\frac{e(-\sqrt{C}x)}{\sqrt{C}d-ex}\right) \log(d+ex) + \log(d+ex) \log\left(\frac{e\sqrt{C}x}{\sqrt{C}d+ex}\right) + \text{PolyLog}\left(2, \frac{\sqrt{C}d+ex}{\sqrt{C}d+ex}\right) - \text{PolyLog}\left(2, \frac{\sqrt{C}d+ex}{\sqrt{C}d-ex}\right) - \text{PolyLog}\left(2, \frac{\sqrt{C}d+ex}{\sqrt{C}d+ex}\right) + \text{PolyLog}\left(2, \frac{\sqrt{C}d+ex}{\sqrt{C}d+ex}\right) \right)}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x^2])/(d + e*x), x]

[Out] (a*Log[d + e*x])/e + (b*(2*ArcTanh[c*x^2]*Log[d + e*x] - Log[(e*(I - Sqrt[c]*x))/(Sqrt[c]*d + I*e)]*Log[d + e*x] - Log[-((e*(I + Sqrt[c]*x))/(Sqrt[c]*d - I*e))]*Log[d + e*x] + Log[-((e*(1 + Sqrt[c]*x))/(Sqrt[c]*d - e))]*Log[d + e*x] + Log[d + e*x]*Log[(e - Sqrt[c]*e*x)/(Sqrt[c]*d + e)] + PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - e)] - PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - I*e)] - PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + I*e)] + PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + e)])/(2*e)

Maple [A]

time = 0.14, size = 362, normalized size = 1.11

method	result
default	$\frac{a \ln(ex+d)}{e} + \frac{b \ln(ex+d) \operatorname{arctanh}(cx^2)}{e} + \frac{b \ln(ex+d) \ln\left(\frac{e\sqrt{C} - (ex+d)c+dc}{e\sqrt{C} + dc}\right)}{2e} + \frac{b \ln(ex+d) \ln\left(\frac{e\sqrt{C} + (ex+d)c-dc}{e\sqrt{C} - dc}\right)}{2e} + \frac{b \operatorname{dilog}\left(\frac{e\sqrt{C} - (ex+d)c+dc}{e\sqrt{C} + dc}\right)}{2e} + \frac{b \operatorname{dilog}\left(\frac{e\sqrt{C} + (ex+d)c-dc}{e\sqrt{C} - dc}\right)}{2e}$
risch	$\frac{a \ln(ex+d)}{e} - \frac{b \ln(ex+d) \ln(-cx^2+1)}{2e} + \frac{b \ln(ex+d) \ln\left(\frac{e\sqrt{C} - (ex+d)c+dc}{e\sqrt{C} + dc}\right)}{2e} + \frac{b \ln(ex+d) \ln\left(\frac{e\sqrt{C} + (ex+d)c-dc}{e\sqrt{C} - dc}\right)}{2e} + \frac{b \operatorname{dilog}\left(\frac{e\sqrt{C} - (ex+d)c+dc}{e\sqrt{C} + dc}\right)}{2e} + \frac{b \operatorname{dilog}\left(\frac{e\sqrt{C} + (ex+d)c-dc}{e\sqrt{C} - dc}\right)}{2e}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x^2))/(e*x+d), x, method=_RETURNVERBOSE)

[Out] a*ln(e*x+d)/e+b*ln(e*x+d)/e*arctanh(c*x^2)+1/2*b/e*ln(e*x+d)*ln((e*c^(1/2)-(e*x+d)*c+d*c)/(e*c^(1/2)+d*c))+1/2*b/e*ln(e*x+d)*ln((e*c^(1/2)+(e*x+d)*c-d*c)/(e*c^(1/2)-d*c))+1/2*b/e*dilog((e*c^(1/2)-(e*x+d)*c+d*c)/(e*c^(1/2)+d*c))+1/2*b/e*dilog((e*c^(1/2)+(e*x+d)*c-d*c)/(e*c^(1/2)-d*c))-1/2*b/e*ln(e*x+d)*ln((e*(-c)^(1/2)-(e*x+d)*c+d*c)/(e*(-c)^(1/2)+d*c))-1/2*b/e*ln(e*x+d)*ln((e*(-c)^(1/2)+(e*x+d)*c-d*c)/(e*(-c)^(1/2)-d*c))-1/2*b/e*dilog((e*(-c)^(1/2)-(e*x+d)*c+d*c)/(e*(-c)^(1/2)+d*c))-1/2*b/e*dilog((e*(-c)^(1/2)+(e*x+d)*c-d*c)/(e*(-c)^(1/2)-d*c))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^2))/(e*x+d),x, algorithm="maxima")

[Out] a*e^(-1)*log(x*e + d) + 1/2*b*integrate((log(c*x^2 + 1) - log(-c*x^2 + 1))/(x*e + d), x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^2))/(e*x+d),x, algorithm="fricas")

[Out] integral((b*arctanh(c*x^2) + a)/(x*e + d), x)

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**2))/(e*x+d),x)

[Out] Timed out

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^2))/(e*x+d),x, algorithm="giac")

[Out] integrate((b*arctanh(c*x^2) + a)/(e*x + d), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{atanh}(c x^2)}{d + e x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^2))/(d + e*x),x)

[Out] int((a + b*atanh(c*x^2))/(d + e*x), x)

$$3.27 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{(d+ex)^2} dx$$

Optimal. Leaf size=166

$$\frac{b\sqrt{c} \operatorname{ArcTan}(\sqrt{c} x)}{cd^2 + e^2} - \frac{b\sqrt{c} \tanh^{-1}(\sqrt{c} x)}{cd^2 - e^2} - \frac{a + b \tanh^{-1}(cx^2)}{e(d+ex)} + \frac{2bcde \log(d+ex)}{c^2d^4 - e^4} - \frac{bcd \log(1 - cx^2)}{2e(cd^2 - e^2)} + \frac{bcd}{2e}$$

[Out] (-a-b*arctanh(c*x^2))/e/(e*x+d)+2*b*c*d*e*ln(e*x+d)/(c^2*d^4-e^4)-1/2*b*c*d*ln(-c*x^2+1)/e/(c*d^2-e^2)+1/2*b*c*d*ln(c*x^2+1)/e/(c*d^2+e^2)+b*arctan(x*c^(1/2))*c^(1/2)/(c*d^2+e^2)-b*arctanh(x*c^(1/2))*c^(1/2)/(c*d^2-e^2)

Rubi [A]

time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6071, 6857, 649, 213, 266, 209}

$$-\frac{a + b \tanh^{-1}(cx^2)}{e(d+ex)} + \frac{b\sqrt{c} \operatorname{ArcTan}(\sqrt{c} x)}{cd^2 + e^2} + \frac{2bcde \log(d+ex)}{c^2d^4 - e^4} - \frac{bcd \log(1 - cx^2)}{2e(cd^2 - e^2)} + \frac{bcd \log(cx^2 + 1)}{2e(cd^2 + e^2)} - \frac{b\sqrt{c} \tanh^{-1}(\sqrt{c} x)}{cd^2 - e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x^2])/(d + e*x)^2,x]

[Out] (b*Sqrt[c]*ArcTan[Sqrt[c]*x])/(c*d^2 + e^2) - (b*Sqrt[c]*ArcTanh[Sqrt[c]*x])/(c*d^2 - e^2) - (a + b*ArcTanh[c*x^2])/(e*(d + e*x)) + (2*b*c*d*e*Log[d + e*x])/(c^2*d^4 - e^4) - (b*c*d*Log[1 - c*x^2])/(2*e*(c*d^2 - e^2)) + (b*c*d*Log[1 + c*x^2])/(2*e*(c*d^2 + e^2))

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 6071

```
Int[((a_) + ArcTanh[(c_)*(x_)^(n_)])*(b_)*((d_) + (e_)*(x_))^(m_), x_Symbol] := Simp[(d + e*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(e*(m + 1))), x] - Dist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx^2)}{(d + ex)^2} dx &= -\frac{a + b \tanh^{-1}(cx^2)}{e(d + ex)} + \frac{b \int \frac{2cx}{(d+ex)(1-c^2x^4)} dx}{e} \\
 &= -\frac{a + b \tanh^{-1}(cx^2)}{e(d + ex)} + \frac{(2bc) \int \frac{x}{(d+ex)(1-c^2x^4)} dx}{e} \\
 &= -\frac{a + b \tanh^{-1}(cx^2)}{e(d + ex)} + \frac{(2bc) \int \left(-\frac{de^3}{(-c^2d^4 + e^4)(d+ex)} + \frac{e-cdx}{2(cd^2 - e^2)(-1+cx^2)} + \frac{e+cdx}{2(cd^2 + e^2)(1+cx^2)} \right) dx}{e} \\
 &= -\frac{a + b \tanh^{-1}(cx^2)}{e(d + ex)} + \frac{2bcde \log(d + ex)}{c^2d^4 - e^4} + \frac{(bc) \int \frac{e-cdx}{-1+cx^2} dx}{e(cd^2 - e^2)} + \frac{(bc) \int \frac{e+cdx}{1+cx^2} dx}{e(cd^2 + e^2)} \\
 &= -\frac{a + b \tanh^{-1}(cx^2)}{e(d + ex)} + \frac{2bcde \log(d + ex)}{c^2d^4 - e^4} + \frac{(bc) \int \frac{1}{-1+cx^2} dx}{cd^2 - e^2} - \frac{(bc^2d) \int \frac{x}{-1+cx^2} dx}{e(cd^2 - e^2)} \\
 &= \frac{b\sqrt{c} \tan^{-1}(\sqrt{c}x)}{cd^2 + e^2} - \frac{b\sqrt{c} \tanh^{-1}(\sqrt{c}x)}{cd^2 - e^2} - \frac{a + b \tanh^{-1}(cx^2)}{e(d + ex)} + \frac{2bcde \log(d + ex)}{c^2d^4 - e^4}
 \end{aligned}$$

Mathematica [A]

time = 0.24, size = 261, normalized size = 1.57

$$\frac{1}{2} \left(-\frac{2a}{e(d+ex)} + \frac{2b\sqrt{c} \operatorname{ArcTan}(\sqrt{c}x)}{cd^2 + e^2} - \frac{2b \tanh^{-1}(cx^2)}{e(d+ex)} + \frac{b\sqrt{c} (c^{3/2}d^3 - cd^2e - e^3) \log(1 - \sqrt{c}x)}{-c^2d^4e + e^5} + \frac{b\sqrt{c} (c^{3/2}d^3 + cd^2e + e^3) \log(1 + \sqrt{c}x)}{-c^2d^4e + e^5} + \frac{4bcde \log(d+ex)}{c^2d^4 - e^4} + \frac{bc^2d^3 \log(1+cx^2)}{c^2d^4 - e^4} - \frac{bcde \log(1-c^2x^4)}{c^2d^4 - e^4} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d + e*x)^2, x]
```

[Out] $((-2*a)/(e*(d + e*x)) + (2*b*\text{Sqrt}[c]*\text{ArcTan}[\text{Sqrt}[c]*x])/(c*d^2 + e^2) - (2*b*\text{ArcTanh}[c*x^2])/(e*(d + e*x)) + (b*\text{Sqrt}[c]*(c^{(3/2)}*d^3 - c*d^2*e - e^3)*\text{Log}[1 - \text{Sqrt}[c]*x])/(-(c^2*d^4*e) + e^5) + (b*\text{Sqrt}[c]*(c^{(3/2)}*d^3 + c*d^2*e + e^3)*\text{Log}[1 + \text{Sqrt}[c]*x])/(-(c^2*d^4*e) + e^5) + (4*b*c*d*e*\text{Log}[d + e*x])/(c^2*d^4 - e^4) + (b*c^2*d^3*\text{Log}[1 + c*x^2])/(c^2*d^4*e - e^5) - (b*c*d*e*\text{Log}[1 - c^2*x^4])/(c^2*d^4 - e^4))/2$

Maple [A]

time = 0.33, size = 181, normalized size = 1.09

method	result
default	$-\frac{a}{(ex+d)e} - \frac{b \operatorname{arctanh}(cx^2)}{(ex+d)e} - \frac{bcd \ln(cx^2-1)}{e(2cd^2-2e^2)} - \frac{2b\sqrt{C} \operatorname{arctanh}(x\sqrt{C})}{2cd^2-2e^2} + \frac{bcd \ln(cx^2+1)}{e(2cd^2+2e^2)} + \frac{2b\sqrt{C} \operatorname{arctan}(x\sqrt{C})}{2cd^2+2e^2} +$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^2))/(e*x+d)^2,x,method=_RETURNVERBOSE)`

[Out] $-a/(e*x+d)/e - b/(e*x+d)/e*\operatorname{arctanh}(c*x^2) - b/e*c/(2*c*d^2-2*e^2)*d*\ln(c*x^2-1) - 2*b*c^{(1/2)}/(2*c*d^2-2*e^2)*\operatorname{arctanh}(x*c^{(1/2)}) + b/e*c/(2*c*d^2+2*e^2)*d*\ln(c*x^2+1) + 2*b*c^{(1/2)}/(2*c*d^2+2*e^2)*\operatorname{arctan}(x*c^{(1/2)}) + 2*b*e*c*d/(c*d^2-e^2)/(c*d^2+e^2)*\ln(e*x+d)$

Maxima [A]

time = 0.46, size = 171, normalized size = 1.03

$$\frac{1}{2} \left(\left(\frac{4de \log(xe+d)}{c^2d^4-e^4} + \frac{d \log(cx^2+1)}{cd^2e+e^3} - \frac{d \log(cx^2-1)}{cd^2e-e^3} + \frac{2 \arctan(\sqrt{c}x)}{(cd^2+e^2)\sqrt{c}} + \frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{(cd^2-e^2)\sqrt{c}} \right) c - \frac{2 \operatorname{arctanh}(cx^2)}{xe^2+de} \right) b - \frac{a}{xe^2+de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/(e*x+d)^2,x, algorithm="maxima")`

[Out] $1/2*((4*d*e*\log(x*e + d)/(c^2*d^4 - e^4) + d*\log(c*x^2 + 1)/(c*d^2*e + e^3) - d*\log(c*x^2 - 1)/(c*d^2*e - e^3) + 2*\operatorname{arctan}(\operatorname{sqrt}(c)*x)/((c*d^2 + e^2)*\operatorname{sqrt}(c)) + \log((c*x - \operatorname{sqrt}(c))/(c*x + \operatorname{sqrt}(c)))/((c*d^2 - e^2)*\operatorname{sqrt}(c)))*c - 2*\operatorname{arctanh}(c*x^2)/(x*e^2 + d*e))*b - a/(x*e^2 + d*e)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 885 vs. 2(149) = 298.

time = 2.57, size = 1762, normalized size = 10.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/(e*x+d)^2,x, algorithm="fricas")`


```
[Out] [-1/2*(2*a*c^2*d^4 - 2*a*cosh(1)^4 - 8*a*cosh(1)^3*sinh(1) - 12*a*cosh(1)^2
*sinh(1)^2 - 8*a*cosh(1)*sinh(1)^3 - 2*a*sinh(1)^4 - 2*(b*c*d^2*x*cosh(1)^2
+ b*c*d^3*cosh(1) - b*x*cosh(1)^4 - b*x*sinh(1)^4 - b*d*cosh(1)^3 - (4*b*x
*cosh(1) + b*d)*sinh(1)^3 + (b*c*d^2*x - 6*b*x*cosh(1)^2 - 3*b*d*cosh(1))*s
inh(1)^2 + (2*b*c*d^2*x*cosh(1) + b*c*d^3 - 4*b*x*cosh(1)^3 - 3*b*d*cosh(1)
^2)*sinh(1))*sqrt(c)*arctan(sqrt(c)*x) + (b*c*d^2*x*cosh(1)^2 + b*c*d^3*cos
h(1) + b*x*cosh(1)^4 + b*x*sinh(1)^4 + b*d*cosh(1)^3 + (4*b*x*cosh(1) + b*d
)*sinh(1)^3 + (b*c*d^2*x + 6*b*x*cosh(1)^2 + 3*b*d*cosh(1))*sinh(1)^2 + (2*
b*c*d^2*x*cosh(1) + b*c*d^3 + 4*b*x*cosh(1)^3 + 3*b*d*cosh(1)^2)*sinh(1))*s
qrt(c)*log((c*x^2 + 2*sqrt(c)*x + 1)/(c*x^2 - 1)) - (b*c^2*d^3*x*cosh(1) +
b*c^2*d^4 - b*c*d*x*cosh(1)^3 - b*c*d*x*sinh(1)^3 - b*c*d^2*cosh(1)^2 - (3*
b*c*d*x*cosh(1) + b*c*d^2)*sinh(1)^2 + (b*c^2*d^3*x - 3*b*c*d*x*cosh(1)^2 -
2*b*c*d^2*cosh(1))*sinh(1))*log(c*x^2 + 1) + (b*c^2*d^3*x*cosh(1) + b*c^2*
d^4 + b*c*d*x*cosh(1)^3 + b*c*d*x*sinh(1)^3 + b*c*d^2*cosh(1)^2 + (3*b*c*d*
x*cosh(1) + b*c*d^2)*sinh(1)^2 + (b*c^2*d^3*x + 3*b*c*d*x*cosh(1)^2 + 2*b*c
*d^2*cosh(1))*sinh(1))*log(c*x^2 - 1) - 4*(b*c*d*x*cosh(1)^3 + b*c*d*x*sinh
(1)^3 + b*c*d^2*cosh(1)^2 + (3*b*c*d*x*cosh(1) + b*c*d^2)*sinh(1)^2 + (3*b*
c*d*x*cosh(1)^2 + 2*b*c*d^2*cosh(1))*sinh(1))*log(x*cosh(1) + x*sinh(1) + d
) + (b*c^2*d^4 - b*cosh(1)^4 - 4*b*cosh(1)^3*sinh(1) - 6*b*cosh(1)^2*sinh(1)
)^2 - 4*b*cosh(1)*sinh(1)^3 - b*sinh(1)^4)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/(
c^2*d^4*x*cosh(1)^2 + c^2*d^5*cosh(1) - x*cosh(1)^6 - x*sinh(1)^6 - d*cosh(
1)^5 - (6*x*cosh(1) + d)*sinh(1)^5 - 5*(3*x*cosh(1)^2 + d*cosh(1))*sinh(1)^
4 - 10*(2*x*cosh(1)^3 + d*cosh(1)^2)*sinh(1)^3 + (c^2*d^4*x - 15*x*cosh(1)^
4 - 10*d*cosh(1)^3)*sinh(1)^2 + (2*c^2*d^4*x*cosh(1) + c^2*d^5 - 6*x*cosh(1)
)^5 - 5*d*cosh(1)^4)*sinh(1)), -1/2*(2*a*c^2*d^4 - 2*a*cosh(1)^4 - 8*a*cosh
(1)^3*sinh(1) - 12*a*cosh(1)^2*sinh(1)^2 - 8*a*cosh(1)*sinh(1)^3 - 2*a*sinh
(1)^4 - 2*(b*c*d^2*x*cosh(1)^2 + b*c*d^3*cosh(1) + b*x*cosh(1)^4 + b*x*sinh
(1)^4 + b*d*cosh(1)^3 + (4*b*x*cosh(1) + b*d)*sinh(1)^3 + (b*c*d^2*x + 6*b*
x*cosh(1)^2 + 3*b*d*cosh(1))*sinh(1)^2 + (2*b*c*d^2*x*cosh(1) + b*c*d^3 + 4
*b*x*cosh(1)^3 + 3*b*d*cosh(1)^2)*sinh(1))*sqrt(-c)*arctan(sqrt(-c)*x) - (b
*c*d^2*x*cosh(1)^2 + b*c*d^3*cosh(1) - b*x*cosh(1)^4 - b*x*sinh(1)^4 - b*d*
cosh(1)^3 - (4*b*x*cosh(1) + b*d)*sinh(1)^3 + (b*c*d^2*x - 6*b*x*cosh(1)^2
- 3*b*d*cosh(1))*sinh(1)^2 + (2*b*c*d^2*x*cosh(1) + b*c*d^3 - 4*b*x*cosh(1)
^3 - 3*b*d*cosh(1)^2)*sinh(1))*sqrt(-c)*log((c*x^2 + 2*sqrt(-c)*x - 1)/(c*x
^2 + 1)) - (b*c^2*d^3*x*cosh(1) + b*c^2*d^4 - b*c*d*x*cosh(1)^3 - b*c*d*x*s
inh(1)^3 - b*c*d^2*cosh(1)^2 - (3*b*c*d*x*cosh(1) + b*c*d^2)*sinh(1)^2 + (b
*c^2*d^3*x - 3*b*c*d*x*cosh(1)^2 - 2*b*c*d^2*cosh(1))*sinh(1))*log(c*x^2 +
1) + (b*c^2*d^3*x*cosh(1) + b*c^2*d^4 + b*c*d*x*cosh(1)^3 + b*c*d*x*sinh(1)
^3 + b*c*d^2*cosh(1)^2 + (3*b*c*d*x*cosh(1) + b*c*d^2)*sinh(1)^2 + (b*c^2*d
^3*x + 3*b*c*d*x*cosh(1)^2 + 2*b*c*d^2*cosh(1))*sinh(1))*log(c*x^2 - 1) - 4
*(b*c*d*x*cosh(1)^3 + b*c*d*x*sinh(1)^3 + b*c*d^2*cosh(1)^2 + (3*b*c*d*x*co
sh(1) + b*c*d^2)*sinh(1)^2 + (3*b*c*d*x*cosh(1)^2 + 2*b*c*d^2*cosh(1))*sinh
(1))*log(x*cosh(1) + x*sinh(1) + d) + (b*c^2*d^4 - b*cosh(1)^4 - 4*b*cosh(1)
)^3*sinh(1) - 6*b*cosh(1)^2*sinh(1)^2 - 4*b*cosh(1)*sinh(1)^3 - b*sinh(1)^4
)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/(c^2*d^4*x*cosh(1)^2 + c^2*d^5*cosh(1) - x
```

```
*cosh(1)^6 - x*sinh(1)^6 - d*cosh(1)^5 - (6*x*cosh(1) + d)*sinh(1)^5 - 5*(3
*x*cosh(1)^2 + d*cosh(1))*sinh(1)^4 - 10*(2*x*cosh(1)^3 + d*cosh(1)^2)*sinh
(1)^3 + (c^2*d^4*x - 15*x*cosh(1)^4 - 10*d*cosh(1)^3)*sinh(1)^2 + (2*c^2*d^
4*x*cosh(1) + c^2*d^5 - 6*x*cosh(1)^5 - 5*d*cosh(1)^4)*sinh(1))]
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(e*x+d)**2,x)
```

[Out] Timed out

Giac [A]

time = 0.45, size = 269, normalized size = 1.62

$$\frac{1}{2} \left(\frac{cd \log \left(c - \frac{2cd}{ex+d} + \frac{cd^2}{(ex+d)^2} + \frac{e^2}{(ex+d)^2} \right)}{cd^2e + e^3} - \frac{cd \log \left(c - \frac{2cd}{ex+d} + \frac{cd^2}{(ex+d)^2} - \frac{e^2}{(ex+d)^2} \right)}{cd^2e - e^3} + \frac{2c \arctan \left(\frac{cd - \frac{cd^2}{ex+d} + \frac{e^2}{ex+d}}{\sqrt{-c}e} \right)}{(cd^2 - e^2)\sqrt{-c}} + \frac{2\sqrt{c} \arctan \left(\frac{cd - \frac{cd^2}{ex+d} - \frac{e^2}{ex+d}}{\sqrt{c}e} \right)}{cd^2 + e^2} - \frac{\log \left(-\frac{ex^2+1}{ex^2-1} \right)}{(ex+d)e} \right) b - \frac{a}{(ex+d)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d*log(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + e^2/(e*x + d)^2)/(c*
d^2*e + e^3) - c*d*log(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 - e^2/(e*x +
d)^2)/(c*d^2*e - e^3) + 2*c*arctan((c*d - c*d^2/(e*x + d) + e^2/(e*x + d)
)/(sqrt(-c)*e))/(c*d^2 - e^2)*sqrt(-c) + 2*sqrt(c)*arctan((c*d - c*d^2/(e*
x + d) - e^2/(e*x + d))/(sqrt(c)*e))/(c*d^2 + e^2) - log(-(c*x^2 + 1)/(c*x^
2 - 1))/((e*x + d)*e))*b - a/((e*x + d)*e)
```

Mupad [B]

time = 2.60, size = 727, normalized size = 4.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d + e*x)^2,x)
```

```
[Out] (log((16*b^4*c^10*x)/e - ((b*(-c)^(1/2)*e + b*c*d)*(8*b^3*c^9*e - ((b*(-c)^(
1/2)*e + b*c*d)*(((b*(-c)^(1/2)*e + b*c*d)*((8*x*(4*b*c^11*d^5*e^2 + 52*b*
c^9*d*e^6))/e^2 - ((b*(-c)^(1/2)*e + b*c*d)*(384*c^8*d*e^6 + (8*x*(40*c^8*e
```

$$\begin{aligned}
& \frac{c^9 + 24c^{10}d^4e^5}{e^2 + 128c^{10}d^5e^2} \Big/ (2(e^3 + cd^2e)) + 320b \\
& \frac{c^9d^2e^3}{2(e^3 + cd^2e)} - 64b^2c^{10}d^2ex \Big/ (2(e^3 + cd^2e)) \\
& + \frac{8b^3c^{11}d^3x}{e^2} \Big/ (2(e^3 + cd^2e)) * (b(-c)^{1/2}e + bcd) \\
& \Big/ (2(e^3 + cd^2e)) - a/(d+e) - (\log((b(-c)^{1/2}e - bcd) * \\
& 8b^3c^9e - ((b(-c)^{1/2}e - bcd) * ((b(-c)^{1/2}e - bcd) * (8x(4 \\
& b^{11}d^5e^2 + 52b^9d^6e^6)) / e^2 + ((b(-c)^{1/2}e - bcd) * (384c^8 \\
& d^6e^6 + (8x(40c^8e^9 + 24c^{10}d^4e^5)) / e^2 + 128c^{10}d^5e^2)) / (2(e^3 + cd^2e)) \\
& + 320b^9d^2e^3) \Big/ (2(e^3 + cd^2e)) + 64b^2c^{10}d^2 \\
& ex \Big/ (2(e^3 + cd^2e)) + \frac{8b^3c^{11}d^3x}{e^2} \Big/ (2(e^3 + cd^2e)) + \\
& \frac{16b^4c^{10}x}{e} * (b(-c)^{1/2}e - bcd) \Big/ (2(e^3 + cd^2e)) - (b \log(c^2 + 1)) \Big/ (2e(d + ex)) \\
& - (bc^{1/2} \log(c^{1/2}x - 1)) \Big/ (2(e^2 + c^{1/2}de)) + (bc^{1/2} \log(c^{1/2}x + 1)) \Big/ (2(e^2 - c^{1/2}de)) \\
& + (b \log(1 - cx^2)) \Big/ (e(2d + 2ex)) - (2bcd \log(d + ex)) \Big/ (e^4 - c^2d^4)
\end{aligned}$$

$$3.28 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{(d+ex)^3} dx$$

Optimal. Leaf size=226

$$-\frac{bcde}{(c^2d^4 - e^4)(d + ex)} + \frac{bc^{3/2}d \operatorname{ArcTan}(\sqrt{c}x)}{(cd^2 + e^2)^2} - \frac{bc^{3/2}d \tanh^{-1}(\sqrt{c}x)}{(cd^2 - e^2)^2} - \frac{a + b \tanh^{-1}(cx^2)}{2e(d + ex)^2} + \frac{bce(3c^2d^4 + e^4) \log(d + ex)}{(c^2d^4 - e^4)(d + ex)}$$

[Out] $-b*c*d*e/(c^2*d^4-e^4)/(e*x+d)+b*c^{(3/2)*d*arctan(x*c^{(1/2)})/(c*d^2+e^2)^2+1/2*(-a-b*arctanh(c*x^2))/e/(e*x+d)^2-b*c^{(3/2)*d*arctanh(x*c^{(1/2)})/(c*d^2-e^2)^2+b*c*e*(3*c^2*d^4+e^4)*\ln(e*x+d)/(c^2*d^4-e^4)^2-1/4*b*c*(c*d^2+e^2)*\ln(-c*x^2+1)/e/(c*d^2-e^2)^2+1/4*b*c*(c*d^2-e^2)*\ln(c*x^2+1)/e/(c*d^2+e^2)^2$

Rubi [A]

time = 0.26, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6071, 6857, 649, 212, 266, 209}

$$-\frac{a + b \tanh^{-1}(cx^2)}{2e(d + ex)^2} + \frac{bc^{3/2}d \operatorname{ArcTan}(\sqrt{c}x)}{(cd^2 + e^2)^2} - \frac{bc^{3/2}d \tanh^{-1}(\sqrt{c}x)}{(cd^2 - e^2)^2} - \frac{bcde}{(c^2d^4 - e^4)(d + ex)} + \frac{bce(3c^2d^4 + e^4) \log(d + ex)}{(c^2d^4 - e^4)^2} - \frac{bc(cd^2 + e^2) \log(1 - cx^2)}{4e(cd^2 - e^2)^2} + \frac{bc(cd^2 - e^2) \log(cx^2 + 1)}{4e(cd^2 + e^2)^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b \operatorname{ArcTanh}[c*x^2])/(d + e*x)^3, x]$

[Out] $-(b*c*d*e)/((c^2*d^4 - e^4)*(d + e*x)) + (b*c^{(3/2)*d*ArcTan[Sqrt[c]*x]}/(c*d^2 + e^2)^2 - (b*c^{(3/2)*d*ArcTanh[Sqrt[c]*x]}/(c*d^2 - e^2)^2 - (a + b*ArcTanh[c*x^2])/(2*e*(d + e*x)^2) + (b*c*e*(3*c^2*d^4 + e^4)*Log[d + e*x])/(c^2*d^4 - e^4)^2 - (b*c*(c*d^2 + e^2)*Log[1 - c*x^2])/(4*e*(c*d^2 - e^2)^2) + (b*c*(c*d^2 - e^2)*Log[1 + c*x^2])/(4*e*(c*d^2 + e^2)^2)$

Rule 209

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{GtQ}[b, 0])$

Rule 212

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))* \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 266

$\operatorname{Int}(x_)^{(m_)} / ((a_) + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \operatorname{FreeQ}\{a, b, m, n\}, x] \&\& \operatorname{EqQ}[m, n - 1]$

Rule 649

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rule 6071

Int[((a_) + ArcTanh[(c_)*(x_)^(n_)])*(b_)*((d_) + (e_)*(x_)^(m_)), x_Symbol] := Simp[(d + e*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(e*(m + 1))), x] - Dist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]

Rule 6857

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\int \frac{a + b \tanh^{-1}(cx^2)}{(d + ex)^3} dx = \int \left(\frac{a}{(d + ex)^3} + \frac{b \tanh^{-1}(cx^2)}{(d + ex)^3} \right) dx$$

$$= -\frac{a}{2e(d + ex)^2} + b \int \frac{\tanh^{-1}(cx^2)}{(d + ex)^3} dx$$

Mathematica [A]

time = 0.46, size = 379, normalized size = 1.68

$$\frac{1}{4} \left(\frac{2a}{e(d+ex)^2} - \frac{4bcde}{(c^2d^2 - e^2)(d+ex)} + \frac{4bc^2d \operatorname{ArcTan}(\sqrt{cx})}{(ae^2 + e^2)^2} - \frac{2b \tanh^{-1}(cx^2)}{e(d+ex)^2} - \frac{bc^2d(c^2d^2e - 2c^2de - 4ae^2e + 3\sqrt{c}de^2 - 2e^2) \log(1 - \sqrt{cx})}{e(-c^2d^2 + e^2)^2} - \frac{bc^2d(c^2d^2e + 2c^2de + 4ae^2e + 3\sqrt{c}de^2 + 2e^2) \log(1 + \sqrt{cx})}{e(-c^2d^2 + e^2)^2} + \frac{4bc(3c^2d^2 + e^2) \log(d+ex)}{(-c^2d^2 + e^2)^2} + \frac{bc^2(c^2d^2 + 3d^2e^2) \log(1+cx^2)}{e(-c^2d^2 + e^2)^2} - \frac{bc(3c^2d^2 + e^2) \log(1-cx^2)}{(-c^2d^2 + e^2)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x^2])/(d + e*x)^3, x]

[Out] ((-2*a)/(e*(d + e*x)^2) - (4*b*c*d*e)/((c^2*d^4 - e^4)*(d + e*x)) + (4*b*c^(3/2)*d*ArcTan[Sqrt[c]*x])/(c*d^2 + e^2)^2 - (2*b*ArcTanh[c*x^2])/(e*(d + e*x)^2) - (b*c^(3/2)*d*(c^(5/2)*d^5 - 2*c^2*d^4*e - 4*c*d^2*e^3 + 3*Sqrt[c]*d*e^4 - 2*e^5)*Log[1 - Sqrt[c]*x])/(e*(-(c^2*d^4) + e^4)^2) - (b*c^(3/2)*d*(c^(5/2)*d^5 + 2*c^2*d^4*e + 4*c*d^2*e^3 + 3*Sqrt[c]*d*e^4 + 2*e^5)*Log[1 + Sqrt[c]*x])/(e*(-(c^2*d^4) + e^4)^2) + (4*b*c*e*(3*c^2*d^4 + e^4)*Log[d + e*x])/(-(c^2*d^4) + e^4)^2 + (b*c^2*(c^2*d^6 + 3*d^2*e^4)*Log[1 + c*x^2])/(e*(-(c^2*d^4) + e^4)^2) - (b*c*e*(3*c^2*d^4 + e^4)*Log[1 - c^2*x^4])/(-(c^2*d^4) + e^4)^2)/4

Maple [A]

time = 0.34, size = 310, normalized size = 1.37

method	result
default	$-\frac{a}{2(ex+d)^2e} - \frac{b \operatorname{arctanh}(cx^2)}{2(ex+d)^2e} - \frac{bc^2 \ln(cx^2-1)d^2}{4e(cd^2-e^2)^2} - \frac{bec \ln(cx^2-1)}{4(cd^2-e^2)^2} - \frac{bc^{\frac{3}{2}} d \operatorname{arctanh}(x\sqrt{c})}{(cd^2-e^2)^2} + \frac{bc^2 \ln(cx^2+1)d^2}{4e(cd^2+e^2)^2} - \frac{bec \ln(cx^2+1)}{4e(cd^2+e^2)^2}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((a+b*arctanh(c*x^2))/(e*x+d)^3,x,method=_RETURNVERBOSE)`

```
[Out] -1/2*a/(e*x+d)^2/e-1/2*b/(e*x+d)^2/e*arctanh(c*x^2)-1/4*b/e*c^2/(c*d^2-e^2)^2*ln(c*x^2-1)*d^2-1/4*b*e*c/(c*d^2-e^2)^2*ln(c*x^2-1)-b*c^(3/2)*d*arctanh(x*c^(1/2))/(c*d^2-e^2)^2+1/4*b/e*c^2/(c*d^2+e^2)^2*ln(c*x^2+1)*d^2-1/4*b*e*c/(c*d^2+e^2)^2*ln(c*x^2+1)+b*c^(3/2)*d*arctan(x*c^(1/2))/(c*d^2+e^2)^2-b*e*c*d/(c*d^2-e^2)/(c*d^2+e^2)/(e*x+d)+3*b*e*c^3/(c*d^2-e^2)^2/(c*d^2+e^2)^2*ln(e*x+d)*d^4+b*e^5*c/(c*d^2-e^2)^2/(c*d^2+e^2)^2*ln(e*x+d)
```

Maxima [A]

time = 0.48, size = 300, normalized size = 1.33

$$\frac{1}{4} \left(\left(\frac{4\sqrt{c}d \operatorname{arctan}(\sqrt{c}x)}{c^2d^4+2cd^2e^2+e^4} + \frac{2\sqrt{c}d \log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^2d^4-2cd^2e^2+e^4} - \frac{4de}{c^2d^4+(c^2d^4-e^5)x-de^4} + \frac{(cd^2-e^2)\log(cx^2+1)}{c^2d^4e+2cd^2e^3+e^5} - \frac{(cd^2+e^2)\log(cx^2-1)}{c^2d^4e-2cd^2e^3+e^5} + \frac{4(3c^2d^4e+e^5)\log(xe+d)}{c^2d^8-2c^2d^4e^4+e^8} \right) c - \frac{2 \operatorname{arctan}(cx^2)}{x^2e^3+2dxe^2+d^2e} \right) b - \frac{a}{2(x^2e^3+2dxe^2+d^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+b*arctanh(c*x^2))/(e*x+d)^3,x, algorithm="maxima")`

```
[Out] 1/4*((4*sqrt(c)*d*arctan(sqrt(c)*x)/(c^2*d^4 + 2*c*d^2*e^2 + e^4) + 2*sqrt(c)*d*log((c*x - sqrt(c))/(c*x + sqrt(c)))/(c^2*d^4 - 2*c*d^2*e^2 + e^4) - 4*d*e/(c^2*d^5 + (c^2*d^4*e - e^5)*x - d*e^4) + (c*d^2 - e^2)*log(c*x^2 + 1)/(c^2*d^4*e + 2*c*d^2*e^3 + e^5) - (c*d^2 + e^2)*log(c*x^2 - 1)/(c^2*d^4*e - 2*c*d^2*e^3 + e^5) + 4*(3*c^2*d^4*e + e^5)*log(x*e + d)/(c^4*d^8 - 2*c^2*d^4*e^4 + e^8))*c - 2*arctanh(c*x^2)/(x^2*e^3 + 2*d*x*e^2 + d^2*e)*b - 1/2*a/(x^2*e^3 + 2*d*x*e^2 + d^2*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4393 vs. 2(205) = 410.

time = 19.41, size = 8795, normalized size = 38.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+b*arctanh(c*x^2))/(e*x+d)^3,x, algorithm="fricas")`

```
[Out] [-1/4*(2*a*c^4*d^8 + 4*b*c^3*d^5*x*cosh(1)^3 + 4*b*c^3*d^6*cosh(1)^2 - 4*a*c^2*d^4*cosh(1)^4 - 4*b*c*d*x*cosh(1)^7 - 4*b*c*d^2*cosh(1)^6 + 2*a*cosh(1)
```

$$\begin{aligned}
&^8 + 2*a*\sinh(1)^8 - 4*(b*c*d*x - 4*a*cosh(1))*\sinh(1)^7 - 4*(7*b*c*d*x*cos \\
&h(1) + b*c*d^2 - 14*a*cosh(1)^2)*\sinh(1)^6 - 4*(21*b*c*d*x*cosh(1)^2 + 6*b* \\
&c*d^2*cosh(1) - 28*a*cosh(1)^3)*\sinh(1)^5 - 4*(a*c^2*d^4 + 35*b*c*d*x*cosh(\\
&1)^3 + 15*b*c*d^2*cosh(1)^2 - 35*a*cosh(1)^4)*\sinh(1)^4 + 4*(b*c^3*d^5*x - \\
&4*a*c^2*d^4*cosh(1) - 35*b*c*d*x*cosh(1)^4 - 20*b*c*d^2*cosh(1)^3 + 28*a*co \\
&sh(1)^5)*\sinh(1)^3 + 4*(3*b*c^3*d^5*x*cosh(1) + b*c^3*d^6 - 6*a*c^2*d^4*cos \\
&h(1)^2 - 21*b*c*d*x*cosh(1)^5 - 15*b*c*d^2*cosh(1)^4 + 14*a*cosh(1)^6)*\sinh \\
&(1)^2 - 4*(2*b*c^3*d^6*x*cosh(1)^2 + b*c^3*d^7*cosh(1) + 4*b*c^2*d^4*x*cosh \\
&(1)^4 + b*c*d*x^2*cosh(1)^7 + b*c*d*x^2*\sinh(1)^7 + 2*b*c*d^2*x*cosh(1)^6 + \\
&(7*b*c*d*x^2*cosh(1) + 2*b*c*d^2*x)*\sinh(1)^6 + (2*b*c^2*d^3*x^2 + b*c*d^3 \\
&)*cosh(1)^5 + (2*b*c^2*d^3*x^2 + 21*b*c*d*x^2*cosh(1)^2 + 12*b*c*d^2*x*cosh \\
&(1) + b*c*d^3)*\sinh(1)^5 + (4*b*c^2*d^4*x + 35*b*c*d*x^2*cosh(1)^3 + 30*b*c \\
&*d^2*x*cosh(1)^2 + 5*(2*b*c^2*d^3*x^2 + b*c*d^3)*cosh(1))*\sinh(1)^4 + (b*c^ \\
&3*d^5*x^2 + 2*b*c^2*d^5)*cosh(1)^3 + (b*c^3*d^5*x^2 + 16*b*c^2*d^4*x*cosh(1 \\
&)) + 35*b*c*d*x^2*cosh(1)^4 + 2*b*c^2*d^5 + 40*b*c*d^2*x*cosh(1)^3 + 10*(2*b \\
&*c^2*d^3*x^2 + b*c*d^3)*cosh(1)^2)*\sinh(1)^3 + (2*b*c^3*d^6*x + 24*b*c^2*d^ \\
&4*x*cosh(1)^2 + 21*b*c*d*x^2*cosh(1)^5 + 30*b*c*d^2*x*cosh(1)^4 + 10*(2*b*c \\
&^2*d^3*x^2 + b*c*d^3)*cosh(1)^3 + 3*(b*c^3*d^5*x^2 + 2*b*c^2*d^5)*cosh(1))* \\
&\sinh(1)^2 + (4*b*c^3*d^6*x*cosh(1) + b*c^3*d^7 + 16*b*c^2*d^4*x*cosh(1)^3 + \\
&7*b*c*d*x^2*cosh(1)^6 + 12*b*c*d^2*x*cosh(1)^5 + 5*(2*b*c^2*d^3*x^2 + b*c \\
&d^3)*cosh(1)^4 + 3*(b*c^3*d^5*x^2 + 2*b*c^2*d^5)*cosh(1)^2)*\sinh(1))*\sqrt{- \\
&c}*\arctan(\sqrt{-c}*x) - 2*(2*b*c^3*d^6*x*cosh(1)^2 + b*c^3*d^7*cosh(1) - 4* \\
&b*c^2*d^4*x*cosh(1)^4 + b*c*d*x^2*cosh(1)^7 + b*c*d*x^2*\sinh(1)^7 + 2*b*c*d \\
&^2*x*cosh(1)^6 + (7*b*c*d*x^2*cosh(1) + 2*b*c*d^2*x)*\sinh(1)^6 - (2*b*c^2*d \\
&^3*x^2 - b*c*d^3)*cosh(1)^5 - (2*b*c^2*d^3*x^2 - 21*b*c*d*x^2*cosh(1)^2 - 1 \\
&2*b*c*d^2*x*cosh(1) - b*c*d^3)*\sinh(1)^5 - (4*b*c^2*d^4*x - 35*b*c*d*x^2*co \\
&sh(1)^3 - 30*b*c*d^2*x*cosh(1)^2 + 5*(2*b*c^2*d^3*x^2 - b*c*d^3)*cosh(1))*s \\
&inh(1)^4 + (b*c^3*d^5*x^2 - 2*b*c^2*d^5)*cosh(1)^3 + (b*c^3*d^5*x^2 - 16*b* \\
&c^2*d^4*x*cosh(1) + 35*b*c*d*x^2*cosh(1)^4 - 2*b*c^2*d^5 + 40*b*c*d^2*x*cos \\
&h(1)^3 - 10*(2*b*c^2*d^3*x^2 - b*c*d^3)*cosh(1)^2)*\sinh(1)^3 + (2*b*c^3*d^6 \\
&*x - 24*b*c^2*d^4*x*cosh(1)^2 + 21*b*c*d*x^2*cosh(1)^5 + 30*b*c*d^2*x*cosh(\\
&1)^4 - 10*(2*b*c^2*d^3*x^2 - b*c*d^3)*cosh(1)^3 + 3*(b*c^3*d^5*x^2 - 2*b*c^ \\
&2*d^5)*cosh(1))*\sinh(1)^2 + (4*b*c^3*d^6*x*cosh(1) + b*c^3*d^7 - 16*b*c^2*d \\
&^4*x*cosh(1)^3 + 7*b*c*d*x^2*cosh(1)^6 + 12*b*c*d^2*x*cosh(1)^5 - 5*(2*b*c^ \\
&2*d^3*x^2 - b*c*d^3)*cosh(1)^4 + 3*(b*c^3*d^5*x^2 - 2*b*c^2*d^5)*cosh(1)^2) \\
&*\sinh(1))*\sqrt{-c}*\log((c*x^2 + 2*\sqrt{-c}*x - 1)/(c*x^2 + 1)) - (2*b*c^4*d \\
&^7*x*cosh(1) + b*c^4*d^8 - 6*b*c^3*d^5*x*cosh(1)^3 + 6*b*c^2*d^3*x*cosh(1)^ \\
&5 - b*c*x^2*cosh(1)^8 - b*c*x^2*\sinh(1)^8 - 2*b*c*d*x*cosh(1)^7 - 2*(4*b*c* \\
&x^2*cosh(1) + b*c*d*x)*\sinh(1)^7 + (3*b*c^2*d^2*x^2 - b*c*d^2)*cosh(1)^6 + \\
&(3*b*c^2*d^2*x^2 - 28*b*c*x^2*cosh(1)^2 - 14*b*c*d*x*cosh(1) - b*c*d^2)*sin \\
&h(1)^6 + 2*(3*b*c^2*d^3*x - 28*b*c*x^2*cosh(1)^3 - 21*b*c*d*x*cosh(1)^2 + 3 \\
&*(3*b*c^2*d^2*x^2 - b*c*d^2)*cosh(1))*\sinh(1)^5 - 3*(b*c^3*d^4*x^2 - b*c^2* \\
&d^4)*cosh(1)^4 - (3*b*c^3*d^4*x^2 - 30*b*c^2*d^3*x*cosh(1) + 70*b*c*x^2*cos \\
&h(1)^4 - 3*b*c^2*d^4 + 70*b*c*d*x*cosh(1)^3 - 15*(3*b*c^2*d^2*x^2 - b*c*d^2 \\
&)*cosh(1)^2)*\sinh(1)^4 - 2*(3*b*c^3*d^5*x - 30*b*c^2*d^3*x*cosh(1)^2 + 28*b
\end{aligned}$$

```

*c*x^2*cosh(1)^5 + 35*b*c*d*x*cosh(1)^4 - 10*(3*b*c^2*d^2*x^2 - b*c*d^2)*cosh(1)^3 + 6*(b*c^3*d^4*x^2 - b*c^2*d^4)*cosh(1)*sinh(1)^3 + (b*c^4*d^6*x^2 - 3*b*c^3*d^6)*cosh(1)^2 + (b*c^4*d^6*x^2 - 18*b*c^3*d^5*x*cosh(1) - 3*b*c^3*d^6 + 60*b*c^2*d^3*x*cosh(1)^3 - 28*b*c*x^2*cosh(1)^6 - 42*b*c*d*x*cosh(1)^5 + 15*(3*b*c^2*d^2*x^2 - b*c*d^2)*cosh(1)^4 - 18*(b*c^3*d^4*x^2 - b*c^2*d^4)*cosh(1)^2*sinh(1)^2 + 2*(b*c^4*d^7*x - 9*b*c^3*d^5*x*cosh(1)^2 + 15*b*c^2*d^3*x*cosh(1)^4 - 4*b*c*x^2*cosh(1)^7 - 7*b*c*d*x*cosh(1)^6 + 3*(3*b*c^2*d^2*x^2 - b*c*d^2)*cosh(1)^5 - 6*(b*c^3*d^4*x^2 - b*c^2*d^4)*cosh(1)^3 + (b*c^4*d^6*x^2 - 3*b*c^3*d^6)*cosh(1)*sinh(1))*log(c*x^2 + 1) + (2*b*c^4*d^7*x*cosh(1) + b*c^4*d^8 + 6*b*c^3*d^5*x*cosh(1)^3 + 6*b*c^2*d^3*x*cosh(1)^5 + b*c*x^2*cosh(1)^8 + b*c*x^2*sinh(1)^8 + 2*b*c*d*x*cosh(1)^7 + 2*(4*b*c*x^2*cosh(1) + b*c*d*x)*sinh(1)^7 + (3*b*c^2*d^2*x^2 + b*c*d^2)*cosh(1)^6 + (3*b*c^2*d^2*x^2 + 28*b*c*x^2*cosh(1)^2 + 14*b*c*d*x*cosh(1) + b*c*d^2)*sinh(1)^6 + 2*(3*b*c^2*d^3*x + 28*b*c*x^2*cosh(1)^3 + 21*b*c*d*x*cosh(1)^2 + 3*(3*b*c^2*d^2*x^2 + b*c*d^2)*cosh(1))*sinh(1)^5 + 3*(b*c^3*d^4*x^2 + b*c^2*d^4)*cosh(1)^4 + (3*b*c^3*d^4*x^2 + 30*b*c^2*d^3*x*cosh(1) + 70*b*c*x^2*cosh(1)^4 + 3*b*c^2*d^4 + 70*b*c*d*x*cosh(1)^3 + 15*(3*b*c^2*d^2*x^2 + b*c*d^2)*cosh(1)^2)*sinh(1)^4 + 2*(3*b*c^3*d^5*x + 30*b*c^2*d^3*x*cosh(1)^2 + 28*b*c*x^2*cosh(1)^5 + 35*b*c*d*x*cosh(1)^4 + 10*(3*b*c^2*d^2*x^2 + b*c*d^2)*cosh(1)^3 + 6*(b*c^3*d^4*x^2 + b*c^2*d^4)*cosh(...)

```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**2))/(e*x+d)**3,x)

[Out] Timed out

Giac [A]

time = 22.47, size = 362, normalized size = 1.60

$$\frac{bc^3 d \arctan(\sqrt{c} x)}{c^2 d^4 + 2 c d^2 e^2 + e^4} + \frac{b c^2 d \arctan\left(\frac{\sqrt{c} x}{\sqrt{-c}}\right)}{(c^2 d^4 - 2 c d^2 e^2 + e^4) \sqrt{-c}} + \frac{(b c^2 d^2 - b c e^2) \log(c x^2 + 1)}{4 (c^2 d^4 e + 2 c d^2 e^3 + e^5)} - \frac{(b c^2 d^2 + b c e^2) \log(-c x^2 + 1)}{4 (c^2 d^4 e - 2 c d^2 e^3 + e^5)} + \frac{(3 b c^2 d^3 e + b c e^3) \log(e x + d)}{c^2 d^3 - 2 c^2 d^4 e^4 + e^8} - \frac{b \log\left(\frac{-c x^2 + 1}{c x^2 + 1}\right)}{4 (e^3 x^2 + 2 d e^2 x + d^2 e)} - \frac{a c^2 d^4 + 2 b c d e^3 x + 2 b c d^2 e^2 - a e^4}{2 (c^2 d^4 e^3 x^2 + 2 c^2 d^3 e^2 x + c^2 d^3 e - e^2 x^2 - 2 d e^2 x - d^2 e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^2))/(e*x+d)^3,x, algorithm="giac")

[Out] b*c^(3/2)*d*arctan(sqrt(c)*x)/(c^2*d^4 + 2*c*d^2*e^2 + e^4) + b*c^2*d*arctan(c*x/sqrt(-c))/((c^2*d^4 - 2*c*d^2*e^2 + e^4)*sqrt(-c)) + 1/4*(b*c^2*d^2 - b*c*e^2)*log(c*x^2 + 1)/(c^2*d^4*e + 2*c*d^2*e^3 + e^5) - 1/4*(b*c^2*d^2 + b*c*e^2)*log(-c*x^2 + 1)/(c^2*d^4*e - 2*c*d^2*e^3 + e^5) + (3*b*c^3*d^4*e + b*c*e^5)*log(e*x + d)/(c^4*d^8 - 2*c^2*d^4*e^4 + e^8) - 1/4*b*log(-(c*x^2 + 1)/(c*x^2 - 1))/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/2*(a*c^2*d^4 + 2*b*c*d

$$\frac{e^{3x} + 2bcd^2e^2 - ae^4}{(c^2d^4e^3x^2 + 2c^2d^5e^2x + c^2d^6e - e^7x^2 - 2de^6x - d^2e^5)}$$

Mupad [B]

time = 5.28, size = 2016, normalized size = 8.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b \cdot \text{atanh}(c \cdot x^2)) / (d + e \cdot x)^3, x)$

[Out]
$$\frac{(a \cdot c^2 d^4 - a e^4 + 2 b c d^2 e^2) / (2(e^4 - c^2 d^4)) + (b c d e^3 x) / (e^4 - c^2 d^4)}{(d^2 e + e^3 x^2 + 2 d e^2 x) + (\log(c d^{28} (-c^3)^{(13/2)} + c^{21} d^{28} x + 100 c^7 e^{28} x - 100 c^2 e^{28} (-c^3)^{(3/2)} + 496 d^2 e^{26} (-c^3)^{(5/2)} - 7398 d^8 e^{20} (-c^3)^{(7/2)} + 17176 d^{14} e^{14} (-c^3)^{(9/2)} - 2703 d^{20} e^8 (-c^3)^{(11/2)} - 20 d^{26} e^2 (-c^3)^{(13/2)} + 496 c^8 d^2 e^{26} x + 1473 c^9 d^4 e^{24} x + 3692 c^{10} d^6 e^{22} x + 7398 c^{11} d^8 e^{20} x + 11868 c^{12} d^{10} e^{18} x + 16015 c^{13} d^{12} e^{16} x + 17176 c^{14} d^{14} e^{14} x + 13192 c^{15} d^{16} e^{12} x + 6984 c^{16} d^{18} e^{10} x + 2703 c^{17} d^{20} e^8 x + 764 c^{18} d^{22} e^6 x + 78 c^{19} d^{24} e^4 x - 20 c^{20} d^{26} e^2 x + 3692 c^2 d^6 e^{22} (-c^3)^{(5/2)} - 16015 c^2 d^{12} e^{16} (-c^3)^{(7/2)} + 6984 c^2 d^{18} e^{10} (-c^3)^{(9/2)} - 78 c^2 d^{24} e^4 (-c^3)^{(11/2)} + 1473 c d^4 e^{24} (-c^3)^{(5/2)} - 11868 c d^{10} e^{18} (-c^3)^{(7/2)} + 13192 c d^{16} e^{12} (-c^3)^{(9/2)} - 764 c d^{22} e^6 (-c^3)^{(11/2)}) \cdot (b c^2 d^2 - b c e^2 + 2 b d e (-c^3)^{(1/2)})}{(4(e^5 + 2 c d^2 e^3 + c^2 d^4 e)) - (\log(c^{21} d^{28} x - c d^{28} (-c^3)^{(13/2)} + 100 c^7 e^{28} x + 100 c^2 e^{28} (-c^3)^{(3/2)} - 496 d^2 e^{26} (-c^3)^{(5/2)} + 7398 d^8 e^{20} (-c^3)^{(7/2)} - 17176 d^{14} e^{14} (-c^3)^{(9/2)} + 2703 d^{20} e^8 (-c^3)^{(11/2)} + 20 d^{26} e^2 (-c^3)^{(13/2)} + 496 c^8 d^2 e^{26} x + 1473 c^9 d^4 e^{24} x + 3692 c^{10} d^6 e^{22} x + 7398 c^{11} d^8 e^{20} x + 11868 c^{12} d^{10} e^{18} x + 16015 c^{13} d^{12} e^{16} x + 17176 c^{14} d^{14} e^{14} x + 13192 c^{15} d^{16} e^{12} x + 6984 c^{16} d^{18} e^{10} x + 2703 c^{17} d^{20} e^8 x + 764 c^{18} d^{22} e^6 x + 78 c^{19} d^{24} e^4 x - 20 c^{20} d^{26} e^2 x - 3692 c^2 d^6 e^{22} (-c^3)^{(5/2)} + 16015 c^2 d^{12} e^{16} (-c^3)^{(7/2)} - 6984 c^2 d^{18} e^{10} (-c^3)^{(9/2)} + 78 c^2 d^{24} e^4 (-c^3)^{(11/2)} - 1473 c d^4 e^{24} (-c^3)^{(5/2)} + 11868 c d^{10} e^{18} (-c^3)^{(7/2)} - 13192 c d^{16} e^{12} (-c^3)^{(9/2)} + 764 c d^{22} e^6 (-c^3)^{(11/2)}) \cdot (b c e^2 - b c^2 d^2 + 2 b d e (-c^3)^{(1/2)})}{(4(e^5 + 2 c d^2 e^3 + c^2 d^4 e)) - (\log(100 e^{28} (c^3)^{(7/2)} + c^2 d^{28} (c^3)^{(15/2)} - 3692 d^6 e^{22} (c^3)^{(9/2)} + 16015 d^{12} e^{16} (c^3)^{(11/2)} - 6984 d^{18} e^{10} (c^3)^{(13/2)} + 78 d^{24} e^4 (c^3)^{(15/2)} + c^{25} d^{28} x + 100 c^{11} e^{28} x - 496 c^{12} d^2 e^{26} x + 1473 c^{13} d^4 e^{24} x - 3692 c^{14} d^6 e^{22} x + 7398 c^{15} d^8 e^{20} x - 11868 c^{16} d^{10} e^{18} x + 16015 c^{17} d^{12} e^{16} x - 17176 c^{18} d^{14} e^{14} x + 13192 c^{19} d^{16} e^{12} x - 6984 c^{20} d^{18} e^{10} x + 2703 c^{21} d^{20} e^8 x - 764 c^{22} d^{22} e^6 x + 78 c^{23} d^{24} e^4 x + 20 c^{24} d^{26} e^2 x - 496 c d^2 e^{26} (c^3)^{(7/2)} + 7398 c d^8 e^{20} (c^3)^{(9/2)} - 17176 c d^{14} e^{14} (c^3)^{(11/2)} + 2703 c d^{20} e^8 (c^3)^{(13/2)} + 20 c d^{26} e^2 (c^3)^{(15/2)} + 1473 c^2 d^4 e^{24} x$$

$$\begin{aligned}
& (c^3)^{(7/2)} - 11868*c^2*d^{10}*e^{18}*(c^3)^{(9/2)} + 13192*c^2*d^{16}*e^{12}*(c^3)^{(11/2)} - 764*c^2*d^{22}*e^6*(c^3)^{(13/2)}*(b*c^2*d^2 + b*c*e^2 + 2*b*d*e*(c^3)^{(1/2)))/(4*(e^5 - 2*c*d^2*e^3 + c^2*d^4*e)) - (\log(100*e^{28}*(c^3)^{(7/2)} + c^2*d^{28}*(c^3)^{(15/2)} - 3692*d^6*e^{22}*(c^3)^{(9/2)} + 16015*d^{12}*e^{16}*(c^3)^{(11/2)} - 6984*d^{18}*e^{10}*(c^3)^{(13/2)} + 78*d^{24}*e^4*(c^3)^{(15/2)} - c^{25}*d^{28}*x - 100*c^{11}*e^{28}*x + 496*c^{12}*d^2*e^{26}*x - 1473*c^{13}*d^4*e^{24}*x + 3692*c^{14}*d^6*e^{22}*x - 7398*c^{15}*d^8*e^{20}*x + 11868*c^{16}*d^{10}*e^{18}*x - 16015*c^{17}*d^{12}*e^{16}*x + 17176*c^{18}*d^{14}*e^{14}*x - 13192*c^{19}*d^{16}*e^{12}*x + 6984*c^{20}*d^{18}*e^{10}*x - 2703*c^{21}*d^{20}*e^8*x + 764*c^{22}*d^{22}*e^6*x - 78*c^{23}*d^{24}*e^4*x - 20*c^{24}*d^{26}*e^2*x - 496*c*d^2*e^{26}*(c^3)^{(7/2)} + 7398*c*d^8*e^{20}*(c^3)^{(9/2)} - 17176*c*d^{14}*e^{14}*(c^3)^{(11/2)} + 2703*c*d^{20}*e^8*(c^3)^{(13/2)} + 20*c*d^{26}*e^2*(c^3)^{(15/2)} + 1473*c^2*d^4*e^{24}*(c^3)^{(7/2)} - 11868*c^2*d^{10}*e^{18}*(c^3)^{(9/2)} + 13192*c^2*d^{16}*e^{12}*(c^3)^{(11/2)} - 764*c^2*d^{22}*e^6*(c^3)^{(13/2)}*(b*c^2*d^2 + b*c*e^2 - 2*b*d*e*(c^3)^{(1/2)))/(4*(e^5 - 2*c*d^2*e^3 + c^2*d^4*e)) + (\log(d + e*x)*(b*c*e^5 + 3*b*c^3*d^4*e))/(e^8 + c^4*d^8 - 2*c^2*d^4*e^4) - (b*log(c*x^2 + 1))/(4*e*(d^2 + e^2*x^2 + 2*d*e*x)) + (b*log(1 - c*x^2))/(2*e*(2*d^2 + 2*e^2*x^2 + 4*d*e*x))
\end{aligned}$$

3.29 $\int (d + ex) (a + b \tanh^{-1}(cx^2))^2 dx$

Optimal. Leaf size=1085

$$a^2 dx + \frac{2abd \operatorname{ArcTan}(\sqrt{c}x)}{\sqrt{c}} + \frac{ib^2 d \operatorname{ArcTan}(\sqrt{c}x)^2}{\sqrt{c}} - \frac{2abd \tanh^{-1}(\sqrt{c}x)}{\sqrt{c}} - \frac{b^2 d \tanh^{-1}(\sqrt{c}x)^2}{\sqrt{c}} + \frac{e(a + b \tanh^{-1}(cx^2))^2}{2}$$

[Out] $-1/2*b^2*e*polylog(2,1-2/(-c*x^2+1))/c+2*b^2*d*arctan(x*c^{(1/2)})*\ln(2/(1+I*x*c^{(1/2)}))/c^{(1/2)}-2*b^2*d*arctanh(x*c^{(1/2)})*\ln(2/(1+x*c^{(1/2)}))/c^{(1/2)}-1/2*I*b^2*d*polylog(2,1-(1+I)*(1-x*c^{(1/2)})/(1-I*x*c^{(1/2)}))/c^{(1/2)}-1/2*I*b^2*d*polylog(2,1+(-1+I)*(1+x*c^{(1/2)})/(1-I*x*c^{(1/2)}))/c^{(1/2)}-b*e*(a+b*arctanh(c*x^2))*\ln(2/(-c*x^2+1))/c-a*b*d*x*\ln(-c*x^2+1)+a*b*d*x*\ln(c*x^2+1)+I*b^2*d*arctan(x*c^{(1/2)})^2/c^{(1/2)}-b^2*d*arctan(x*c^{(1/2)})*\ln(-c*x^2+1)/c^{(1/2)}+b^2*d*arctanh(x*c^{(1/2)})*\ln(-c*x^2+1)/c^{(1/2)}+b^2*d*arctan(x*c^{(1/2)})*\ln(c*x^2+1)/c^{(1/2)}-b^2*d*arctanh(x*c^{(1/2)})*\ln(c*x^2+1)/c^{(1/2)}+b^2*d*arctan(x*c^{(1/2)})*\ln((1+I)*(1-x*c^{(1/2)})/(1-I*x*c^{(1/2)}))/c^{(1/2)}+b^2*d*arctanh(x*c^{(1/2)})*\ln(-2*(1-x*(-c)^{(1/2)})*c^{(1/2)}/((-c)^{(1/2)}-c^{(1/2)})/(1+x*c^{(1/2)}))/c^{(1/2)}+b^2*d*arctanh(x*c^{(1/2)})*\ln(2*(1+x*(-c)^{(1/2)})*c^{(1/2)}/((-c)^{(1/2)}+c^{(1/2)})/(1+x*c^{(1/2)}))/c^{(1/2)}+b^2*d*arctan(x*c^{(1/2)})*\ln((1-I)*(1+x*c^{(1/2)})/(1-I*x*c^{(1/2)}))/c^{(1/2)}+I*b^2*d*polylog(2,1-2/(1-I*x*c^{(1/2)}))/c^{(1/2)}+I*b^2*d*polylog(2,1-2/(1+I*x*c^{(1/2)}))/c^{(1/2)}-1/2*b^2*d*x*\ln(-c*x^2+1)*\ln(c*x^2+1)+2*a*b*d*arctan(x*c^{(1/2)})/c^{(1/2)}-2*a*b*d*arctanh(x*c^{(1/2)})/c^{(1/2)}+2*b^2*d*arctanh(x*c^{(1/2)})*\ln(2/(1-x*c^{(1/2)}))/c^{(1/2)}-2*b^2*d*arctan(x*c^{(1/2)})*\ln(2/(1-I*x*c^{(1/2)}))/c^{(1/2)}+a^2*d*x+1/4*b^2*d*x*\ln(-c*x^2+1)^2+1/4*b^2*d*x*\ln(c*x^2+1)^2-1/2*b^2*d*polylog(2,1+2*(1-x*(-c)^{(1/2)})*c^{(1/2)}/((-c)^{(1/2)}-c^{(1/2)})/(1+x*c^{(1/2)}))/c^{(1/2)}-1/2*b^2*d*polylog(2,1-2*(1+x*(-c)^{(1/2)})*c^{(1/2)}/((-c)^{(1/2)}+c^{(1/2)})/(1+x*c^{(1/2)}))/c^{(1/2)}-b^2*d*arctanh(x*c^{(1/2)})^2/c^{(1/2)}+b^2*d*polylog(2,1-2/(1-x*c^{(1/2)}))/c^{(1/2)}+b^2*d*polylog(2,1-2/(1+x*c^{(1/2)}))/c^{(1/2)}+1/2*e*(a+b*arctanh(c*x^2))^2/c+1/2*e*x^2*(a+b*arctanh(c*x^2))^2$

Rubi [A]

time = 1.34, antiderivative size = 1085, normalized size of antiderivative = 1.00, number of steps used = 77, number of rules used = 24, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {6073, 6023, 2498, 327, 212, 2500, 2526, 2520, 12, 6131, 6055, 2449, 2352, 209, 2636, 6139, 6057, 2497, 5048, 4966, 5040, 4964, 6039, 6021}

Antiderivative was successfully verified.

[In] Int[(d + e*x)*(a + b*ArcTanh[c*x^2])^2,x]

[Out] $a^2*d*x + (2*a*b*d*ArcTan[Sqrt[c]*x])/Sqrt[c] + (I*b^2*d*ArcTan[Sqrt[c]*x]^2)/Sqrt[c] - (2*a*b*d*ArcTanh[Sqrt[c]*x])/Sqrt[c] - (b^2*d*ArcTanh[Sqrt[c]*$

$$\begin{aligned}
& x]^2)/\text{Sqrt}[c] + (e*(a + b*\text{ArcTanh}[c*x^2])^2)/(2*c) + (e*x^2*(a + b*\text{ArcTanh}[\\
& c*x^2])^2)/2 + (2*b^2*d*\text{ArcTanh}[\text{Sqrt}[c]*x]*\text{Log}[2/(1 - \text{Sqrt}[c]*x)])/\text{Sqrt}[c] \\
& - (2*b^2*d*\text{ArcTan}[\text{Sqrt}[c]*x]*\text{Log}[2/(1 - I*\text{Sqrt}[c]*x)])/\text{Sqrt}[c] + (b^2*d*\text{Arc} \\
& \text{Tan}[\text{Sqrt}[c]*x]*\text{Log}[(1 + I)*(1 - \text{Sqrt}[c]*x)/(1 - I*\text{Sqrt}[c]*x)])/\text{Sqrt}[c] + \\
& (2*b^2*d*\text{ArcTan}[\text{Sqrt}[c]*x]*\text{Log}[2/(1 + I*\text{Sqrt}[c]*x)])/\text{Sqrt}[c] - (2*b^2*d*\text{Arc} \\
& \text{Tanh}[\text{Sqrt}[c]*x]*\text{Log}[2/(1 + \text{Sqrt}[c]*x)])/\text{Sqrt}[c] + (b^2*d*\text{ArcTanh}[\text{Sqrt}[c]*x] \\
& * \text{Log}[(-2*\text{Sqrt}[c]*(1 - \text{Sqrt}[-c]*x))/((\text{Sqrt}[-c] - \text{Sqrt}[c])*(1 + \text{Sqrt}[c]*x))]) \\
& / \text{Sqrt}[c] + (b^2*d*\text{ArcTanh}[\text{Sqrt}[c]*x]*\text{Log}[(2*\text{Sqrt}[c]*(1 + \text{Sqrt}[-c]*x))/((\text{Sqr} \\
& \text{t}[-c] + \text{Sqrt}[c])*(1 + \text{Sqrt}[c]*x))])/\text{Sqrt}[c] + (b^2*d*\text{ArcTan}[\text{Sqrt}[c]*x]*\text{Log}[\\
& ((1 - I)*(1 + \text{Sqrt}[c]*x))/(1 - I*\text{Sqrt}[c]*x)])/\text{Sqrt}[c] - (b*e*(a + b*\text{ArcTanh} \\
& [c*x^2])* \text{Log}[2/(1 - c*x^2)]/c - a*b*d*x*\text{Log}[1 - c*x^2] - (b^2*d*\text{ArcTan}[\text{Sqr} \\
& \text{t}[c]*x]*\text{Log}[1 - c*x^2])/\text{Sqrt}[c] + (b^2*d*\text{ArcTanh}[\text{Sqrt}[c]*x]*\text{Log}[1 - c*x^2]) \\
& / \text{Sqrt}[c] + (b^2*d*x*\text{Log}[1 - c*x^2]^2)/4 + a*b*d*x*\text{Log}[1 + c*x^2] + (b^2*d*A \\
& \text{rcTan}[\text{Sqrt}[c]*x]*\text{Log}[1 + c*x^2])/\text{Sqrt}[c] - (b^2*d*\text{ArcTanh}[\text{Sqrt}[c]*x]*\text{Log}[1 \\
& + c*x^2])/\text{Sqrt}[c] - (b^2*d*x*\text{Log}[1 - c*x^2]*\text{Log}[1 + c*x^2])/2 + (b^2*d*x*Lo \\
& \text{g}[1 + c*x^2]^2)/4 + (b^2*d*\text{PolyLog}[2, 1 - 2/(1 - \text{Sqrt}[c]*x)])/\text{Sqrt}[c] + (I* \\
& b^2*d*\text{PolyLog}[2, 1 - 2/(1 - I*\text{Sqrt}[c]*x)])/\text{Sqrt}[c] - ((I/2)*b^2*d*\text{PolyLog}[2 \\
& , 1 - ((1 + I)*(1 - \text{Sqrt}[c]*x))/(1 - I*\text{Sqrt}[c]*x)])/\text{Sqrt}[c] + (I*b^2*d*\text{Poly} \\
& \text{Log}[2, 1 - 2/(1 + I*\text{Sqrt}[c]*x)])/\text{Sqrt}[c] + (b^2*d*\text{PolyLog}[2, 1 - 2/(1 + \text{Sqr} \\
& \text{t}[c]*x)])/\text{Sqrt}[c] - (b^2*d*\text{PolyLog}[2, 1 + (2*\text{Sqrt}[c]*(1 - \text{Sqrt}[-c]*x))/((\text{Sqr} \\
& \text{t}[-c] - \text{Sqrt}[c])*(1 + \text{Sqrt}[c]*x)))/(2*\text{Sqrt}[c]) - (b^2*d*\text{PolyLog}[2, 1 - (2 \\
& *\text{Sqrt}[c]*(1 + \text{Sqrt}[-c]*x))/((\text{Sqrt}[-c] + \text{Sqrt}[c])*(1 + \text{Sqrt}[c]*x)))/(2*\text{Sqrt} \\
& [c]) - ((I/2)*b^2*d*\text{PolyLog}[2, 1 - ((1 - I)*(1 + \text{Sqrt}[c]*x))/(1 - I*\text{Sqrt}[c] \\
& *x)])/\text{Sqrt}[c] - (b^2*e*\text{PolyLog}[2, 1 - 2/(1 - c*x^2)])/(2*c)
\end{aligned}$$
Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n
- 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*(m - n + 1)/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

$x] /; \text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n - 1] \&\& \text{NeQ}[m + n*p + 1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2352

$\text{Int}[\text{Log}[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] \rightarrow \text{Simp}[(-e^{(-1)})*\text{PolyLog}[2, 1 - c*x], x] /; \text{FreeQ}\{c, d, e\}, x\} \&\& \text{EqQ}[e + c*d, 0]$

Rule 2449

$\text{Int}[\text{Log}[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[-e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x\} \&\& \text{EqQ}[c, 2*d] \&\& \text{EqQ}[e^2*f + d^2*g, 0]$

Rule 2497

$\text{Int}[\text{Log}[u]*(Pq_)^{(m_.)}, x_Symbol] \rightarrow \text{With}\{C = \text{FullSimplify}[Pq^m*((1 - u)/D[u, x])]\}, \text{Simp}[C*\text{PolyLog}[2, 1 - u], x] /; \text{FreeQ}[C, x] /; \text{IntegerQ}[m] \&\& \text{PolyQ}[Pq, x] \&\& \text{RationalFunctionQ}[u, x] \&\& \text{LeQ}[\text{RationalFunctionExponents}[u, x][[2]], \text{Expon}[Pq, x]]$

Rule 2498

$\text{Int}[\text{Log}[(c_.)*((d_) + (e_.)*(x_)^{(n_)})^{(p_.)}], x_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*(d + e*x^n)^p], x] - \text{Dist}[e*n*p, \text{Int}[x^n/(d + e*x^n), x], x] /; \text{FreeQ}\{c, d, e, n, p\}, x]$

Rule 2500

$\text{Int}[(a_. + \text{Log}[(c_.)*((d_) + (e_.)*(x_)^{(n_)})^{(p_.)}]*(b_.))^{(q_.)}, x_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{Log}[c*(d + e*x^n)^p])^q, x] - \text{Dist}[b*e*n*p*q, \text{Int}[x^n*(a + b*\text{Log}[c*(d + e*x^n)^p])^{(q - 1)}/(d + e*x^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x\} \&\& \text{IGtQ}[q, 0] \&\& (\text{EqQ}[q, 1] \parallel \text{IntegerQ}[n])$

Rule 2520

$\text{Int}[(a_. + \text{Log}[(c_.)*((d_) + (e_.)*(x_)^{(n_)})^{(p_.)}]*(b_.))/((f_) + (g_.)*(x_)^2), x_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[1/(f + g*x^2), x]\}, \text{Simp}[u*(a + b*\text{Log}[c*(d + e*x^n)^p]), x] - \text{Dist}[b*e*n*p, \text{Int}[u*(x^{(n - 1)})/(d + e*x^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, p\}, x\} \&\& \text{IntegerQ}[n]$

Rule 2526

$\text{Int}[(a_. + \text{Log}[(c_.)*((d_) + (e_.)*(x_)^{(n_)})^{(p_.)}]*(b_.))^{(q_.)}*(x_)^{(m_.)}*((f_) + (g_.)*(x_)^{(s_)})^{(r_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{Log}[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; \text{FreeQ}\{a, b, c, d, e$

, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]

Rule 2636

Int[Log[v_]*Log[w_], x_Symbol] := Simp[x*Log[v]*Log[w], x] + (-Int[SimplifyIntegrand[x*Log[w]*(D[v, x]/v), x], x] - Int[SimplifyIntegrand[x*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]

Rule 4964

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(- (a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]

Rule 4966

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(- (a + b*ArcTan[c*x])*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]

Rule 5040

Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]

Rule 5048

Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])

Rule 6021

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && (EqQ[n, 1] || EqQ[p, 1])

Rule 6023

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]

Rule 6039

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify[(m + 1)/n]]

Rule 6055

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(- (a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 6057

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(- (a + b*ArcTanh[c*x])*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 6073

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))^(p_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x^n])^p, (d + e*x)^m, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 1] && IGtQ[m, 0]

Rule 6131

Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_.) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6139

Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))*(x_)^(m_.)/((d_.) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],

```
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

Mathematica [A]

time = 2.13, size = 684, normalized size = 0.63

Antiderivative was successfully verified.

`[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x^2])^2,x]`

```
[Out] (2*a^2*c*d*x^2 + a^2*c*e*x^3 + 4*a*b*c*d*x^2*ArcTanh[c*x^2] + 4*a*b*d*Sqrt[
c*x^2]*(ArcTan[Sqrt[c*x^2]] - ArcTanh[Sqrt[c*x^2]]) + b^2*e*x*ArcTanh[c*x^2
]*((-1 + c*x^2)*ArcTanh[c*x^2] - 2*Log[1 + E^(-2*ArcTanh[c*x^2])]) + a*b*e*
x*(2*c*x^2*ArcTanh[c*x^2] + Log[1 - c^2*x^4]) + b^2*e*x*PolyLog[2, -E^(-2*A
rcTanh[c*x^2])] - b^2*d*Sqrt[c*x^2]*((2*I)*ArcTan[Sqrt[c*x^2]]^2 - 4*ArcTan
[Sqrt[c*x^2]]*ArcTanh[c*x^2] - 2*Sqrt[c*x^2]*ArcTanh[c*x^2]^2 - 2*ArcTan[Sq
rt[c*x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c*x^2]])] - 2*ArcTanh[c*x^2]*Log[1
- Sqrt[c*x^2]] + Log[2]*Log[1 - Sqrt[c*x^2]] - Log[1 - Sqrt[c*x^2]]^2/2 + L
og[1 - Sqrt[c*x^2]]*Log[(1/2 + I/2)*(-1 + Sqrt[c*x^2])] + 2*ArcTanh[c*x^2]*
Log[1 + Sqrt[c*x^2]] - Log[2]*Log[1 + Sqrt[c*x^2]] - Log[((1 + I) - (1 - I)
*Sqrt[c*x^2])/2]*Log[1 + Sqrt[c*x^2]] - Log[(-1/2 - I/2)*(1 + Sqrt[c*x^2])]
*Log[1 + Sqrt[c*x^2]] + Log[1 + Sqrt[c*x^2]]^2/2 + Log[1 - Sqrt[c*x^2]]*Log
[((1 + I) + (1 - I)*Sqrt[c*x^2])/2] + (I/2)*PolyLog[2, -E^((4*I)*ArcTan[Sqr
t[c*x^2]])] - PolyLog[2, (1 - Sqrt[c*x^2])/2] + PolyLog[2, (-1/2 - I/2)*(-1
+ Sqrt[c*x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c*x^2])] + PolyLog[2,
(1 + Sqrt[c*x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c*x^2])] - PolyLog
[2, (1/2 + I/2)*(1 + Sqrt[c*x^2])])/(2*c*x)
```

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int (ex + d) (a + b \operatorname{arctanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((e*x+d)*(a+b*arctanh(c*x^2))^2,x)``[Out] int((e*x+d)*(a+b*arctanh(c*x^2))^2,x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((e*x+d)*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")`

```
[Out] 1/2*a^2*x^2*e + (c*(2*arctan(sqrt(c)*x)/c^(3/2) + log((c*x - sqrt(c))/(c*x
+ sqrt(c)))/c^(3/2)) + 2*x*arctanh(c*x^2))*a*b*d + a^2*d*x + 1/2*(2*c*x^2*a
rctanh(c*x^2) + log(-c^2*x^4 + 1))*a*b*e/c + 1/8*(b^2*x^2*e + 2*b^2*d*x)*lo
g(-c*x^2 + 1)^2 - integrate(-1/4*((b^2*c*x^3*e + b^2*c*d*x^2 - b^2*x*e - b^
2*d)*log(c*x^2 + 1)^2 - 2*(b^2*c*x^3*e + 2*b^2*c*d*x^2 + (b^2*c*x^3*e + b^2
*c*d*x^2 - b^2*x*e - b^2*d)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 - 1), x
)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x*e + a^2*d + (b^2*x*e + b^2*d)*arctanh(c*x^2)^2 + 2*(a*b*x*e
+ a*b*d)*arctanh(c*x^2), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx^2))^2 (d + ex) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2*(d + e*x), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arctanh(c*x^2) + a)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{atanh}(cx^2))^2 (d + ex) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2*(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x^2))^2*(d + e*x), x)
```

$$3.30 \quad \int \frac{(a+b \tanh^{-1}(cx^2))^2}{d+ex} dx$$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b \tanh^{-1}(cx^2))^2}{d+ex}, x\right)$$

[Out] Unintegrable((a+b*arctanh(c*x^2))^2/(e*x+d), x)

Rubi [A]

time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{d+ex} dx$$

Verification is not applicable to the result.

[In] Int[(a + b*ArcTanh[c*x^2])^2/(d + e*x), x]

[Out] Defer[Int] [(a + b*ArcTanh[c*x^2])^2/(d + e*x), x]

Rubi steps

$$\begin{aligned} \int \frac{(a+b \tanh^{-1}(cx^2))^2}{d+ex} dx &= \int \left(\frac{a^2}{d+ex} + \frac{2ab \tanh^{-1}(cx^2)}{d+ex} + \frac{b^2 \tanh^{-1}(cx^2)^2}{d+ex} \right) dx \\ &= \frac{a^2 \log(d+ex)}{e} + (2ab) \int \frac{\tanh^{-1}(cx^2)}{d+ex} dx + b^2 \int \frac{\tanh^{-1}(cx^2)^2}{d+ex} dx \end{aligned}$$

Mathematica [A]

time = 30.39, size = 0, normalized size = 0.00

$$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{d+ex} dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*ArcTanh[c*x^2])^2/(d + e*x), x]

[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/(d + e*x), x]

Maple [A]

time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a+b \operatorname{arctanh}(cx^2))^2}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^2))^2/(e*x+d),x)`

[Out] `int((a+b*arctanh(c*x^2))^2/(e*x+d),x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^2/(e*x+d),x, algorithm="maxima")`

[Out] `a^2*e^(-1)*log(x*e + d) + integrate(1/4*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2/(x*e + d) + a*b*(log(c*x^2 + 1) - log(-c*x^2 + 1))/(x*e + d), x)`

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^2/(e*x+d),x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(c*x^2))^2 + 2*a*b*arctanh(c*x^2) + a^2)/(x*e + d), x)`

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**2))**2/(e*x+d),x)`

[Out] Timed out

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^2/(e*x+d),x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^2) + a)^2/(e*x + d), x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a + b \operatorname{atanh}(c x^2))^2}{d + e x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^2))^2/(d + e*x),x)

[Out] int((a + b*atanh(c*x^2))^2/(d + e*x), x)

$$3.31 \quad \int \frac{(a+b \tanh^{-1}(cx^2))^2}{(d+ex)^2} dx$$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b \tanh^{-1}(cx^2))^2}{(d+ex)^2}, x\right)$$

[Out] Unintegrable((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x)

Rubi [A]

time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{(d+ex)^2} dx$$

Verification is not applicable to the result.

[In] Int[(a + b*ArcTanh[c*x^2])^2/(d + e*x)^2,x]

[Out] Defer[Int] [(a + b*ArcTanh[c*x^2])^2/(d + e*x)^2, x]

Rubi steps

$$\begin{aligned} \int \frac{(a+b \tanh^{-1}(cx^2))^2}{(d+ex)^2} dx &= \int \left(\frac{a^2}{(d+ex)^2} + \frac{2ab \tanh^{-1}(cx^2)}{(d+ex)^2} + \frac{b^2 \tanh^{-1}(cx^2)^2}{(d+ex)^2} \right) dx \\ &= -\frac{a^2}{e(d+ex)} + (2ab) \int \frac{\tanh^{-1}(cx^2)}{(d+ex)^2} dx + b^2 \int \frac{\tanh^{-1}(cx^2)^2}{(d+ex)^2} dx \\ &= -\frac{a^2}{e(d+ex)} - \frac{2ab \tanh^{-1}(cx^2)}{e(d+ex)} + b^2 \int \frac{\tanh^{-1}(cx^2)^2}{(d+ex)^2} dx + \frac{(2ab) \int \frac{2cx}{(d+ex)(1-cx^2)}}{e} \\ &= -\frac{a^2}{e(d+ex)} - \frac{2ab \tanh^{-1}(cx^2)}{e(d+ex)} + b^2 \int \frac{\tanh^{-1}(cx^2)^2}{(d+ex)^2} dx + \frac{(4abc) \int \frac{x}{(d+ex)(1-cx^2)}}{e} \\ &= -\frac{a^2}{e(d+ex)} - \frac{2ab \tanh^{-1}(cx^2)}{e(d+ex)} + b^2 \int \frac{\tanh^{-1}(cx^2)^2}{(d+ex)^2} dx + \frac{(4abc) \int \left(-\frac{1}{(-c^2d+cx^2)} \right)}{e} \\ &= -\frac{a^2}{e(d+ex)} - \frac{2ab \tanh^{-1}(cx^2)}{e(d+ex)} + \frac{4abcde \log(d+ex)}{c^2d^4 - e^4} + b^2 \int \frac{\tanh^{-1}(cx^2)^2}{(d+ex)^2} dx \\ &= -\frac{a^2}{e(d+ex)} - \frac{2ab \tanh^{-1}(cx^2)}{e(d+ex)} + \frac{4abcde \log(d+ex)}{c^2d^4 - e^4} + b^2 \int \frac{\tanh^{-1}(cx^2)^2}{(d+ex)^2} dx \\ &= -\frac{a^2}{e(d+ex)} + \frac{2ab\sqrt{c} \tan^{-1}(\sqrt{c}x)}{cd^2 + e^2} - \frac{2ab\sqrt{c} \tanh^{-1}(\sqrt{c}x)}{cd^2 - e^2} - \frac{2ab \tanh^{-1}(cx^2)}{e(d+ex)} \end{aligned}$$

Mathematica [A]

time = 25.99, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{(d + ex)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/(d + e*x)^2,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/(d + e*x)^2, x]
```

Maple [A]

time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arctanh}(cx^2))^2}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x)
```

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] ((4*d*e*log(x*e + d)/(c^2*d^4 - e^4) + d*log(c*x^2 + 1)/(c*d^2*e + e^3) - d
*log(c*x^2 - 1)/(c*d^2*e - e^3) + 2*arctan(sqrt(c)*x)/((c*d^2 + e^2)*sqrt(c
)) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/((c*d^2 - e^2)*sqrt(c)))*c - 2*ar
ctanh(c*x^2)/(x*e^2 + d*e))*a*b - 1/4*b^2*(log(-c*x^2 + 1)^2/(x*e^2 + d*e)
+ integrate(-((c*x^2*e - e)*log(c*x^2 + 1)^2 + 2*(2*c*x^2*e + 2*c*d*x - (c
*x^2*e - e)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^4*e^3 + 2*c*d*x^3*e^2 + (c
*d^2*e - e^3)*x^2 - 2*d*x*e^2 - d^2*e), x)) - a^2/(x*e^2 + d*e)
```

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x, algorithm="fricas")
```


[Out] `integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/(x^2*e^2 + 2*d*x*e + d^2), x)`

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**2))**2/(e*x+d)**2,x)`

[Out] Timed out

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^2) + a)^2/(e*x + d)^2, x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a + b \operatorname{atanh}(c x^2))^2}{(d + e x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c*x^2))^2/(d + e*x)^2,x)`

[Out] `int((a + b*atanh(c*x^2))^2/(d + e*x)^2, x)`

3.32 $\int (d + ex)^2 (a + b \tanh^{-1}(cx^3)) dx$

Optimal. Leaf size=336

$$-\frac{\sqrt{3} bde \operatorname{ArcTan}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{c} x}{\sqrt{3}}\right)}{2c^{2/3}} + \frac{\sqrt{3} bde \operatorname{ArcTan}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{c} x}{\sqrt{3}}\right)}{2c^{2/3}} + \frac{\sqrt{3} bd^2 \operatorname{ArcTan}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \frac{bde \tanh^{-1}(cx^3)}{c}$$

[Out] $-b*d*e*\operatorname{arctanh}(c^{(1/3)*x})/c^{(2/3)}+1/3*(e*x+d)^3*(a+b*\operatorname{arctanh}(c*x^3))/e+1/2*b*d^2*\ln(1-c^{(2/3)*x^2})/c^{(1/3)}+1/4*b*d*e*\ln(1-c^{(1/3)*x+c^{(2/3)*x^2}})/c^{(2/3)}-1/4*b*d*e*\ln(1+c^{(1/3)*x+c^{(2/3)*x^2}})/c^{(2/3)}+1/6*b*(c*d^3+e^3)*\ln(-c*x^3+1)/c/e-1/6*b*(c*d^3-e^3)*\ln(c*x^3+1)/c/e-1/4*b*d^2*\ln(1+c^{(2/3)*x^2}+c^{(4/3)*x^4})/c^{(1/3)}+1/2*b*d*e*\operatorname{arctan}(-1/3*3^{(1/2)}+2/3*c^{(1/3)*x*3^{(1/2)}})*3^{(1/2)}/c^{(2/3)}+1/2*b*d*e*\operatorname{arctan}(1/3*3^{(1/2)}+2/3*c^{(1/3)*x*3^{(1/2)}})*3^{(1/2)}/c^{(2/3)}+1/2*b*d^2*\operatorname{arctan}(1/3*(1+2*c^{(2/3)*x^2})*3^{(1/2)})*3^{(1/2)}/c^{(1/3)}$

Rubi [A]

time = 0.53, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {6071, 1845, 281, 298, 31, 648, 631, 210, 642, 302, 632, 212, 1483, 647}

$$\frac{(d+ex)^2(a+b\tanh^{-1}(cx^3))}{3e} + \frac{\sqrt{3}bd^2\operatorname{ArcTan}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{2c^{2/3}} - \frac{\sqrt{3}bde\operatorname{ArcTan}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{2c^{2/3}} + \frac{\sqrt{3}bde\operatorname{ArcTan}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{2c^{2/3}} + \frac{bd^2\log(1-c^{2/3}x^2)}{2\sqrt[3]{c}} - \frac{bd^2\log(c^{2/3}x^2+c^{4/3}x^4+1)}{4\sqrt[3]{c}} + \frac{bde\log(c^{2/3}x^2-\sqrt[3]{c}x+1)}{4c^{2/3}} - \frac{bde\log(c^{2/3}x^2+\sqrt[3]{c}x+1)}{4c^{2/3}} - \frac{bde\tanh^{-1}(\sqrt[3]{c}x)}{c^{2/3}} + \frac{b(cd^3+e^3)\log(1-cx^3)}{6c} - \frac{b(cd^3-e^3)\log(cx^3+1)}{6c}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(d + e*x)^2*(a + b*\operatorname{ArcTanh}[c*x^3]), x]$

[Out] $-1/2*(\operatorname{Sqrt}[3]*b*d*e*\operatorname{ArcTan}[1/\operatorname{Sqrt}[3] - (2*c^{(1/3)*x})/\operatorname{Sqrt}[3]])/c^{(2/3)} + (\operatorname{Sqrt}[3]*b*d*e*\operatorname{ArcTan}[1/\operatorname{Sqrt}[3] + (2*c^{(1/3)*x})/\operatorname{Sqrt}[3]])/(2*c^{(2/3)}) + (\operatorname{Sqrt}[3]*b*d^2*\operatorname{ArcTan}[(1 + 2*c^{(2/3)*x^2})/\operatorname{Sqrt}[3]])/(2*c^{(1/3)}) - (b*d*e*\operatorname{ArcTanh}[c^{(1/3)*x}])/c^{(2/3)} + ((d + e*x)^3*(a + b*\operatorname{ArcTanh}[c*x^3]))/(3*e) + (b*d^2*\operatorname{Log}[1 - c^{(2/3)*x^2}])/(2*c^{(1/3)}) + (b*d*e*\operatorname{Log}[1 - c^{(1/3)*x} + c^{(2/3)*x^2}])/(4*c^{(2/3)}) - (b*d*e*\operatorname{Log}[1 + c^{(1/3)*x} + c^{(2/3)*x^2}])/(4*c^{(2/3)}) + (b*(c*d^3 + e^3)*\operatorname{Log}[1 - c*x^3])/(6*c*e) - (b*(c*d^3 - e^3)*\operatorname{Log}[1 + c*x^3])/(6*c*e) - (b*d^2*\operatorname{Log}[1 + c^{(2/3)*x^2} + c^{(4/3)*x^4}])/(4*c^{(1/3)})$

Rule 31

$\operatorname{Int}[(a + (b*x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /;$ $\operatorname{FreeQ}\{a, b\}, x]$

Rule 210

$\operatorname{Int}[(a + (b*x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /;$ $\operatorname{FreeQ}\{a, b\}, x]$ && $\operatorname{PosQ}[a/b]$ & $\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
+ 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
n - 1] && NegQ[a/b]
```

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x])/b], x] /; FreeQ[{a, b, c, d,
```

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 647

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[(-a)*c]

Rule 648

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1483

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rule 1845

Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 6071

Int[((a_) + ArcTanh[(c_)*(x_)^(n_)]*(b_))*((d_) + (e_)*(x_)^(m_)), x_Symbol] := Simp[(d + e*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(e*(m + 1))), x] - Dist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int (d + ex)^2 (a + b \tanh^{-1}(cx^3)) dx &= \int (a(d + ex)^2 + b(d + ex)^2 \tanh^{-1}(cx^3)) dx \\
&= \frac{a(d + ex)^3}{3e} + b \int (d + ex)^2 \tanh^{-1}(cx^3) dx \\
&= \frac{a(d + ex)^3}{3e} + b \int (d^2 \tanh^{-1}(cx^3) + 2dex \tanh^{-1}(cx^3) + e^2 x^2 \tanh^{-1}(cx^3)) dx \\
&= \frac{a(d + ex)^3}{3e} + (bd^2) \int \tanh^{-1}(cx^3) dx + (2bde) \int x \tanh^{-1}(cx^3) dx \\
&= \frac{a(d + ex)^3}{3e} + bd^2 x \tanh^{-1}(cx^3) + bde x^2 \tanh^{-1}(cx^3) + \frac{1}{3} be^2 x^3 \tanh^{-1}(cx^3) \\
&= \frac{a(d + ex)^3}{3e} + bd^2 x \tanh^{-1}(cx^3) + bde x^2 \tanh^{-1}(cx^3) + \frac{1}{3} be^2 x^3 \tanh^{-1}(cx^3) \\
&= \frac{a(d + ex)^3}{3e} - \frac{bde \tanh^{-1}(\sqrt[3]{c} x)}{c^{2/3}} + bd^2 x \tanh^{-1}(cx^3) + bde x^2 \tanh^{-1}(cx^3) \\
&= \frac{a(d + ex)^3}{3e} - \frac{bde \tanh^{-1}(\sqrt[3]{c} x)}{c^{2/3}} + bd^2 x \tanh^{-1}(cx^3) + bde x^2 \tanh^{-1}(cx^3) \\
&= \frac{a(d + ex)^3}{3e} - \frac{\sqrt{3} bde \tan^{-1}\left(\frac{1-2\sqrt[3]{c} x}{\sqrt{3}}\right)}{2c^{2/3}} + \frac{\sqrt{3} bde \tan^{-1}\left(\frac{1+2\sqrt[3]{c} x}{\sqrt{3}}\right)}{2c^{2/3}} \\
&= \frac{a(d + ex)^3}{3e} - \frac{\sqrt{3} bde \tan^{-1}\left(\frac{1-2\sqrt[3]{c} x}{\sqrt{3}}\right)}{2c^{2/3}} + \frac{\sqrt{3} bde \tan^{-1}\left(\frac{1+2\sqrt[3]{c} x}{\sqrt{3}}\right)}{2c^{2/3}}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 299, normalized size = 0.89

$$\frac{12ac^2x + 12acdx^2 + 4ac^2x^3 + 6\sqrt{3}b\sqrt{c}d(\sqrt{c}d + e) \operatorname{ArcTan}\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right) - 6\sqrt{3}b\sqrt{c}d(\sqrt{c}d - e) \operatorname{ArcTan}\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right) + 4bcx(3d^2 + 3dex + e^2x^2) \tanh^{-1}(cx^3) + 6b\sqrt{c}d(\sqrt{c}d + e) \log(1 - \sqrt{c}x) + 6b\sqrt{c}d(\sqrt{c}d - e) \log(1 + \sqrt{c}x) - 3b\sqrt{c}d(\sqrt{c}d - e) \log(1 - \sqrt{c}x + e^{2/3}x^2) - 3b\sqrt{c}d(\sqrt{c}d + e) \log(1 + \sqrt{c}x + e^{2/3}x^2) + 2be^2 \log(1 - e^2x^2)}{12c}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2*(a + b*ArcTanh[c*x^3]), x]

[Out] (12*a*c*d^2*x + 12*a*c*d*e*x^2 + 4*a*c*e^2*x^3 + 6*Sqrt[3]*b*c^(1/3)*d*(c^(1/3)*d + e)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]] - 6*Sqrt[3]*b*c^(1/3)*d*(c^(1/3)*d - e)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]] + 4*b*c*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcTanh[c*x^3] + 6*b*c^(1/3)*d*(c^(1/3)*d + e)*Log[1 - c^(1/3)*x] + 6*b*c^(1/3)*d*(c^(1/3)*d - e)*Log[1 + c^(1/3)*x] - 3*b*c^(1/3)*d*(c^(1/3)

$(d - e) \cdot \text{Log}[1 - c^{1/3}x + c^{2/3}x^2] - 3b \cdot c^{1/3} \cdot d \cdot (c^{1/3}d + e) \cdot \text{Log}[1 + c^{1/3}x + c^{2/3}x^2] + 2b \cdot e^2 \cdot \text{Log}[1 - c^2x^6] / (12c)$

Maple [A]

time = 0.25, size = 482, normalized size = 1.43

method	result
default	$\frac{(ex+d)^3 a}{3e} + \frac{be^2 \operatorname{arctanh}(cx^3)x^3}{3} + be \operatorname{arctanh}(cx^3)x^2d + b \operatorname{arctanh}(cx^3)xd^2 + \frac{b \operatorname{arctanh}(cx^3)d^3}{3e} + \frac{bd^2 \ln(x)}{2c}$
risch	$\frac{(ex+d)^3 b \ln(cx^3+1)}{6e} - \frac{bde \ln(-cx^3+1)x^2}{2} + \frac{bed\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}-1}\right)}{\frac{1}{c}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}} + \frac{bed\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}+1}\right)}{\frac{1}{c}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{3} \cdot (e \cdot x + d)^3 \cdot a / e + \frac{1}{3} \cdot b \cdot e^2 \cdot \operatorname{arctanh}(c \cdot x^3) \cdot x^3 + b \cdot e \cdot \operatorname{arctanh}(c \cdot x^3) \cdot x^2 \cdot d + b \cdot a \cdot \operatorname{rctanh}(c \cdot x^3) \cdot x \cdot d^2 + \frac{1}{3} \cdot b \cdot e \cdot \operatorname{arctanh}(c \cdot x^3) \cdot d^3 + \frac{1}{2} \cdot b \cdot c \cdot d^2 / (1/c)^{(2/3)} \cdot \ln(x + (1/c)^{(1/3)}) - \frac{1}{4} \cdot b \cdot c \cdot d^2 / (1/c)^{(2/3)} \cdot \ln(x^2 - (1/c)^{(1/3)} \cdot x + (1/c)^{(2/3)}) + \frac{1}{2} \cdot b \cdot c \cdot d^2 / (1/c)^{(2/3)} \cdot 3^{(1/2)} \cdot \arctan(1/3 \cdot 3^{(1/2)} \cdot (2 / (1/c)^{(1/3)} \cdot x - 1)) - \frac{1}{2} \cdot b \cdot e \cdot c \cdot d / (1/c)^{(1/3)} \cdot \ln(x + (1/c)^{(1/3)}) + \frac{1}{4} \cdot b \cdot e \cdot c \cdot d / (1/c)^{(1/3)} \cdot \ln(x^2 - (1/c)^{(1/3)} \cdot x + (1/c)^{(2/3)}) + \frac{1}{2} \cdot b \cdot e \cdot c \cdot d \cdot 3^{(1/2)} / (1/c)^{(1/3)} \cdot \arctan(1/3 \cdot 3^{(1/2)} \cdot (2 / (1/c)^{(1/3)} \cdot x - 1)) - \frac{1}{6} \cdot b \cdot e \cdot \ln(c \cdot x^3 + 1) \cdot d^3 + \frac{1}{6} \cdot b \cdot e^2 / c \cdot \ln(c \cdot x^3 + 1) + \frac{1}{2} \cdot b \cdot c \cdot d^2 / (1/c)^{(2/3)} \cdot \ln(x - (1/c)^{(1/3)}) - \frac{1}{4} \cdot b \cdot c \cdot d^2 / (1/c)^{(2/3)} \cdot \ln(x^2 + (1/c)^{(1/3)} \cdot x + (1/c)^{(2/3)}) - \frac{1}{2} \cdot b \cdot c \cdot d^2 / (1/c)^{(2/3)} \cdot 3^{(1/2)} \cdot \arctan(1/3 \cdot 3^{(1/2)} \cdot (2 / (1/c)^{(1/3)} \cdot x + 1)) + \frac{1}{2} \cdot b \cdot e \cdot c \cdot d / (1/c)^{(1/3)} \cdot \ln(x - (1/c)^{(1/3)}) - \frac{1}{4} \cdot b \cdot e \cdot c \cdot d / (1/c)^{(1/3)} \cdot \ln(x^2 + (1/c)^{(1/3)} \cdot x + (1/c)^{(2/3)}) + \frac{1}{2} \cdot b \cdot e \cdot c \cdot d \cdot 3^{(1/2)} / (1/c)^{(1/3)} \cdot \arctan(1/3 \cdot 3^{(1/2)} \cdot (2 / (1/c)^{(1/3)} \cdot x + 1)) + \frac{1}{6} \cdot b \cdot e \cdot \ln(c \cdot x^3 - 1) \cdot d^3 + \frac{1}{6} \cdot b \cdot e^2 / c \cdot \ln(c \cdot x^3 - 1)$

Maxima [A]

time = 0.47, size = 295, normalized size = 0.88

$$\frac{1}{3} a x^3 + a d x^2 + \frac{1}{3} \left(\frac{2 \sqrt{3} \operatorname{arctan}\left(\frac{\sqrt{3}(1+b \cdot x^3)}{1+b}\right)}{c^3} - \frac{\log(c^2 x^3 + c^2 x + 1)}{c^3} + \frac{2 \log\left(\frac{d \cdot x + 1}{c^2}\right)}{c^3} \right) + 4 x \operatorname{arctanh}(c x^3) b e^2 + a d^2 x + \frac{1}{3} \left(\frac{2 \sqrt{3} \operatorname{arctan}\left(\frac{\sqrt{3}(1+b \cdot x^3)}{1+b}\right)}{c^3} + \frac{2 \sqrt{3} \operatorname{arctan}\left(\frac{\sqrt{3}(1+b \cdot x^3)}{1+b}\right)}{c^3} - \frac{\log(c^2 x^3 + c^2 x + 1)}{c^3} + \frac{\log(c^2 x^3 - c^2 x + 1)}{c^3} + \frac{2 \log\left(\frac{d \cdot x + 1}{c^2}\right)}{c^3} + \frac{2 \log\left(\frac{d \cdot x + 1}{c^2}\right)}{c^3} \right) \left(b d e + \frac{(2 c^2 \operatorname{arctanh}(c x^3) + \log(-c^2 x^3 + 1)) b c^2}{6 c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*(a+b*arctanh(c*x^3)),x, algorithm="maxima")`

[Out] $\frac{1}{3} a x^3 e^2 + a d x^2 e + \frac{1}{4} \cdot (c \cdot (2 \cdot \sqrt{3}) \cdot \arctan(1/3 \cdot \sqrt{3}) \cdot (2 \cdot c^{(4/3)} \cdot x^2 + c^{(2/3)}) / c^{(2/3)}) / c^{(4/3)} - \log(c^{(4/3)} \cdot x^4 + c^{(2/3)} \cdot x^2 + 1) / c^{(4/3)}$

$$3) + 2*\log((c^{(2/3)}*x^2 - 1)/c^{(2/3)})/c^{(4/3)} + 4*x*\operatorname{arctanh}(c*x^3)*b*d^2 + a*d^2*x + 1/4*(4*x^2*\operatorname{arctanh}(c*x^3) + c*(2*\sqrt{3})*\operatorname{arctan}(1/3*\sqrt{3})*(2*c^{(2/3)}*x + c^{(1/3)})/c^{(1/3)})/c^{(5/3)} + 2*\sqrt{3}*\operatorname{arctan}(1/3*\sqrt{3})*(2*c^{(2/3)}*x - c^{(1/3)})/c^{(1/3)})/c^{(5/3)} - \log(c^{(2/3)}*x^2 + c^{(1/3)}*x + 1)/c^{(5/3)} + \log(c^{(2/3)}*x^2 - c^{(1/3)}*x + 1)/c^{(5/3)} - 2*\log((c^{(1/3)}*x + 1)/c^{(1/3)})/c^{(5/3)} + 2*\log((c^{(1/3)}*x - 1)/c^{(1/3)})/c^{(5/3)})*b*d*e + 1/6*(2*c*x^3*\operatorname{arctanh}(c*x^3) + \log(-c^2*x^6 + 1))*b*e^2/c$$

Fricas [C] Result contains complex when optimal does not.

time = 6.56, size = 79566, normalized size = 236.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*(a+b*arctanh(c*x^3)),x, algorithm="fricas")`

[Out]
$$\frac{1}{24}*(8*a*c*x^3*\cosh(1)^2 + 8*a*c*x^3*\sinh(1)^2 + 24*a*c*d*x^2*\cosh(1) + 24*a*c*d^2*x + 2*(2*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*((9*b^2*c*d^3*\cosh(1)^2 + 9*b^2*c*d^3*\sinh(1)^2 + b^2*\sinh(1) - (18*b^2*c*d^3*\sinh(1) - b^2)*\cosh(1))/(c^2*\cosh(1)^3 - 3*c^2*\cosh(1)^2*\sinh(1) + 3*c^2*\cosh(1)*\sinh(1)^2 - c^2*\sinh(1)^3) - b^2/(c*\cosh(1)^2 - 2*c*\cosh(1)*\sinh(1) + c*\sinh(1)^2)^2)/(((27*c^2*d^6 + 1)*b^3*\cosh(1)^3 - 3*(27*c^2*d^6 - 1)*b^3*\cosh(1)^2*\sinh(1) + 3*(27*c^2*d^6 + 1)*b^3*\cosh(1)*\sinh(1)^2 - (27*c^2*d^6 - 1)*b^3*\sinh(1)^3)/(c^3*\cosh(1)^3 - 3*c^3*\cosh(1)^2*\sinh(1) + 3*c^3*\cosh(1)*\sinh(1)^2 - c^3*\sinh(1)^3) - 3*(9*b^2*c*d^3*\cosh(1)^2 + 9*b^2*c*d^3*\sinh(1)^2 + b^2*\sinh(1) - (18*b^2*c*d^3*\sinh(1) - b^2)*\cosh(1))*b/((c^2*\cosh(1)^3 - 3*c^2*\cosh(1)^2*\sinh(1) + 3*c^2*\cosh(1)*\sinh(1)^2 - c^2*\sinh(1)^3)*(c*\cosh(1)^2 - 2*c*\cosh(1)*\sinh(1) + c*\sinh(1)^2)) + 2*b^3/(c*\cosh(1)^2 - 2*c*\cosh(1)*\sinh(1) + c*\sinh(1)^2)^3 - \sqrt{729*(\cosh(1)^{10} - 10*\cosh(1)^9*\sinh(1) + 45*\cosh(1)^8*\sinh(1)^2 - 120*\cosh(1)^7*\sinh(1)^3 + 210*\cosh(1)^6*\sinh(1)^4 - 252*\cosh(1)^5*\sinh(1)^5 + 210*\cosh(1)^4*\sinh(1)^6 - 120*\cosh(1)^3*\sinh(1)^7 + 45*\cosh(1)^2*\sinh(1)^8 - 10*\cosh(1)*\sinh(1)^9 + \sinh(1)^{10})*c^4*d^{12} + 1458*(\cosh(1)^7 - 7*\cosh(1)^6*\sinh(1) + 21*\cosh(1)^5*\sinh(1)^2 - 35*\cosh(1)^4*\sinh(1)^3 + 35*\cosh(1)^3*\sinh(1)^4 - 21*\cosh(1)^2*\sinh(1)^5 + 7*\cosh(1)*\sinh(1)^6 - \sinh(1)^7)*c^3*d^9 + \cosh(1)^{10} - 8*\cosh(1)^3*\sinh(1)^7 - 3*\cosh(1)^2*\sinh(1)^8 + 2*\cosh(1)*\sinh(1)^9 + \sinh(1)^{10} + 27*(2*\cosh(1)^{10} - 16*\cosh(1)^3*\sinh(1)^7 - 6*\cosh(1)^2*\sinh(1)^8 + 8*\cosh(1)*\sinh(1)^9 - 2*\sinh(1)^{10} + 2*(14*\cosh(1)^4 - 15)*\sinh(1)^6 + 30*\cosh(1)^6 + 120*\cosh(1)*\sinh(1)^5 - (28*\cosh(1)^6 + 150*\cosh(1)^2 + 5)*\sinh(1)^4 - 5*\cosh(1)^4 + 4*(4*\cosh(1)^7 + 5*\cosh(1))*\sinh(1)^3 + 6*(\cosh(1)^8 + 25*\cosh(1)^4 - 5*\cosh(1)^2)*\sinh(1)^2 - 4*(2*\cosh(1)^9 + 30*\cosh(1)^5 - 5*\cosh(1)^3)*\sinh(1))*c^2*d^6 + 2*(\cosh(1)^4 - 3)*\sinh(1)^6 - 6*\cosh(1)^6 + 12*(\cosh(1)^5 - \cosh(1))*\sinh(1)^5 - 54*(\cosh(1)^7 - \cosh(1)*\sinh(1)^6 + \sinh(1)^7 - (3*\cosh(1)^2 - 2)*\sinh(1)^5 - 2*\cosh(1)^5 + (3*\cosh(1)^3 - 2*\cosh(1))*\sinh(1)^4 + (3*\cosh(1)^4 - 4*\cosh(1)^2 + 1)*\sinh(1)^3 + \cosh(1)^3 - (3*\cosh(1)^5 - 4*\cosh(1)^3 + \cosh(1))*\sinh(1)^2 -$$

```

(cosh(1)^6 - 2*cosh(1)^4 + cosh(1)^2)*sinh(1))*c*d^3 + 2*(cosh(1)^6 + 3*cos
h(1)^2 - 4)*sinh(1)^4 + 8*cosh(1)^4 - 8*(cosh(1)^7 - 3*cosh(1)^3 + 2*cosh(1
))*sinh(1)^3 - 3*(cosh(1)^8 - 2*cosh(1)^4 + 1)*sinh(1)^2 - 3*cosh(1)^2 + 2*
(cosh(1)^9 - 6*cosh(1)^5 + 8*cosh(1)^3 - 3*cosh(1))*sinh(1))*b^3/(c^3*(cosh
(1) - sinh(1))^5))^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(((27*c^2*d^6 + 1)*b
^3*cosh(1)^3 - 3*(27*c^2*d^6 - 1)*b^3*cosh(1)^2*sinh(1) + 3*(27*c^2*d^6 + 1
)*b^3*cosh(1)*sinh(1)^2 - (27*c^2*d^6 - 1)*b^3*sinh(1)^3)/(c^3*cosh(1)^3 -
3*c^3*cosh(1)^2*sinh(1) + 3*c^3*cosh(1)*sinh(1)^2 - c^3*sinh(1)^3) - 3*(9*b
^2*c*d^3*cosh(1)^2 + 9*b^2*c*d^3*sinh(1)^2 + b^2*sinh(1) - (18*b^2*c*d^3*si
nh(1) - b^2)*cosh(1))*b/((c^2*cosh(1)^3 - 3*c^2*cosh(1)^2*sinh(1) + 3*c^2*c
osh(1)*sinh(1)^2 - c^2*sinh(1)^3)*(c*cosh(1)^2 - 2*c*cosh(1)*sinh(1) + c*si
nh(1)^2)) + 2*b^3/(c*cosh(1)^2 - 2*c*cosh(1)*sinh(1) + c*sinh(1)^2)^3 - sqr
t(729*(cosh(1)^10 - 10*cosh(1)^9*sinh(1) + 45*cosh(1)^8*sinh(1)^2 - 120*cos
h(1)^7*sinh(1)^3 + 210*cosh(1)^6*sinh(1)^4 - 252*cosh(1)^5*sinh(1)^5 + 210*
cosh(1)^4*sinh(1)^6 - 120*cosh(1)^3*sinh(1)^7 + 45*cosh(1)^2*sinh(1)^8 - 10
*cosh(1)*sinh(1)^9 + sinh(1)^10)*c^4*d^12 + 1458*(cosh(1)^7 - 7*cosh(1)^6*s
inh(1) + 21*cosh(1)^5*sinh(1)^2 - 35*cosh(1)^4*sinh(1)^3 + 35*cosh(1)^3*si
nh(1)^4 - 21*cosh(1)^2*sinh(1)^5 + 7*cosh(1)*sinh(1)^6 - sinh(1)^7)*c^3*d^9
+ cosh(1)^10 - 8*cosh(1)^3*sinh(1)^7 - 3*cosh(1)^2*sinh(1)^8 + 2*cosh(1)*si
nh(1)^9 + sinh(1)^10 + 27*(2*cosh(1)^10 - 16*cosh(1)^3*sinh(1)^7 - 6*cosh(1
)^2*sinh(1)^8 + 8*cosh(1)*sinh(1)^9 - 2*sinh(1)^10 + 2*(14*cosh(1)^4 - 15)*
sinh(1)^6 + 30*cosh(1)^6 + 120*cosh(1)*sinh(1)^5 - (28*cosh(1)^6 + 150*cosh
(1)^2 + 5)*sinh(1)^4 - 5*cosh(1)^4 + 4*(4*cosh(1)^7 + 5*cosh(1))*sinh(1)^3
+ 6*(cosh(1)^8 + 25*cosh(1)^4 - 5*cosh(1)^2)*sinh(1)^2 - 4*(2*cosh(1)^9 + 3
0*cosh(1)^5 - 5*cosh(1)^3)*sinh(1))*c^2*d^6 + 2*(cosh(1)^4 - 3)*sinh(1)^6 -
6*cosh(1)^6 + 12*(cosh(1)^5 - cosh(1))*sinh(1)^5 - 54*(cosh(1)^7 - cosh(1)
)*sinh(1)^6 + sinh(1)^7 - (3*cosh(1)^2 - 2)*sinh(1)^5 - 2*cosh(1)^5 + (3*cos
h(1)^3 - 2*cosh(1))*sinh(1)^4 + (3*cosh(1)^4 - 4*cosh(1)^2 + 1)*sinh(1)^3 +
cosh(1)^3 - (3*cosh(1)^5 - 4*cosh(1)^3 + cosh(1))*sinh(1)^2 - (cosh(1)^6 -
2*cosh(1)^4 + cosh(1)^2)*sinh(1))*c*d^3 + 2*(cosh(1)^6 + 3*cosh(1)^2 - 4)*
sinh(1)^4 + 8*cosh(1)^4 - 8*(cosh(1)^7 - 3*cosh(1)^3 + 2*cosh(1))*sinh(1)^3
- 3*(cosh(1)^8 - 2*cosh(1)^4 + 1)*sinh(1)^2 - 3*cosh(1)^2 + 2*(cosh(1)^9 -
6*cosh(1)^5 + 8*cosh(1)^3 - 3*cosh(1))*sinh(1))*b^3/(c^3*(cosh(1) - sinh(1
))^5))^(1/3) + 2*b/(c*cosh(1)^2 - 2*c*cosh(1)*sinh(1) + c*sinh(1)^2))*c*log
(9*b^2*c^2*d^5*x + 9*b^2*c*d^2*x*cosh(1)^3 + 15*b^2*c*d^3*cosh(1)^2 + b^2*c
osh(1)^5 + 5*b^2*cosh(1)*sinh(1)^4 + b^2*sinh(1)^5 + (9*b^2*c*d^2*x + 10*b^
2*cosh(1)^2)*sinh(1)^3 + 1/4*(c^2*cosh(1) + c^2*sinh(1))*(2*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*((9*b^2*c*d^3*cosh(1)^2 + 9*b^2*c...

```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*(a+b*atanh(c*x**3)),x)

[Out] Timed out

Giac [A]

time = 11.89, size = 346, normalized size = 1.03

$$\frac{1}{3} a^2 e^{2x} + a d e^{2x} + \frac{1}{2} (3 a^2 d^2 + 3 b^2 d^2) \log\left(\frac{e^{2x} + 1}{e^{2x} - 1}\right) - \frac{\sqrt{3} (b^2 d^2 - b d^2 c^2) \arctan\left(\frac{1}{\sqrt{3}} \left(2x + \frac{1}{c^3}\right) c^3\right)}{2 c^2} - \frac{\sqrt{3} (b^2 d^2 + b d^2 c^2) \arctan\left(\frac{1}{\sqrt{3}} \left(2x - \frac{1}{c^3}\right) c^3\right)}{2 c^2} - \frac{(3 b^2 d^2 + 3 b d^2 c^2 - 2 b^2 d^2) \log\left(e^x + \frac{1}{c^3}\right)}{12 c^2} - \frac{(3 b^2 d^2 - 3 b d^2 c^2 - 2 b^2 d^2) \log\left(e^x - \frac{1}{c^3}\right)}{12 c^2} - \frac{(3 b^2 d^2 - 3 b d^2 c^2 + b^2 d^2) \log\left(\left|x + \frac{1}{c^3}\right|\right)}{6 c^2} - \frac{(3 b^2 d^2 + 3 b d^2 c^2 + b^2 d^2) \log\left(\left|x - \frac{1}{c^3}\right|\right)}{6 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x^3)),x, algorithm="giac")

[Out] $\frac{1}{3} a e^{2x^3} + a d e^{x^3} + a d^2 x + \frac{1}{6} (b e^{2x^3} + 3 b^2 d e^{x^3} + 3 b^2 d^2 x^2) \log\left(-\frac{c x^3 + 1}{c x^3 - 1}\right) - \frac{1}{2} \sqrt{3} (b c d^2 \operatorname{abs}(c)^{2/3} - b c^2 d e \operatorname{abs}(c)^{1/3}) \operatorname{arctan}\left(\frac{1}{3} \sqrt{3} (2x + 1/\operatorname{abs}(c)^{1/3}) \operatorname{abs}(c)^{1/3}\right) / c^2 + \frac{1}{2} \sqrt{3} (b c d^2 \operatorname{abs}(c)^{2/3} + b c^2 d e \operatorname{abs}(c)^{1/3}) \operatorname{arctan}\left(\frac{1}{3} \sqrt{3} (2x - 1/\operatorname{abs}(c)^{1/3}) \operatorname{abs}(c)^{1/3}\right) / c^2 - \frac{1}{12} (3 b^2 c d^2 \operatorname{abs}(c)^{2/3} + 3 b^2 c d e \operatorname{abs}(c)^{1/3} - 2 b^2 c e^2) \log(x^2 + x/\operatorname{abs}(c)^{1/3} + 1/\operatorname{abs}(c)^{2/3}) / c^2 - \frac{1}{12} (3 b^2 c d^2 \operatorname{abs}(c)^{2/3} - 3 b^2 c d e \operatorname{abs}(c)^{1/3} - 2 b^2 c e^2) \log(x^2 - x/\operatorname{abs}(c)^{1/3} + 1/\operatorname{abs}(c)^{2/3}) / c^2 + \frac{1}{6} (3 b^2 c d^2 a \operatorname{abs}(c)^{2/3} - 3 b^2 c d e \operatorname{abs}(c)^{1/3} + b^2 c e^2) \log(\operatorname{abs}(x + 1/\operatorname{abs}(c)^{1/3})) / c^2 + \frac{1}{6} (3 b^2 c d^2 a \operatorname{abs}(c)^{2/3} + 3 b^2 c d e \operatorname{abs}(c)^{1/3} + b^2 c e^2) \log(\operatorname{abs}(x - 1/\operatorname{abs}(c)^{1/3})) / c^2$

Mupad [B]

time = 1.82, size = 1081, normalized size = 3.22

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^3))*(d + e*x)^2,x)

[Out] $\operatorname{symsum}\left(\log(x(6b^5c^7d^2e^8 + 162b^5c^9d^8e^2)) + \operatorname{root}(216c^3z^3 - 108b^2c^2e^2z^2 - 162b^2c^2d^3e^2z + 18b^2c^2e^4z - 27b^3c^2d^6 - b^3e^6, z, k) \cdot (x(486b^4c^10d^8 - 90b^4c^8d^2e^6)) + \operatorname{root}(216c^3z^3 - 108b^2c^2e^2z^2 - 162b^2c^2d^3e^2z + 18b^2c^2e^4z - 27b^3c^2d^6 - b^3e^6, z, k) \cdot (\operatorname{root}(216c^3z^3 - 108b^2c^2e^2z^2 - 162b^2c^2d^3e^2z + 18b^2c^2e^4z - 27b^3c^2d^6 - b^3e^6, z, k) \cdot (3888b^2c^10d^3e - 3888 \operatorname{root}(216c^3z^3 - 108b^2c^2e^2z^2 - 162b^2c^2d^3e^2z + 18b^2c^2e^4z - 27b^3c^2d^6 - b^3e^6, z, k) \cdot b c^{11} d^2 x + 648b^2c^{10} d^2 e^2 x) - 972b^3c^9d^3e^3 + 324b^3c^9d^2e^4 x)\right) + 243b^5c^9d^9 e + 9b^5c^7d^3e^7) \operatorname{root}(216c^3z^3 - 108b^2c^2e^2z^2 - 162b^2c^2d^3e^2z + 18b^2c^2e^4z - 27b^3c^2d^6 - b^3e^6, z, k), k, 1, 3) + \operatorname{symsum}\left(\log(x(6b^5c^7d^2e^8 + 162b^5c^9d^8e^2)) + \operatorname{root}(216c^3z^3 - 108b^2c^2e^2z^2 + 162b^2c^2d^3e^2z + 18b^2c^2e^4z - 27b^3c^2d^6 - b^3e^6, z, k) \cdot (x(486b^4c^10d^8 - 90b^4c^8d^2e^6)) + \operatorname{root}(216c^3z^3 - 108b^2c^2e^2z^2 - 162b^2c^2d^3e^2z + 18b^2c^2e^4z - 27b^3c^2d^6 - b^3e^6, z, k) \cdot (\operatorname{root}(216c^3z^3 - 108b^2c^2e^2z^2 - 162b^2c^2d^3e^2z + 18b^2c^2e^4z - 27b^3c^2d^6 - b^3e^6, z, k) \cdot (3888b^2c^10d^3e - 3888 \operatorname{root}(216c^3z^3 - 108b^2c^2e^2z^2 - 162b^2c^2d^3e^2z + 18b^2c^2e^4z - 27b^3c^2d^6 - b^3e^6, z, k) \cdot b c^{11} d^2 x + 648b^2c^{10} d^2 e^2 x) - 972b^3c^9d^3e^3 + 324b^3c^9d^2e^4 x)\right) + 243b^5c^9d^9 e + 9b^5c^7d^3e^7) \operatorname{root}(216c^3z^3 - 108b^2c^2e^2z^2 - 162b^2c^2d^3e^2z + 18b^2c^2e^4z - 27b^3c^2d^6 - b^3e^6, z, k), k, 1, 3)$

$$\begin{aligned}
& - 108*b*c^2*e^2*z^2 + 162*b^2*c^2*d^3*e*z + 18*b^2*c*e^4*z - 27*b^3*c^2*d^6 \\
& - b^3*e^6, z, k)*(root(216*c^3*z^3 - 108*b*c^2*e^2*z^2 + 162*b^2*c^2*d^3*e \\
& *z + 18*b^2*c*e^4*z - 27*b^3*c^2*d^6 - b^3*e^6, z, k)*(3888*b^2*c^10*d^3*e \\
& - 3888*root(216*c^3*z^3 - 108*b*c^2*e^2*z^2 + 162*b^2*c^2*d^3*e*z + 18*b^2* \\
& c*e^4*z - 27*b^3*c^2*d^6 - b^3*e^6, z, k)*b*c^11*d^2*x + 648*b^2*c^10*d^2*e \\
& ^2*x) - 972*b^3*c^9*d^3*e^3 + 324*b^3*c^9*d^2*e^4*x)) + 243*b^5*c^9*d^9*e + \\
& 9*b^5*c^7*d^3*e^7)*root(216*c^3*z^3 - 108*b*c^2*e^2*z^2 + 162*b^2*c^2*d^3* \\
& e*z + 18*b^2*c*e^4*z - 27*b^3*c^2*d^6 - b^3*e^6, z, k), k, 1, 3) + \log(c*x^ \\
& 3 + 1)*((b*e^2*x^3)/6 + (b*d^2*x)/2 + (b*d*e*x^2)/2) - \log(1 - c*x^3)*((b*e \\
& ^2*x^3)/6 + (b*d^2*x)/2 + (b*d*e*x^2)/2) + (a*e^2*x^3)/3 + a*d^2*x + a*d*e* \\
& x^2
\end{aligned}$$

3.33 $\int (d + ex) (a + b \tanh^{-1}(cx^3)) dx$

Optimal. Leaf size=285

$$-\frac{\sqrt{3} b e \operatorname{ArcTan}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{c} x}{\sqrt{3}}\right)}{4c^{2/3}} + \frac{\sqrt{3} b e \operatorname{ArcTan}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{c} x}{\sqrt{3}}\right)}{4c^{2/3}} + \frac{\sqrt{3} b d \operatorname{ArcTan}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \frac{b e \tanh^{-1}}{2c^{2/3}}$$

[Out] $-1/2*b*e*\operatorname{arctanh}(c^{(1/3)}*x)/c^{(2/3)}-1/2*b*d^2*\operatorname{arctanh}(c*x^3)/e+1/2*(e*x+d)^2*(a+b*\operatorname{arctanh}(c*x^3))/e+1/2*b*d*\ln(1-c^{(2/3)}*x^2)/c^{(1/3)}+1/8*b*e*\ln(1-c^{(1/3)}*x+c^{(2/3)}*x^2)/c^{(2/3)}-1/8*b*e*\ln(1+c^{(1/3)}*x+c^{(2/3)}*x^2)/c^{(2/3)}-1/4*b*d*\ln(1+c^{(2/3)}*x^2+c^{(4/3)}*x^4)/c^{(1/3)}+1/4*b*e*\operatorname{arctan}(-1/3*3^{(1/2)}+2/3*c^{(1/3)}*x*3^{(1/2)})*3^{(1/2)}/c^{(2/3)}+1/4*b*e*\operatorname{arctan}(1/3*3^{(1/2)}+2/3*c^{(1/3)}*x*3^{(1/2)})*3^{(1/2)}/c^{(2/3)}+1/2*b*d*\operatorname{arctan}(1/3*(1+2*c^{(2/3)}*x^2)*3^{(1/2)})*3^{(1/2)}/c^{(1/3)}$

Rubi [A]

time = 0.31, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6071, 1845, 281, 212, 298, 31, 648, 631, 210, 642, 302, 632}

$$\frac{(d+ex)^2(a+b\tanh^{-1}(cx^3))}{2e} + \frac{\sqrt{3}bd\operatorname{ArcTan}\left(\frac{2c^{2/3}x+1}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \frac{\sqrt{3}be\operatorname{ArcTan}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}} + \frac{\sqrt{3}be\operatorname{ArcTan}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{4c^{2/3}} + \frac{bd\log(1-c^{2/3}x^2)}{2\sqrt[3]{c}} - \frac{bd\log(c^{4/3}x^4+c^{2/3}x^2+1)}{4\sqrt[3]{c}} + \frac{be\log(c^{2/3}x^2-\sqrt[3]{c}x+1)}{8c^{2/3}} - \frac{be\log(c^{2/3}x^2+\sqrt[3]{c}x+1)}{8c^{2/3}} - \frac{be\tanh^{-1}(\sqrt[3]{c}x)}{2c^{2/3}} - \frac{bd^2\tanh^{-1}(cx^3)}{2e}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(d + e*x)*(a + b*\operatorname{ArcTanh}[c*x^3]), x]$

[Out] $-1/4*(\operatorname{Sqrt}[3]*b*e*\operatorname{ArcTan}[1/\operatorname{Sqrt}[3] - (2*c^{(1/3)}*x)/\operatorname{Sqrt}[3]])/c^{(2/3)} + (\operatorname{Sqrt}[3]*b*e*\operatorname{ArcTan}[1/\operatorname{Sqrt}[3] + (2*c^{(1/3)}*x)/\operatorname{Sqrt}[3]])/(4*c^{(2/3)}) + (\operatorname{Sqrt}[3]*b*d*\operatorname{ArcTan}[(1 + 2*c^{(2/3)}*x^2)/\operatorname{Sqrt}[3]])/(2*c^{(1/3)}) - (b*e*\operatorname{ArcTanh}[c^{(1/3)}*x])/(2*c^{(2/3)}) - (b*d^2*\operatorname{ArcTanh}[c*x^3])/(2*e) + ((d + e*x)^2*(a + b*\operatorname{ArcTanh}[c*x^3]))/(2*e) + (b*d*\operatorname{Log}[1 - c^{(2/3)}*x^2])/(2*c^{(1/3)}) + (b*e*\operatorname{Log}[1 - c^{(1/3)}*x + c^{(2/3)}*x^2])/(8*c^{(2/3)}) - (b*e*\operatorname{Log}[1 + c^{(1/3)}*x + c^{(2/3)}*x^2])/(8*c^{(2/3)}) - (b*d*\operatorname{Log}[1 + c^{(2/3)}*x^2 + c^{(4/3)}*x^4])/(4*c^{(1/3)})$

Rule 31

$\operatorname{Int}[(a_ + (b_)*(x_))^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

Rule 210

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 281

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 298

Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 302

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[a/b]

Rule 631

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 632

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x])/b], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(c^ii*(a + b*x^n))], {ii, 0, n/2 - 1}}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 6071

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.) + (e_.)*(x_)^(m_.)), x_Symbol] := Simp[(d + e*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(e*(m + 1))), x] - Dist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)(a+b \tanh^{-1}(cx^3)) dx &= \frac{(d+ex)^2(a+b \tanh^{-1}(cx^3))}{2e} - \frac{b \int \frac{3cx^2(d+ex)^2}{1-c^2x^6} dx}{2e} \\
&= \frac{(d+ex)^2(a+b \tanh^{-1}(cx^3))}{2e} - \frac{(3bc) \int \frac{x^2(d+ex)^2}{1-c^2x^6} dx}{2e} \\
&= \frac{(d+ex)^2(a+b \tanh^{-1}(cx^3))}{2e} - \frac{(3bc) \int \left(\frac{d^2x^2}{1-c^2x^6} + \frac{2dex^3}{1-c^2x^6} + \frac{e^2x^4}{1-c^2x^6} \right) dx}{2e} \\
&= \frac{(d+ex)^2(a+b \tanh^{-1}(cx^3))}{2e} - (3bcd) \int \frac{x^3}{1-c^2x^6} dx - \frac{(3bcd^2) \int \frac{x}{1-c^2x^6} dx}{2e} \\
&= \frac{(d+ex)^2(a+b \tanh^{-1}(cx^3))}{2e} - \frac{1}{2}(3bcd) \text{Subst} \left(\int \frac{x}{1-c^2x^3} dx, x, x^2 \right) \\
&= -\frac{be \tanh^{-1}(\sqrt[3]{c}x)}{2c^{2/3}} - \frac{bd^2 \tanh^{-1}(cx^3)}{2e} + \frac{(d+ex)^2(a+b \tanh^{-1}(cx^3))}{2e} \\
&= -\frac{be \tanh^{-1}(\sqrt[3]{c}x)}{2c^{2/3}} - \frac{bd^2 \tanh^{-1}(cx^3)}{2e} + \frac{(d+ex)^2(a+b \tanh^{-1}(cx^3))}{2e} \\
&= -\frac{\sqrt{3} be \tan^{-1} \left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}} \right)}{4c^{2/3}} + \frac{\sqrt{3} be \tan^{-1} \left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}} \right)}{4c^{2/3}} - \frac{be \tanh^{-1}(\sqrt[3]{c}x)}{2c^{2/3}} \\
&= -\frac{\sqrt{3} be \tan^{-1} \left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}} \right)}{4c^{2/3}} + \frac{\sqrt{3} be \tan^{-1} \left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}} \right)}{4c^{2/3}} + \frac{\sqrt{3} bd \tan^{-1}(cx^3)}{2c^{2/3}}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 333, normalized size = 1.17

$$\frac{bdx + \frac{1}{2}ex^2 + \frac{\sqrt{3}be \text{ArcTan}\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}} + \frac{\sqrt{3}be \text{ArcTan}\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}} + bdx \tanh^{-1}(cx^3) + \frac{1}{2}bd^2 \tanh^{-1}(cx^3) + \frac{be \log(1-\sqrt[3]{c}x)}{4c^{2/3}} + \frac{be \log(1+\sqrt[3]{c}x)}{4c^{2/3}} + \frac{be \log(1-\sqrt[3]{c}x + c^{2/3}x^2)}{8c^{2/3}} + \frac{be \log(1+\sqrt[3]{c}x + c^{2/3}x^2)}{8c^{2/3}} - \frac{b(-2\sqrt{3} \text{ArcTan}\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right) + 2\sqrt{3} \text{ArcTan}\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right) - 2\log(1-\sqrt[3]{c}x) - 2\log(1+\sqrt[3]{c}x) + \log(1-\sqrt[3]{c}x + c^{2/3}x^2) + \log(1+\sqrt[3]{c}x + c^{2/3}x^2))}{4c^2}}{4c^2}$$

Antiderivative was successfully verified.

`[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x^3]), x]`

```

[Out] a*d*x + (a*e*x^2)/2 + (Sqrt[3]*b*e*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (Sqrt[3]*b*e*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + b*d*x*ArcTanh[c*x^3] + (b*e*x^2*ArcTanh[c*x^3])/2 + (b*e*Log[1 - c^(1/3)*x])/(4*c^(2/3)) - (b*e*Log[1 + c^(1/3)*x])/(4*c^(2/3)) + (b*e*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*e*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*d*(-2*Sqrt[3]*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]] - 2*Log[1 - c^(1/3)*x] - 2*Log[1 + c^(1/3)*x])/(4*c^2)

```

] + Log[1 - c^(1/3)*x + c^(2/3)*x^2] + Log[1 + c^(1/3)*x + c^(2/3)*x^2]))/(4*c^(1/3))

Maple [A]

time = 0.14, size = 363, normalized size = 1.27

method	result
default	$a\left(\frac{1}{2}e x^2 + dx\right) + \frac{b \operatorname{arctanh}(c x^3) x^2 e}{2} + b \operatorname{arctanh}(c x^3) dx + \frac{bd \ln\left(x + \left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{2}{3}}} - \frac{bd \ln\left(x^2 - \left(\frac{1}{c}\right)^{\frac{1}{3}} x + \left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{2}{3}}} + \dots$
risch	$\frac{b\left(\frac{1}{2}e x^2 + dx\right) \ln(c x^3 + 1)}{2} + \frac{ae x^2}{2} + adx - \frac{bdx \ln(-c x^3 + 1)}{2} + \frac{bd \ln\left(x - \left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{2}{3}}} - \frac{bd \ln\left(x^2 + \left(\frac{1}{c}\right)^{\frac{1}{3}} x + \left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{2}{3}}} - \dots$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)

[Out] a*(1/2*e*x^2+d*x)+1/2*b*arctanh(c*x^3)*x^2*e+b*arctanh(c*x^3)*d*x+1/2*b*d/c/(1/c)^(2/3)*ln(x+(1/c)^(1/3))-1/4*b*d/c/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/2*b*d/c/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))-1/4*b*e/c/(1/c)^(1/3)*ln(x+(1/c)^(1/3))+1/8*b*e/c/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*e*3^(1/2)/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))+1/2*b*d/c/(1/c)^(2/3)*ln(x-(1/c)^(1/3))-1/4*b*d/c/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))-1/2*b*d/c/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))+1/4*b*e/c/(1/c)^(1/3)*ln(x-(1/c)^(1/3))-1/8*b*e/c/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*e*3^(1/2)/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))

Maxima [A]

time = 0.47, size = 250, normalized size = 0.88

$$\frac{1}{2} a x^2 e + \frac{1}{4} \left(c \left(\frac{2 \sqrt{3} \arctan\left(\frac{\sqrt{3}(2x^2+1)}{3x}\right) - \log\left(\frac{c^2 x^2 + c^2 x + 1}{c^2}\right) - 2 \log\left(\frac{c^2 x + 1}{c^2}\right)}{c^2} \right) + 4 x \operatorname{arctanh}(c x^3) \right) b d + a d x + \frac{1}{8} \left(4 x^2 \operatorname{arctanh}(c x^3) + c \left(\frac{2 \sqrt{3} \arctan\left(\frac{\sqrt{3}(2x^2+1)}{3x}\right) - 2 \sqrt{3} \arctan\left(\frac{\sqrt{3}(2x^2-1)}{3x}\right)}{c^2} + \frac{\log\left(\frac{c^2 x^2 + c^2 x + 1}{c^2}\right) - \log\left(\frac{c^2 x^2 - c^2 x + 1}{c^2}\right) - 2 \log\left(\frac{c^2 x + 1}{c^2}\right) + 2 \log\left(\frac{c^2 x - 1}{c^2}\right)}{c^2} \right) \right) b e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arctanh(c*x^3)),x, algorithm="maxima")

[Out] 1/2*a*x^2*e + 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3)))/c^(2/3))/c^(4/3) - log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1)/c^(4/3) + 2*log((c^(2/3)*x^2 - 1)/c^(2/3))/c^(4/3) + 4*x*arctanh(c*x^3))*b*d + a*d*x + 1/8*(4*x^2*arctanh(c*x^3) + c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))

$$\left. \right)/c^{(1/3)}/c^{(5/3)} + 2*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*c^{(2/3)*x} - c^{(1/3)})/c^{(1/3)})/c^{(5/3)} - \log(c^{(2/3)*x^2} + c^{(1/3)*x} + 1)/c^{(5/3)} + \log(c^{(2/3)*x^2} - c^{(1/3)*x} + 1)/c^{(5/3)} - 2*\log((c^{(1/3)*x} + 1)/c^{(1/3)})/c^{(5/3)} + 2*\log((c^{(1/3)*x} - 1)/c^{(1/3)})/c^{(5/3)})*b*e$$

Fricas [C] Result contains complex when optimal does not.

time = 1.29, size = 14519, normalized size = 50.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arctanh(c*x^3)),x, algorithm="fricas")

[Out] $\frac{1}{2}a*x^2*\cosh(1) + \frac{1}{2}a*x^2*\sinh(1) + a*d*x + \frac{1}{16}(b^2*d*(-I*\sqrt{3} + 1))/((c*\cosh(1) - c*\sinh(1))*(-1/128*b^3*\sqrt{(64*(\cosh(1)^3 - 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 - \sinh(1)^3)*c^2*d^6 - 16*(\cosh(1)^4 - 2*\cosh(1)^2*\sinh(1)^2 + \sinh(1)^4 - 2)*c*d^3 + \cosh(1)^5 + 3*\cosh(1)^4*\sinh(1) + 2*\cosh(1)^3*\sinh(1)^2 - 2*\cosh(1)^2*\sinh(1)^3 - 3*\cosh(1)*\sinh(1)^4 - \sinh(1)^5)/(\cosh(1) - \sinh(1)))/(c^2*(\cosh(1) - \sinh(1))) - \frac{1}{128}(8*b^3*c*d^3*\sinh(1) + b^3*c*\cosh(1)^2 + b^3*\sinh(1)^2 - 2*(4*b^3*c*d^3 - b^3*\sinh(1))*\cosh(1))/((c^2*\cosh(1) - c^2*\sinh(1)))^{(1/3)} - 8*(I*\sqrt{3} + 1)*(-1/128*b^3*\sqrt{(64*(\cosh(1)^3 - 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 - \sinh(1)^3)*c^2*d^6 - 16*(\cosh(1)^4 - 2*\cosh(1)^2*\sinh(1)^2 + \sinh(1)^4 - 2)*c*d^3 + \cosh(1)^5 + 3*\cosh(1)^4*\sinh(1) + 2*\cosh(1)^3*\sinh(1)^2 - 2*\cosh(1)^2*\sinh(1)^3 - 3*\cosh(1)*\sinh(1)^4 - \sinh(1)^5)/(\cosh(1) - \sinh(1)))/(c^2*(\cosh(1) - \sinh(1))) - \frac{1}{128}(8*b^3*c*d^3*\sinh(1) + b^3*c*\cosh(1)^2 + b^3*\sinh(1)^2 - 2*(4*b^3*c*d^3 - b^3*\sinh(1))*\cosh(1))/((c^2*\cosh(1) - c^2*\sinh(1)))^{(1/3)}* \log(8*b^2*c*d^3*x + b^2*x*\cosh(1)^3 + b^2*x*\sinh(1)^3 + (b^2*d*(-I*\sqrt{3} + 1))/((c*\cosh(1) - c*\sinh(1))*(-1/128*b^3*\sqrt{(64*(\cosh(1)^3 - 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 - \sinh(1)^3)*c^2*d^6 - 16*(\cosh(1)^4 - 2*\cosh(1)^2*\sinh(1)^2 + \sinh(1)^4 - 2)*c*d^3 + \cosh(1)^5 + 3*\cosh(1)^4*\sinh(1) + 2*\cosh(1)^3*\sinh(1)^2 - 2*\cosh(1)^2*\sinh(1)^3 - 3*\cosh(1)*\sinh(1)^4 - \sinh(1)^5)/(\cosh(1) - \sinh(1)))/(c^2*(\cosh(1) - \sinh(1))) - \frac{1}{128}(8*b^3*c*d^3*\sinh(1) + b^3*c*\cosh(1)^2 + b^3*\sinh(1)^2 - 2*(4*b^3*c*d^3 - b^3*\sinh(1))*\cosh(1))/((c^2*\cosh(1) - c^2*\sinh(1)))^{(1/3)} - 8*(I*\sqrt{3} + 1)*(-1/128*b^3*\sqrt{(64*(\cosh(1)^3 - 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 - \sinh(1)^3)*c^2*d^6 - 16*(\cosh(1)^4 - 2*\cosh(1)^2*\sinh(1)^2 + \sinh(1)^4 - 2)*c*d^3 + \cosh(1)^5 + 3*\cosh(1)^4*\sinh(1) + 2*\cosh(1)^3*\sinh(1)^2 - 2*\cosh(1)^2*\sinh(1)^3 - 3*\cosh(1)*\sinh(1)^4 - \sinh(1)^5)/(\cosh(1) - \sinh(1)))/(c^2*(\cosh(1) - \sinh(1))) - \frac{1}{128}(8*b^3*c*d^3*\sinh(1) + b^3*c*\cosh(1)^2 + b^3*\sinh(1)^2 - 2*(4*b^3*c*d^3 - b^3*\sinh(1))*\cosh(1))/((c^2*\cosh(1) - c^2*\sinh(1)))^{(1/3)}*b*c*d^2 + 4*b^2*d*\cosh(1)^2 + \frac{1}{16}(b^2*d*(-I*\sqrt{3} + 1))/((c*\cosh(1) - c*\sinh(1))*(-1/128*b^3*\sqrt{(64*(\cosh(1)^3 - 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 - \sinh(1)^3)*c^2*d^6 - 16*(\cosh(1)^4 - 2*\cosh(1)^2*\sinh(1)^2 + \sinh(1)^4 - 2)*c*d^3 + \cosh(1)^5 + 3*\cosh(1)^4*\sinh(1) + 2*\cosh(1)^3*\sinh(1)^2 - 2*$


```

*cosh(1)^2*sinh(1)^3 - 3*cosh(1)*sinh(1)^4 - sinh(1)^5)/(cosh(1) - sinh(1))
)/(c^2*(cosh(1) - sinh(1))) - 1/128*(8*b^3*c*d^3*sinh(1) + b^3*cosh(1)^2 +
b^3*sinh(1)^2 - 2*(4*b^3*c*d^3 - b^3*sinh(1))*cosh(1))/(c^2*cosh(1) - c^2*s
inh(1)))^(1/3)) - 8*(I*sqrt(3) + 1)*(-1/128*b^3*sqrt((64*(cosh(1)^3 - 3*cos
h(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 - sinh(1)^3)*c^2*d^6 - 16*(cosh(1)^4 -
2*cosh(1)^2*sinh(1)^2 + sinh(1)^4 - 2)*c*d^3 + cosh(1)^5 + 3*cosh(1)^4*sin
h(1) + 2*cosh(1)^3*sinh(1)^2 - 2*cosh(1)^2*sinh(1)^3 - 3*cosh(1)*sinh(1)^4
- sinh(1)^5)/(cosh(1) - sinh(1)))/(c^2*(cosh(1) - sinh(1))) - 1/128*(8*b^3*
c*d^3*sinh(1) + b^3*cosh(1)^2 + b^3*sinh(1)^2 - 2*(4*b^3*c*d^3 - b^3*sinh(1
))*cosh(1))/(c^2*cosh(1) - c^2*sinh(1)))^(1/3))^2*(c*cosh(1) + c*sinh(1)) +
(3*b^2*x*cosh(1) + 4*b^2*d)*sinh(1)^2 + (3*b^2*x*cosh(1)^2 + 8*b^2*d*cosh(
1))*sinh(1) - 1/32*(b^2*d*(-I*sqrt(3) + 1)/((c*cosh(1) - c*sinh(1))*(-1/12
8*b^3*sqrt((64*(cosh(1)^3 - 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 - sin
h(1)^3)*c^2*d^6 - 16*(cosh(1)^4 - 2*cosh(1)^2*sinh(1)^2 + sinh(1)^4 - 2)*c*
d^3 + cosh(1)^5 + 3*cosh(1)^4*sinh(1) + 2*cosh(1)^3*sinh(1)^2 - 2*cosh(1)^2
*sinh(1)^3 - 3*cosh(1)*sinh(1)^4 - sinh(1)^5)/(cosh(1) - sinh(1)))/(c^2*(co
sh(1) - sinh(1))) - 1/128*(8*b^3*c*d^3*sinh(1) + b^3*cosh(1)^2 + b^3*sinh(1
)^2 - 2*(4*b^3*c*d^3 - b^3*sinh(1))*cosh(1))/(c^2*cosh(1) - c^2*sinh(1)))^(
1/3)) - 2*sqrt(3/2)*sqrt(1/2)*sqrt(-(128*b^2*d*cosh(1) + 128*b^2*d*sinh(1)
+ (b^2*d*(-I*sqrt(3) + 1)/((c*cosh(1) - c*sinh(1))*(-1/128*b^3*sqrt((64*(co
sh(1)^3 - 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 - sinh(1)^3)*c^2*d^6 -
16*(cosh(1)^4 - 2*cosh(1)^2*sinh(1)^2 + sinh(1)^4 - 2)*c*d^3 + cosh(1)^5 +
3*cosh(1)^4*sinh(1) + 2*cosh(1)^3*sinh(1)^2 - 2*cosh(1)^2*sinh(1)^3 - 3*cos
h(1)*sinh(1)^4 - sinh(1)^5)/(cosh(1) - sinh(1)))/(c^2*(cosh(1) - sinh(1)))
- 1/128*(8*b^3*c*d^3*sinh(1) + b^3*cosh(1)^2 + b^3*sinh(1)^2 - 2*(4*b^3*c*d
^3 - b^3*sinh(1))*cosh(1))/(c^2*cosh(1) - c^2*sinh(1)))^(1/3)) - 8*(I*sqrt(
3) + 1)*(-1/128*b^3*sqrt((64*(cosh(1)^3 - 3*cosh(1)^2*sinh(1) + 3*cosh(1)*s
inh(1)^2 - sinh(1)^3)*c^2*d^6 - 16*(cosh(1)^4 - 2*cosh(1)^2*sinh(1)^2 + sin
h(1)^4 - 2)*c*d^3 + cosh(1)^5 + 3*cosh(1)^4*sinh(1) + 2*cosh(1)^3*sinh(1)^2
- 2*cosh(1)^2*sinh(1)^3 - 3*cosh(1)*sinh(1)^4 - sinh(1)^5)/(cosh(1) - sinh
(1)))/(c^2*(cosh(1) - sinh(1))) - 1/128*(8*b^3*c*d^3*sinh(1) + b^3*cosh(1)^
2 + b^3*sinh(1)^2 - 2*(4*b^3*c*d^3 - b^3*sinh(1))*cosh(1))/(c^2*cosh(1) - c
^2*sinh(1)))^(1/3))^2*c)/c) - 8*(I*sqrt(3) + 1)*(-1/128*b^3*sqrt((64*(cosh(
1)^3 - 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 - sinh(1)^3)*c^2*d^6 - 16*
(cosh(1)^4 - 2*cosh(1)^2*sinh(1)^2 + sinh(1)^4 ...

```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*atanh(c*x**3)),x)

[Out] Timed out

Giac [A]

time = 1.09, size = 279, normalized size = 0.98

$$\frac{1}{2}ax^2 + adx + \frac{1}{4}(bx^2 + 2bdx) \log\left(\frac{cx^3 + 1}{cx^3 - 1}\right) - \frac{\sqrt{3}(2bd|c|^2 - b|c|^4) \arctan\left(\frac{1}{\sqrt{3}}\sqrt{2x + \frac{1}{c^2}}|c|\right)}{4c^2} + \frac{\sqrt{3}(2bd|c|^2 + b|c|^4) \arctan\left(\frac{1}{\sqrt{3}}\sqrt{2x - \frac{1}{c^2}}|c|\right)}{4c^2} - \frac{(2bd|c|^2 + b|c|^4) \log\left(x^2 + \frac{1}{c^2} + \frac{1}{c^2}\right)}{8c^2} - \frac{(2bd|c|^2 - b|c|^4) \log\left(x^2 - \frac{1}{c^2} + \frac{1}{c^2}\right)}{8c^2} + \frac{(2bd|c|^2 - b|c|^4) \log\left(x + \frac{1}{c^2}\right)}{4c^2} + \frac{(2bd|c|^2 + b|c|^4) \log\left(x - \frac{1}{c^2}\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arctanh(c*x^3)),x, algorithm="giac")

[Out] $\frac{1}{2}ax^2 + adx + \frac{1}{4}(bx^2 + 2bdx) \log\left(\frac{cx^3 + 1}{cx^3 - 1}\right) - \frac{1}{4}\sqrt{3}(2b^2cd^2 \operatorname{abs}(c)^{2/3} - b^2c^2e \operatorname{abs}(c)^{1/3}) \arctan\left(\frac{1}{3}\sqrt{3}(2x + 1/\operatorname{abs}(c)^{1/3}) \operatorname{abs}(c)^{1/3}\right)/c^2 + \frac{1}{4}\sqrt{3}(2b^2cd^2 \operatorname{abs}(c)^{2/3} + b^2c^2e \operatorname{abs}(c)^{1/3}) \arctan\left(\frac{1}{3}\sqrt{3}(2x - 1/\operatorname{abs}(c)^{1/3}) \operatorname{abs}(c)^{1/3}\right)/c^2 - \frac{1}{8}(2b^2cd^2 \operatorname{abs}(c)^{2/3} + b^2c^2e \operatorname{abs}(c)^{1/3}) \log(x^2 + x/\operatorname{abs}(c)^{1/3} + 1/\operatorname{abs}(c)^{2/3})/c^2 - \frac{1}{8}(2b^2cd^2 \operatorname{abs}(c)^{2/3} - b^2c^2e \operatorname{abs}(c)^{1/3}) \log(x^2 - x/\operatorname{abs}(c)^{1/3} + 1/\operatorname{abs}(c)^{2/3})/c^2 + \frac{1}{4}(2b^2cd^2 \operatorname{abs}(c)^{2/3} - b^2c^2e \operatorname{abs}(c)^{1/3}) \log(\operatorname{abs}(x + 1/\operatorname{abs}(c)^{1/3}))/c^2 + \frac{1}{4}(2b^2cd^2 \operatorname{abs}(c)^{2/3} + b^2c^2e \operatorname{abs}(c)^{1/3}) \log(\operatorname{abs}(x - 1/\operatorname{abs}(c)^{1/3}))/c^2$

Mupad [B]

time = 1.03, size = 621, normalized size = 2.18

$$\frac{1}{2}ax^2 + adx + \frac{1}{4}(bx^2 + 2bdx) \log\left(\frac{cx^3 + 1}{cx^3 - 1}\right) - \frac{\sqrt{3}(2bd|c|^2 - b|c|^4) \arctan\left(\frac{1}{\sqrt{3}}\sqrt{2x + \frac{1}{c^2}}|c|\right)}{4c^2} + \frac{\sqrt{3}(2bd|c|^2 + b|c|^4) \arctan\left(\frac{1}{\sqrt{3}}\sqrt{2x - \frac{1}{c^2}}|c|\right)}{4c^2} - \frac{(2bd|c|^2 + b|c|^4) \log\left(x^2 + \frac{1}{c^2} + \frac{1}{c^2}\right)}{8c^2} - \frac{(2bd|c|^2 - b|c|^4) \log\left(x^2 - \frac{1}{c^2} + \frac{1}{c^2}\right)}{8c^2} + \frac{(2bd|c|^2 - b|c|^4) \log\left(x + \frac{1}{c^2}\right)}{4c^2} + \frac{(2bd|c|^2 + b|c|^4) \log\left(x - \frac{1}{c^2}\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^3))*(d + e*x),x)

[Out] $\operatorname{symsum}\left(\log\left(\frac{243b^5c^9d^4e}{2} - \operatorname{root}\left(64c^2z^3 + 24b^2c^2d^3 + b^3e^3, z, k\right)\left(\operatorname{root}\left(64c^2z^3 + 24b^2c^2d^3 + b^3e^3, z, k\right)\left(\operatorname{root}\left(64c^2z^3 + 24b^2c^2d^3 + b^3e^3, z, k\right)\left(486b^2c^{10}e^2x - 1944b^2c^{10}d^2e + 3888\operatorname{root}\left(64c^2z^3 + 24b^2c^2d^3 + b^3e^3, z, k\right)b^2c^{11}d^2x\right) - \frac{243b^3c^9e^3}{2}\right) - 486b^4c^{10}d^4x\right) + \frac{243b^5c^9d^3e^2x}{4}\right)\operatorname{root}\left(64c^2z^3 + 24b^2c^2d^3 + b^3e^3, z, k\right), k, 1, 3\right) + \operatorname{symsum}\left(\log\left(\frac{243b^5c^9d^4e}{2} - \operatorname{root}\left(64c^2z^3 - 24b^2c^2d^3 - b^3e^3, z, k\right)\left(\operatorname{root}\left(64c^2z^3 - 24b^2c^2d^3 - b^3e^3, z, k\right)\left(\operatorname{root}\left(64c^2z^3 - 24b^2c^2d^3 - b^3e^3, z, k\right)\left(486b^2c^{10}e^2x - 1944b^2c^{10}d^2e + 3888\operatorname{root}\left(64c^2z^3 - 24b^2c^2d^3 - b^3e^3, z, k\right)b^2c^{11}d^2x\right) - \frac{243b^3c^9e^3}{2}\right) - 486b^4c^{10}d^4x\right) + \frac{243b^5c^9d^3e^2x}{4}\right)\operatorname{root}\left(64c^2z^3 - 24b^2c^2d^3 - b^3e^3, z, k\right), k, 1, 3\right) + \log(cx^3 + 1) \left(\frac{bdx}{2} + \frac{be^2x^2}{4}\right) - \log(1 - cx^3) \left(\frac{bdx}{2} + \frac{be^2x^2}{4}\right) + adx + \frac{ae^2x^2}{2}$

$$3.34 \quad \int \frac{a+b \tanh^{-1}(cx^3)}{d+ex} dx$$

Optimal. Leaf size=523

$$\frac{(a + b \tanh^{-1}(cx^3)) \log(d + ex)}{e} + \frac{b \log\left(\frac{e(1 - \sqrt[3]{c}x)}{\sqrt[3]{c}d + e}\right) \log(d + ex)}{2e} - \frac{b \log\left(-\frac{e(1 + \sqrt[3]{c}x)}{\sqrt[3]{c}d - e}\right) \log(d + ex)}{2e} +$$

```
[Out] (a+b*arctanh(c*x^3))*ln(e*x+d)/e+1/2*b*ln(e*(1-c^(1/3)*x)/(c^(1/3)*d+e))*ln
(e*x+d)/e-1/2*b*ln(-e*(1+c^(1/3)*x)/(c^(1/3)*d-e))*ln(e*x+d)/e+1/2*b*ln(-e*
((-1)^(1/3)+c^(1/3)*x)/(c^(1/3)*d-(-1)^(1/3)*e))*ln(e*x+d)/e-1/2*b*ln(-e*((
-1)^(2/3)+c^(1/3)*x)/(c^(1/3)*d-(-1)^(2/3)*e))*ln(e*x+d)/e+1/2*b*ln((-1)^(2
/3)*e*(1+(-1)^(1/3)*c^(1/3)*x)/(c^(1/3)*d+(-1)^(2/3)*e))*ln(e*x+d)/e-1/2*b*
ln((-1)^(1/3)*e*(1+(-1)^(2/3)*c^(1/3)*x)/(c^(1/3)*d+(-1)^(1/3)*e))*ln(e*x+d
)/e-1/2*b*polylog(2,c^(1/3)*(e*x+d)/(c^(1/3)*d-e))/e+1/2*b*polylog(2,c^(1/3
)*(e*x+d)/(c^(1/3)*d+e))/e+1/2*b*polylog(2,c^(1/3)*(e*x+d)/(c^(1/3)*d-(-1)^(
1/3)*e))/e-1/2*b*polylog(2,c^(1/3)*(e*x+d)/(c^(1/3)*d+(-1)^(1/3)*e))/e-1/2
*b*polylog(2,c^(1/3)*(e*x+d)/(c^(1/3)*d-(-1)^(2/3)*e))/e+1/2*b*polylog(2,c^(
1/3)*(e*x+d)/(c^(1/3)*d+(-1)^(2/3)*e))/e
```

Rubi [A]

time = 0.71, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6067, 281, 212, 2463, 266, 2441, 2440, 2438}

$\frac{\log(d+ex)(a+b \tanh^{-1}(cx^3))}{e} - \frac{b \log\left(\frac{e(1-\sqrt[3]{c}x)}{\sqrt[3]{c}d+e}\right)}{2e} - \frac{b \log\left(-\frac{e(1+\sqrt[3]{c}x)}{\sqrt[3]{c}d-e}\right)}{2e} - \frac{b \log\left(-\frac{e((-1)^{1/3}+c^{1/3}x)}{\sqrt[3]{c}d-(-1)^{1/3}e}\right)}{2e} - \frac{b \log\left(-\frac{e((-1)^{2/3}+c^{1/3}x)}{\sqrt[3]{c}d-(-1)^{2/3}e}\right)}{2e} - \frac{b \log\left(\frac{e((-1)^{2/3}e(1+(-1)^{1/3}c^{1/3}x)}{\sqrt[3]{c}d+(-1)^{2/3}e}\right)}{2e} - \frac{b \log\left(\frac{e((-1)^{1/3}e(1+(-1)^{2/3}c^{1/3}x)}{\sqrt[3]{c}d+(-1)^{1/3}e}\right)}{2e} - \frac{b \text{PolyLog}\left[2, \frac{c^{1/3}(d+ex)}{c^{1/3}d-e}\right]}{2e} + \frac{b \text{PolyLog}\left[2, \frac{c^{1/3}(d+ex)}{c^{1/3}d+e}\right]}{2e} + \frac{b \text{PolyLog}\left[2, \frac{c^{1/3}(d+ex)}{c^{1/3}d-(-1)^{1/3}e}\right]}{2e} - \frac{b \text{PolyLog}\left[2, \frac{c^{1/3}(d+ex)}{c^{1/3}d+(-1)^{1/3}e}\right]}{2e} - \frac{b \text{PolyLog}\left[2, \frac{c^{1/3}(d+ex)}{c^{1/3}d-(-1)^{2/3}e}\right]}{2e} + \frac{b \text{PolyLog}\left[2, \frac{c^{1/3}(d+ex)}{c^{1/3}d+(-1)^{2/3}e}\right]}{2e}$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x^3])/(d + e*x), x]

```
[Out] ((a + b*ArcTanh[c*x^3])*Log[d + e*x])/e + (b*Log[(e*(1 - c^(1/3)*x))/(c^(1/
3)*d + e])*Log[d + e*x])/(2*e) - (b*Log[-((e*(1 + c^(1/3)*x))/(c^(1/3)*d -
e))]*Log[d + e*x])/(2*e) + (b*Log[-((e*((-1)^(1/3) + c^(1/3)*x))/(c^(1/3)*d
- (-1)^(1/3)*e))]*Log[d + e*x])/(2*e) - (b*Log[-((e*((-1)^(2/3) + c^(1/3)*
x))/(c^(1/3)*d - (-1)^(2/3)*e))]*Log[d + e*x])/(2*e) + (b*Log[((-1)^(2/3)*e
*(1 + (-1)^(1/3)*c^(1/3)*x))/(c^(1/3)*d + (-1)^(2/3)*e])*Log[d + e*x])/(2*e
) - (b*Log[((-1)^(1/3)*e*(1 + (-1)^(2/3)*c^(1/3)*x))/(c^(1/3)*d + (-1)^(1/3
)*e])*Log[d + e*x])/(2*e) - (b*PolyLog[2, (c^(1/3)*(d + e*x))/(c^(1/3)*d -
e])/(2*e) + (b*PolyLog[2, (c^(1/3)*(d + e*x))/(c^(1/3)*d + e])/(2*e) + (b
*PolyLog[2, (c^(1/3)*(d + e*x))/(c^(1/3)*d - (-1)^(1/3)*e])/(2*e) - (b*Pol
yLog[2, (c^(1/3)*(d + e*x))/(c^(1/3)*d + (-1)^(1/3)*e])/(2*e) - (b*PolyLog
[2, (c^(1/3)*(d + e*x))/(c^(1/3)*d - (-1)^(2/3)*e])/(2*e) + (b*PolyLog[2,
(c^(1/3)*(d + e*x))/(c^(1/3)*d + (-1)^(2/3)*e])/(2*e)
```

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 281

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 2438

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 2440

Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))]*(b_))/((f_) + (g_)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]

Rule 2441

Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))/((f_) + (g_)*(x_)), x_Symbol] := Simp[Log[e*((f + g*x)/(e*f - d*g))]*(a + b*Log[c*(d + e*x)^n]/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

Rule 2463

Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))^(p_)*((h_)*(x_)^(m_))*((f_) + (g_)*(x_)^(r_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 6067

Int[((a_) + ArcTanh[(c_)*(x_)^(n_)]*(b_))/((d_) + (e_)*(x_)), x_Symbol] := Simp[Log[d + e*x]*(a + b*ArcTanh[c*x^n]/e), x] - Dist[b*c*(n/e), Int[x^(n - 1)*(Log[d + e*x]/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e

, n}, x] && IntegerQ[n]

Rubi steps

$$\int \frac{a + b \tanh^{-1}(cx^3)}{d + ex} dx = \int \left(\frac{a}{d + ex} + \frac{b \tanh^{-1}(cx^3)}{d + ex} \right) dx$$

$$= \frac{a \log(d + ex)}{e} + b \int \frac{\tanh^{-1}(cx^3)}{d + ex} dx$$

Mathematica [C] Result contains complex when optimal does not.
 time = 69.94, size = 515, normalized size = 0.98

$$\frac{b \operatorname{arctanh}(cx^3)}{e} + \frac{a \log(d + ex)}{e} + \frac{b \operatorname{arctanh}(cx^3) \log(d + ex)}{e} + \frac{b \operatorname{arctanh}(cx^3) \log(d + ex) \operatorname{arctanh}(cx^3)}{e} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x^3])/(d + e*x), x]

[Out] (a*Log[d + e*x])/e + (b*(2*ArcTanh[c*x^3]*Log[d + e*x] - Log[(e*(1 - I*Sqrt[3] - 2*c^(1/3)*x))/(2*c^(1/3)*d + e - I*Sqrt[3]*e)]*Log[d + e*x] + Log[(e*(-I + Sqrt[3] - (2*I)*c^(1/3)*x))/((2*I)*c^(1/3)*d + (-I + Sqrt[3])*e)]*Log[d + e*x] + Log[(e*(I + Sqrt[3] + (2*I)*c^(1/3)*x))/((-2*I)*c^(1/3)*d + (I + Sqrt[3])*e)]*Log[d + e*x] - Log[-((e*(1 + c^(1/3)*x))/(c^(1/3)*d - e))]*Log[d + e*x] - Log[-((e*(-1 - I*Sqrt[3] + 2*c^(1/3)*x))/(2*c^(1/3)*d + e + I*Sqrt[3]*e))]*Log[d + e*x] + Log[d + e*x]*Log[(e - c^(1/3)*e*x)/(c^(1/3)*d + e)] - PolyLog[2, (c^(1/3)*(d + e*x))/(c^(1/3)*d - e)] + PolyLog[2, (c^(1/3)*(d + e*x))/(c^(1/3)*d + e)] + PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3)*d - e - I*Sqrt[3]*e)] - PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3)*d + e - I*Sqrt[3]*e)] + PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3)*d - e + I*Sqrt[3]*e)] - PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3)*d + e + I*Sqrt[3]*e)])))/(2*e)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
 time = 0.18, size = 182, normalized size = 0.35

method	result
default	$\frac{a \ln(ex+d)}{e} + \frac{b \ln(ex+d) \operatorname{arctanh}(cx^3)}{e} - \frac{b \left(\sum_{-R1=\operatorname{RootOf}(c_Z^3-3dc_Z^2+3cd^2_Z-cd^3+e^3)} \left(\ln(ex+d) \ln\left(\frac{-ex+R1-d}{-R1}\right) + d \right) \right)}{2e}$
risch	$\frac{a \ln(ex+d)}{e} - \frac{b \ln(ex+d) \ln(-cx^3+1)}{2e} + \frac{b \left(\sum_{-R1=\operatorname{RootOf}(c_Z^3-3dc_Z^2+3cd^2_Z-cd^3+e^3)} \left(\ln(ex+d) \ln\left(\frac{-ex+R1-d}{-R1}\right) + d \right) \right)}{2e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(e*x+d)/e+b*ln(e*x+d)/e*arctanh(c*x^3)-1/2*b/e*sum(ln(e*x+d)*ln((-e*x+_
R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*c-3*_Z^2*c*d+3*_Z*c*d^2-
c*d^3+e^3))+1/2*b/e*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_
R1),_R1=RootOf(_Z^3*c-3*_Z^2*c*d+3*_Z*c*d^2-c*d^3-e^3))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*e^(-1)*log(x*e + d) + 1/2*b*integrate((log(c*x^3 + 1) - log(-c*x^3 + 1))/
(x*e + d), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^3) + a)/(x*e + d), x)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)/(e*x + d), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{atanh}(c x^3)}{d + e x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x^3))/(d + e*x), x)
```

$$3.35 \quad \int \frac{a+b \tanh^{-1}(cx^3)}{(d+ex)^2} dx$$

Optimal. Leaf size=414

$$\frac{\sqrt{3} b \sqrt[3]{c} \operatorname{ArcTan}\left(\frac{1-2\sqrt[3]{c} x}{\sqrt{3}}\right)}{2(c^{2/3}d^2 + \sqrt[3]{c}de + e^2)} - \frac{\sqrt{3} b \sqrt[3]{c} (\sqrt[3]{c}d + e) \operatorname{ArcTan}\left(\frac{1+2\sqrt[3]{c} x}{\sqrt{3}}\right)}{2(cd^3 + e^3)} - \frac{a + b \tanh^{-1}(cx^3)}{e(d+ex)} + \frac{b \sqrt[3]{c} (\sqrt[3]{c}d + e)}{2e^2}$$

[Out] $(-a-b*\operatorname{arctanh}(c*x^3))/e/(e*x+d)+1/2*b*c^{(1/3)}*(c^{(1/3)*d-e})*\ln(1-c^{(1/3)*x})/(c*d^3+e^3)+1/2*b*c^{(1/3)}*(c^{(1/3)*d+e})*\ln(1+c^{(1/3)*x})/(c*d^3-e^3)-3*b*c*d^2*e^2*\ln(e*x+d)/(c^2*d^6-e^6)-1/4*b*c^{(1/3)}*(c^{(1/3)*d+e})*\ln(1-c^{(1/3)*x}+c^{(2/3)*x^2})/(c*d^3-e^3)-1/4*b*c^{(1/3)}*(c^{(1/3)*d-e})*\ln(1+c^{(1/3)*x}+c^{(2/3)*x^2})/(c*d^3+e^3)-1/2*b*c*d^2*\ln(-c*x^3+1)/e/(c*d^3+e^3)+1/2*b*c*d^2*\ln(c*x^3+1)/e/(c*d^3-e^3)-1/2*b*c^{(1/3)}*\operatorname{arctan}(1/3*(1-2*c^{(1/3)*x})*3^{(1/2)})*3^{(1/2)}/(c^{(2/3)*d^2+c^{(1/3)*d}*e+e^2)-1/2*b*c^{(1/3)}*(c^{(1/3)*d+e})*\operatorname{arctan}(1/3*(1+2*c^{(1/3)*x})*3^{(1/2)})*3^{(1/2)}/(c*d^3+e^3)$

Rubi [A]

time = 0.52, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6071, 6857, 1885, 1875, 31, 648, 631, 210, 642, 266, 1874}

$$\frac{a + b \tanh^{-1}(cx^3)}{e(d+ex)} - \frac{\sqrt{3} b \sqrt[3]{c} \operatorname{ArcTan}\left(\frac{1-2\sqrt[3]{c} x}{\sqrt{3}}\right)}{2(c^{2/3}d^2 + \sqrt[3]{c}de + e^2)} - \frac{\sqrt{3} b \sqrt[3]{c} \operatorname{ArcTan}\left(\frac{1+2\sqrt[3]{c} x}{\sqrt{3}}\right) (\sqrt[3]{c}d + e)}{2(cd^3 + e^3)} - \frac{b \sqrt[3]{c} (\sqrt[3]{c}d + e) \log(c^{2/3}d^2 - \sqrt[3]{c}x + 1)}{4(ad^3 - e^3)} - \frac{b \sqrt[3]{c} (\sqrt[3]{c}d - e) \log(c^{2/3}d^2 + \sqrt[3]{c}x + 1)}{4(ad^3 + e^3)} - \frac{3b \sqrt[3]{c} \log(d + ex)}{e^2 d^2 - e^2} + \frac{b \sqrt[3]{c} (\sqrt[3]{c}d - e) \log(1 - \sqrt[3]{c}x)}{2(ad^3 - e^3)} + \frac{b \sqrt[3]{c} (\sqrt[3]{c}d + e) \log(\sqrt[3]{c}x + 1)}{2(ad^3 + e^3)} - \frac{\operatorname{arctan}\log(1 - cx^3)}{2e(ad^3 + e^3)} + \frac{\operatorname{arctan}\log(cx^3 + 1)}{2e(ad^3 - e^3)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcTanh}[c*x^3])/(d + e*x)^2, x]$

[Out] $-1/2*(\operatorname{Sqrt}[3]*b*c^{(1/3)}*\operatorname{ArcTan}[(1 - 2*c^{(1/3)*x})/\operatorname{Sqrt}[3]])/(c^{(2/3)*d^2} + c^{(1/3)*d*e} + e^2) - (\operatorname{Sqrt}[3]*b*c^{(1/3)}*(c^{(1/3)*d} + e)*\operatorname{ArcTan}[(1 + 2*c^{(1/3)*x})/\operatorname{Sqrt}[3]])/(2*(c*d^3 + e^3)) - (a + b*\operatorname{ArcTanh}[c*x^3])/(e*(d + e*x)) + (b*c^{(1/3)}*(c^{(1/3)*d} - e)*\operatorname{Log}[1 - c^{(1/3)*x}])/(2*(c*d^3 + e^3)) + (b*c^{(1/3)}*(c^{(1/3)*d} + e)*\operatorname{Log}[1 + c^{(1/3)*x}])/(2*(c*d^3 - e^3)) - (3*b*c*d^2*e^2*\operatorname{Log}[d + e*x])/(c^2*d^6 - e^6) - (b*c^{(1/3)}*(c^{(1/3)*d} + e)*\operatorname{Log}[1 - c^{(1/3)*x} + c^{(2/3)*x^2}])/(4*(c*d^3 - e^3)) - (b*c^{(1/3)}*(c^{(1/3)*d} - e)*\operatorname{Log}[1 + c^{(1/3)*x} + c^{(2/3)*x^2}])/(4*(c*d^3 + e^3)) - (b*c*d^2*\operatorname{Log}[1 - c*x^3])/(2*e*(c*d^3 + e^3)) + (b*c*d^2*\operatorname{Log}[1 + c*x^3])/(2*e*(c*d^3 - e^3))$

Rule 31

$\operatorname{Int}[(a_) + (b_)*(x_)^(-1), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /;$ $\operatorname{FreeQ}\{a, b\}, x]$

Rule 210

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^(-1), x_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^(-1))*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /;$ $\operatorname{FreeQ}\{a, b\}, x]$ && $\operatorname{PosQ}[a/b]$ &

& (LtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 631

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1874

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 1875

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Dist[r*((B*r + A*s)/(3*a*s)), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]

Rule 1885

Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di

st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 6071

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[(d + e*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(e*(m + 1))), x] - Dist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]

Rule 6857

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx^3)}{(d + ex)^2} dx &= -\frac{a + b \tanh^{-1}(cx^3)}{e(d + ex)} + \frac{b \int \frac{3cx^2}{(d+ex)(1-c^2x^6)} dx}{e} \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{e(d + ex)} + \frac{(3bc) \int \frac{x^2}{(d+ex)(1-c^2x^6)} dx}{e} \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{e(d + ex)} + \frac{(3bc) \int \left(\frac{d^2e^4}{(-cd^3+e^3)(cd^3+e^3)(d+ex)} + \frac{de-e^2x-cd^2x^2}{2(cd^3+e^3)(-1+cx^3)} + \frac{de-e^2x}{2(cd^3-e^3)} \right) dx}{e} \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{e(d + ex)} - \frac{3bcd^2e^2 \log(d + ex)}{c^2d^6 - e^6} + \frac{(3bc) \int \frac{de-e^2x+cd^2x^2}{1+cx^3} dx}{2e(cd^3 - e^3)} + \frac{(3bc) \int \frac{de}{2e(cd^3 - e^3)} dx}{2e(cd^3 - e^3)} \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{e(d + ex)} - \frac{3bcd^2e^2 \log(d + ex)}{c^2d^6 - e^6} + \frac{(3bc) \int \frac{de-e^2x}{1+cx^3} dx}{2e(cd^3 - e^3)} + \frac{(3bc^2d^2) \int \frac{x^2}{1+cx^3} dx}{2e(cd^3 - e^3)} \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{e(d + ex)} - \frac{3bcd^2e^2 \log(d + ex)}{c^2d^6 - e^6} - \frac{bcd^2 \log(1 - cx^3)}{2e(cd^3 + e^3)} + \frac{bcd^2 \log(1 + cx^3)}{2e(cd^3 - e^3)} \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{e(d + ex)} + \frac{b\sqrt[3]{c} (\sqrt[3]{c} d - e) \log(1 - \sqrt[3]{c} x)}{2(cd^3 + e^3)} + \frac{b\sqrt[3]{c} (\sqrt[3]{c} d + e) \log(1 + \sqrt[3]{c} x)}{2(cd^3 - e^3)} \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{e(d + ex)} + \frac{b\sqrt[3]{c} (\sqrt[3]{c} d - e) \log(1 - \sqrt[3]{c} x)}{2(cd^3 + e^3)} + \frac{b\sqrt[3]{c} (\sqrt[3]{c} d + e) \log(1 + \sqrt[3]{c} x)}{2(cd^3 - e^3)} \\
 &= -\frac{\sqrt{3} b\sqrt[3]{c} \tan^{-1}\left(\frac{1-2\sqrt[3]{c} x}{\sqrt{3}}\right)}{2(c^{2/3}d^2 + \sqrt[3]{c} de + e^2)} - \frac{\sqrt{3} b\sqrt[3]{c} (\sqrt[3]{c} d + e) \tan^{-1}\left(\frac{1+2\sqrt[3]{c} x}{\sqrt{3}}\right)}{2(cd^3 + e^3)} - \frac{a + b \tanh^{-1}(cx^3)}{e(d + ex)}
 \end{aligned}$$

Mathematica [A]

time = 0.38, size = 534, normalized size = 1.29

$$\left(\frac{a}{(d+ex)} + \frac{2\sqrt{3}b\text{ArcTan}\left(\frac{cx^3}{\sqrt{3}}\right)}{c^2d^3-2e^3} + \frac{2\sqrt{3}b\sqrt{c^2d^3+e^3}\text{ArcTan}\left(\frac{cx^3}{\sqrt{3}}\right)}{c^2d^3+e^3} + \frac{bd\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{(2cd^3-2e^3)\left(\frac{1}{c}\right)^{\frac{2}{3}}} - \frac{bd\ln\left(x^2-\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{2(2cd^3-2e^3)\left(\frac{1}{c}\right)^{\frac{2}{3}}} + \frac{bd\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\sqrt{3}}-1\right)}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{(2cd^3-2e^3)\left(\frac{1}{c}\right)^{\frac{2}{3}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x^3])/(d + e*x)^2,x]

[Out] ((-4*a)/(e*(d + e*x)) + (2*Sqrt[3]*b*c^(1/3)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/(c^(2/3)*d^2 + c^(1/3)*d*e + e^2) - (2*Sqrt[3]*b*c^(1/3)*(c^(1/3)*d + e)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(c*d^3 + e^3) - (4*b*ArcTanh[c*x^3])/(e*(d + e*x)) + (2*b*c^(1/3)*(c^(5/3)*d^5 - c^(4/3)*d^4*e + c*d^3*e^2 + c^(1/3)*d*e^4 - e^5)*Log[1 - c^(1/3)*x])/(-c^2*d^6*e) + e^7) - (2*b*c^(1/3)*(c^(5/3)*d^5 + c^(4/3)*d^4*e + c*d^3*e^2 + c^(1/3)*d*e^4 + e^5)*Log[1 + c^(1/3)*x])/(-c^2*d^6*e) + e^7) - (12*b*c*d^2*e^2*Log[d + e*x])/(c^2*d^6 - e^6) + (b*c^(1/3)*(2*c^(5/3)*d^5 - c^(4/3)*d^4*e - c*d^3*e^2 - c^(1/3)*d*e^4 - e^5)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(c^2*d^6*e - e^7) + (b*c^(1/3)*(2*c^(5/3)*d^5 + c^(4/3)*d^4*e - c*d^3*e^2 - c^(1/3)*d*e^4 + e^5)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(-c^2*d^6*e) + e^7) + (2*b*c*d^2*e^2*Log[1 - c^2*x^6])/(c^2*d^6 - e^6))/4

Maple [A]

time = 0.30, size = 591, normalized size = 1.43

method	result
default	$-\frac{a}{(ex+d)e} - \frac{b\text{arctanh}(cx^3)}{(ex+d)e} + \frac{bd\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{(2cd^3-2e^3)\left(\frac{1}{c}\right)^{\frac{2}{3}}} - \frac{bd\ln\left(x^2-\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{2(2cd^3-2e^3)\left(\frac{1}{c}\right)^{\frac{2}{3}}} + \frac{bd\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\sqrt{3}}-1\right)}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{(2cd^3-2e^3)\left(\frac{1}{c}\right)^{\frac{2}{3}}} +$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x^3))/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -a/(e*x+d)/e-b/(e*x+d)/e*arctanh(c*x^3)+b/(2*c*d^3-2*e^3)*d/(1/c)^(2/3)*ln(x+(1/c)^(1/3))-1/2*b/(2*c*d^3-2*e^3)*d/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+b/(2*c*d^3-2*e^3)*d/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))+b*e/(2*c*d^3-2*e^3)/(1/c)^(1/3)*ln(x+(1/c)^(1/3))-1/2*b*e/(2*c*d^3-2*e^3)/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))-b*e/(2*c*d^3-2*e^3)*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))+b/e*c/(2*c*d^3-2*e^3)*d^2*ln(c*x^3+1)+b/(2*c*d^3+2*e^3)*d/(1/c)^(2/3)*ln(x-(1/c)^(1/3))-1/2*b/(2*c*d^3+2*e^3)*d/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))-b/(2*c*d^3+2*e^3)*d/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-b*e/(2*c*d^3+2*e^3)/(1/c)^(1/3)*ln(x-(1/c)^(1/3))+1/2*b*e/(2*c*d^3+2*e^3)

$$\begin{aligned} &)/(1/c)^{(1/3)} * \ln(x^2 + (1/c)^{(1/3)} * x + (1/c)^{(2/3)}) - b * e / (2 * c * d^3 + 2 * e^3) * 3^{(1/2)} \\ &/ (1/c)^{(1/3)} * \arctan(1/3 * 3^{(1/2)} * (2 / (1/c)^{(1/3)} * x + 1)) - b / e * c / (2 * c * d^3 + 2 * e^3) * \\ &d^2 * \ln(c * x^3 - 1) - 3 * b * e^2 * c * d^2 / (c * d^3 + e^3) / (c * d^3 - e^3) * \ln(e * x + d) \end{aligned}$$

Maxima [A]

time = 0.47, size = 393, normalized size = 0.95

$$\frac{1}{4} \left(\frac{12d^2 \log(xe+d)}{c^2d^2 - e^2} + \frac{2\sqrt{3}(cde+ct^2) \arctan\left(\frac{\sqrt{3}(2,3x+t)}{3,t}\right)}{(c^2de+ce^2)t} - \frac{2\sqrt{3}(cde-ct^2) \arctan\left(\frac{\sqrt{3}(2,3x-t)}{3,t}\right)}{(c^2de-ce^2)t} + \frac{(2cd^2+ctde-ct^2) \log(ct^2+ctx+1)}{c^2de+ce^2} - \frac{(2cd^2-ctde-ct^2) \log(ct^2-ctx+1)}{c^2de-ce^2} - \frac{2(cd^2+ctde+ct^2) \log\left(\frac{d^2+1}{3}\right)}{c^2de+ce^2} + \frac{2(cd^2-ctde+ct^2) \log\left(\frac{d^2+1}{3}\right)}{c^2de-ce^2} + \frac{4 \operatorname{arctanh}\left(\frac{ce^2}{xe^2+de}\right)}{xe^2+de} \right) b - \frac{a}{xe^2+de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^3))/(e*x+d)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} &-1/4 * ((12*d^2*e^2*log(x*e + d)/(c^2*d^6 - e^6) + 2*sqrt(3)*(c*d*e + c^(2/3) \\ &*e^2)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/3)))/((c^2*d^3*e + c*e \\ &^4)*c^(1/3)) - 2*sqrt(3)*(c*d*e - c^(2/3)*e^2)*arctan(1/3*sqrt(3)*(2*c^(2/3) \\ &)*x - c^(1/3))/c^(1/3))/((c^2*d^3*e - c*e^4)*c^(1/3)) + (2*c*d^2 + c^(2/3)* \\ &d*e - c^(1/3)*e^2)*log(c^(2/3)*x^2 + c^(1/3)*x + 1)/(c^2*d^3*e + c*e^4) - (\\ &2*c*d^2 - c^(2/3)*d*e - c^(1/3)*e^2)*log(c^(2/3)*x^2 - c^(1/3)*x + 1)/(c^2* \\ &d^3*e - c*e^4) - 2*(c*d^2 + c^(2/3)*d*e + c^(1/3)*e^2)*log((c^(1/3)*x + 1)/ \\ &c^(1/3))/((c^2*d^3*e - c*e^4) + 2*(c*d^2 - c^(2/3)*d*e + c^(1/3)*e^2)*log((c \\ &^(1/3)*x - 1)/c^(1/3)))/(c^2*d^3*e + c*e^4))*c + 4*arctanh(c*x^3)/(x*e^2 + d \\ &*e))*b - a/(x*e^2 + d*e) \end{aligned}$$

Fricas [C] Result contains complex when optimal does not.

time = 84.38, size = 36636, normalized size = 88.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^3))/(e*x+d)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} &-1/16*(16*a*c^2*d^6 - 16*a*\cosh(1)^6 - 96*a*\cosh(1)^5*\sinh(1) - 240*a*\cosh(\\ &1)^4*\sinh(1)^2 - 320*a*\cosh(1)^3*\sinh(1)^3 - 240*a*\cosh(1)^2*\sinh(1)^4 - 96 \\ &*a*\cosh(1)*\sinh(1)^5 - 16*a*\sinh(1)^6 + 2*(c^2*d^6*x*\cosh(1)^2 + c^2*d^7*co \\ &sh(1) - x*\cosh(1)^8 - x*\sinh(1)^8 - d*\cosh(1)^7 - (8*x*\cosh(1) + d)*\sinh(1) \\ &^7 - 7*(4*x*\cosh(1)^2 + d*\cosh(1))*\sinh(1)^6 - 7*(8*x*\cosh(1)^3 + 3*d*\cosh(\\ &1)^2)*\sinh(1)^5 - 35*(2*x*\cosh(1)^4 + d*\cosh(1)^3)*\sinh(1)^4 - 7*(8*x*\cosh(\\ &1)^5 + 5*d*\cosh(1)^4)*\sinh(1)^3 + (c^2*d^6*x - 28*x*\cosh(1)^6 - 21*d*\cosh(1) \\ &^5)*\sinh(1)^2 + (2*c^2*d^6*x*\cosh(1) + c^2*d^7 - 8*x*\cosh(1)^7 - 7*d*\cosh(\\ &1)^6)*\sinh(1))*((-I*sqrt(3) + 1)*((b*c*d^2*cosh(1) - b*c*d^2*sinh(1))^2/(c* \\ &d^3 + cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)^2 \\ &- (b^2*c*d*cosh(1)^2 - 2*b^2*c*d*cosh(1)*sinh(1) + b^2*c*d*sinh(1)^2)/(c*d^ \\ &3 + cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3))/(-1 \\ &/16*(cosh(1)^2 - sinh(1)^2)^3*b^3*c/(c*d^3 + cosh(1)^3 + 3*cosh(1)^2*sinh(1) \\ &) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)^2 - 1/8*(b*c*d^2*cosh(1) - b*c*d^2*sin \end{aligned}$$

$$\begin{aligned}
& h(1))^3/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^3 + 3/16*(b^2*c*d*\cosh(1)^2 - 2*b^2*c*d*\cosh(1)*\sinh(1) + b^2*c*d*\sinh(1)^2)*(b*c*d^2*\cosh(1) - b*c*d^2*\sinh(1))/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^2 - 1/16*(b^3*c*\cosh(1)^3 - 3*b^3*c*\cosh(1)^2*\sinh(1) + 3*b^3*c*\cosh(1)*\sinh(1)^2 - b^3*c*\sinh(1)^3)/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3))^{(1/3)} + 4*(-1/16*(\cosh(1)^2 - \sinh(1)^2)^3*b^3*c/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^2 - 1/8*(b*c*d^2*\cosh(1) - b*c*d^2*\sinh(1))^3/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^3 + 3/16*(b^2*c*d*\cosh(1)^2 - 2*b^2*c*d*\cosh(1)*\sinh(1) + b^2*c*d*\sinh(1)^2)*(b*c*d^2*\cosh(1) - b*c*d^2*\sinh(1))/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^2 - 1/16*(b^3*c*\cosh(1)^3 - 3*b^3*c*\cosh(1)^2*\sinh(1) + 3*b^3*c*\cosh(1)*\sinh(1)^2 - b^3*c*\sinh(1)^3)/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3))^{(1/3)}*(I*\sqrt{3} + 1) + 4*(b*c*d^2*\cosh(1) - b*c*d^2*\sinh(1))/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3))*\log(b^2*c*x*\cosh(1) + b^2*c*x*\sinh(1) - 2*b^2*c*d - 1/16*(c*d^3*\cosh(1)^2 + \cosh(1)^5 + 10*\cosh(1)^2*\sinh(1)^3 + 5*\cosh(1)*\sinh(1)^4 + \sinh(1)^5 + (c*d^3 + 10*\cosh(1)^3)*\sinh(1)^2 + (2*c*d^3*\cosh(1) + 5*\cosh(1)^4)*\sinh(1))*((-I*\sqrt{3} + 1))*((b*c*d^2*\cosh(1) - b*c*d^2*\sinh(1))^2/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^2 - (b^2*c*d*\cosh(1)^2 - 2*b^2*c*d*\cosh(1)*\sinh(1) + b^2*c*d*\sinh(1)^2)/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3))/(-1/16*(\cosh(1)^2 - \sinh(1)^2)^3*b^3*c/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^2 - 1/8*(b*c*d^2*\cosh(1) - b*c*d^2*\sinh(1))^3/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^3 + 3/16*(b^2*c*d*\cosh(1)^2 - 2*b^2*c*d*\cosh(1)*\sinh(1) + b^2*c*d*\sinh(1)^2)*(b*c*d^2*\cosh(1) - b*c*d^2*\sinh(1))/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^2 - 1/16*(b^3*c*\cosh(1)^3 - 3*b^3*c*\cosh(1)^2*\sinh(1) + 3*b^3*c*\cosh(1)*\sinh(1)^2 - b^3*c*\sinh(1)^3)/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3))^{(1/3)} + 4*(-1/16*(\cosh(1)^2 - \sinh(1)^2)^3*b^3*c/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^2 - 1/8*(b*c*d^2*\cosh(1) - b*c*d^2*\sinh(1))^3/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^3 + 3/16*(b^2*c*d*\cosh(1)^2 - 2*b^2*c*d*\cosh(1)*\sinh(1) + b^2*c*d*\sinh(1)^2)*(b*c*d^2*\cosh(1) - b*c*d^2*\sinh(1))/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^2 - 1/16*(b^3*c*\cosh(1)^3 - 3*b^3*c*\cosh(1)^2*\sinh(1) + 3*b^3*c*\cosh(1)*\sinh(1)^2 - b^3*c*\sinh(1)^3)/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3))^{(1/3)}*(I*\sqrt{3} + 1) + 4*(b*c*d^2*\cosh(1) - b*c*d^2*\sinh(1))/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3))^2 + 3/4*(b*c*d^2*\cosh(1) + b*c*d^2*\sinh(1))*((-I*\sqrt{3} + 1))*((b*c*d^2*\cosh(1) - b*c*d^2*\sinh(1))^2/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^2 - (b^2*c*d*\cosh(1)^2 - 2*b^2*c*d*\cosh(1)*\sinh(1) + b^2*c*d*\sinh(1)^2)/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3))
\end{aligned}$$

$$\begin{aligned} &^2 + \sinh(1)^3)/(-1/16*(\cosh(1)^2 - \sinh(1)^2)^3*b^3*c/(c*d^3 + \cosh(1)^3 \\ &+ 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^2 - 1/8*(b*c*d^2*c \\ &osh(1) - b*c*d^2*\sinh(1))^3/(c*d^3 + \cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*c \\ &sh(1)*\sinh(1)^2 + \sinh(1)^3)^3 + 3/16*(b^2*c*d*cosh(1)^2 - 2*b^2*c*d*cosh(1) \\ &)*\sinh(1) + b^2*c*d*\sinh(1)^2)*(b*c*d^2*cosh(1) - b*c*d^2*\sinh(1))/(c*d^3 + \\ &cosh(1)^3 + 3*\cosh(1)^2*\sinh(1) + 3*\cosh(1)*\sinh(1)^2 + \sinh(1)^3)^2 - 1/1 \\ &6*(b^3*c*cosh(1)^3 - 3*b^3*c*cosh(1)^2*\sinh(1) + 3*b^3*c*cosh(1)*\sinh(1)^2 \\ &- b^3*c*\sinh(1)^3)/(c*d^3 + \cosh(1)^3 + 3*\cosh(\dots \end{aligned}$$

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**3))/(e*x+d)**2,x)

[Out] Timed out

Giac [A]

time = 14.89, size = 554, normalized size = 1.34

$$\frac{3 \operatorname{Re}(\sqrt{3} \log(x+d)) - \operatorname{Re}(\sqrt{3} \log(x^2-1)) - \operatorname{Re}(\sqrt{3} \log(x^2-1))}{2(c^2 d^2 + c^2 d^2)} - \frac{\sqrt{3} \operatorname{Re}(\operatorname{arctan}(\frac{1}{\sqrt{3}} \sqrt{x^2 + \frac{1}{c}}))}{2(c^2 d^2 + c^2 d^2)} - \frac{\sqrt{3} \operatorname{Re}(\operatorname{arctan}(\frac{\sqrt{3}(x+1)^{1/3}}{2(1-x)})}{2(c^2 d^2 + c^2 d^2)} + \frac{\operatorname{Re}(\sqrt{3} \log(x^2-1) - \sqrt{3} \log(x^2-1) + \sqrt{3} \log(x^2-1))}{2(c^2 d^2 + c^2 d^2)} - \frac{\sqrt{3} \operatorname{Re}(\log(x^2-1))}{2(c^2 d^2 + c^2 d^2)} + \frac{(-c)^2 \operatorname{Re}(-(-c)^2 \log(x^2+x(-1)^{1/3} + (-1)^2))}{4(c^2 d^2 + c^2 d^2)} + \frac{(\operatorname{Re}(\sqrt{3} \log(x^2 + \frac{1}{c})))}{2(c^2 d^2 + c^2 d^2)} + \frac{\operatorname{Re}(\frac{-\sqrt{3} \log(x^2-1)}{2(c^2 d^2 + c^2 d^2))}{2(c^2 d^2 + c^2 d^2)} + \frac{\operatorname{Re}(\sqrt{3} \log(x^2-1) - \sqrt{3} \log(x^2-1) - \sqrt{3} \log(x^2-1))}{2(c^2 d^2 + c^2 d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^3))/(e*x+d)^2,x, algorithm="giac")

[Out] $-3*b*c*d^2*e^2*\log(e*x + d)/(c^2*d^6 - e^6) - 1/2*b*c*d^2*\log(\operatorname{abs}(c*x^3 - 1))/(c*d^3*e + e^4) + 1/2*b*c*d^2*\log(\operatorname{abs}(-c*x^3 - 1))/(c*d^3*e - e^4) - 1/2*\sqrt{3}*b*c*\operatorname{abs}(c)^{(2/3)}*\operatorname{arctan}(1/3*\sqrt{3}*c^{(1/3)}*(2*x + 1/c^{(1/3)}))/(c^2*d^2 - c*d*e*\operatorname{abs}(c)^{(2/3)} + e^2*\operatorname{abs}(c)^{(4/3)}) - 1/2*\sqrt{3}*b*c*\operatorname{arctan}(1/3*\sqrt{3}*(2*x + (-1/c)^{(1/3)}))/((-1/c)^{(1/3)})/(c*d*e + (-c^2)^{(2/3)}*d^2 - (-c^2)^{(1/3)}*e^2) + 1/2*(b*c^3*d^3*e^3*(-1/c)^{(1/3)} - b*c^3*d^4*e^2 - b*c^2*e^6*(-1/c)^{(1/3)} + b*c^2*d*e^5)*(-1/c)^{(1/3)}*\log(\operatorname{abs}(x - (-1/c)^{(1/3)}))/(c^3*d^6*e^2 - 2*c^2*d^3*e^5 + c*e^8) + 1/4*((-c^2)^{(1/3)}*b*c*d - (-c^2)^{(2/3)}*b*e)*\log(x^2 + x*(-1/c)^{(1/3)} + (-1/c)^{(2/3)})/(c^2*d^3 - c*e^3) - 1/4*(b*c*d*\operatorname{abs}(c)^{(2/3)} - b*e*\operatorname{abs}(c)^{(4/3)})*\log(x^2 + x/c^{(1/3)} + 1/c^{(2/3)})/(c^2*d^3 + c*e^3) - 1/2*b*\log(-(c*x^3 + 1)/(c*x^3 - 1))/(e^2*x + d*e) + 1/2*(b*c^3*d^4*e^2 - b*c^{(8/3)}*d^3*e^3 + b*c^2*d*e^5 - b*c^{(5/3)}*e^6)*\log(\operatorname{abs}(x - 1/c^{(1/3)}))/((c^3*d^6*e^2 + 2*c^2*d^3*e^5 + c*e^8)*c^{(1/3)}) - a/(e^2*x + d*e)$

Mupad [B]

time = 1.41, size = 2638, normalized size = 6.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& *d^2e^2z^2 + 6b^2cde^2z + b^3c, z, k)^5 * b^5 c^{15} d^9 e^4 + 12636 * \text{root}(8 \\
& *c*d^3e^3z^3 + 8e^6z^3 + 12b*c*d^2e^2z^2 + 6b^2*c*d*e*z + b^3*c, z, \\
& k)^3 * b^3 c^{13} e^9 x + 38880 * \text{root}(8*c*d^3e^3z^3 + 8e^6z^3 + 12*b*c*d^2* \\
& e^2z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^6 * c^{14} d^6 e^9 x + 116640 * \text{root}(8*c*d \\
& ^3e^3z^3 + 8e^6z^3 + 12*b*c*d^2e^2z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^5 \\
& * b^5 c^{13} d^2 e^{11} x + 11664 * \text{root}(8*c*d^3e^3z^3 + 8e^6z^3 + 12*b*c*d^2e \\
& ^2z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^5 * b^5 c^{15} d^8 e^5 x - 11664 * \text{root}(8*c*d \\
& ^3e^3z^3 + 8e^6z^3 + 12*b*c*d^2e^2z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^2 \\
& * b^4 c^{14} d^2 e^5 x - 3888 * \text{root}(8*c*d^3e^3z^3 + 8e^6z^3 + 12*b*c*d^2e \\
& ^2z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^3 * b^3 c^{15} d^6 e^3 x + 38880 * \text{root}(8*c \\
& *d^3e^3z^3 + 8e^6z^3 + 12*b*c*d^2e^2z^2 + 6*b^2*c*d*e*z + b^3*c, z, k) \\
&)^4 * b^2 c^{14} d^4 e^7 x + 243 * \text{root}(8*c*d^3e^3z^3 + 8e^6z^3 + 12*b*c*d^2* \\
& e^2z^2 + 6*b^2*c*d*e*z + b^3*c, z, k) * b^5 c^{15} d^4 e^4 x / e^4 * \text{root}(8*c*d^3* \\
& e^3z^3 + 8e^6z^3 + 12*b*c*d^2e^2z^2 + 6*b^2*c*d*e*z + b^3*c, z, k), k, \\
& 1, 3) - a/(d*e + e^2*x) - (b*log(c*x^3 + 1))/(2*(d*e + e^2*x)) + (b*log(1 \\
& - c*x^3))/(2*d*e + 2*e^2*x) + (3*b*c*d^2e^2*log(d + e*x))/(e^6 - c^2*d^6)
\end{aligned}$$

$$3.36 \quad \int \frac{x^3 \left(a + b \tanh^{-1} \left(c \sqrt{x} \right) \right)}{1 - c^2 x} dx$$

Optimal. Leaf size=195

$$-\frac{11b\sqrt{x}}{6c^7} - \frac{5bx^{3/2}}{18c^5} - \frac{bx^{5/2}}{15c^3} + \frac{11b \tanh^{-1}(c\sqrt{x})}{6c^8} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^6} - \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2c^4} - \frac{x^3(a + b \tanh^{-1}(c\sqrt{x}))}{c^2}$$

[Out] $-5/18*b*x^{(3/2)}/c^5 - 1/15*b*x^{(5/2)}/c^3 + 11/6*b*\operatorname{arctanh}(c*x^{(1/2)})/c^8 - x*(a + b*\operatorname{arctanh}(c*x^{(1/2)}))/c^6 - 1/2*x^2*(a + b*\operatorname{arctanh}(c*x^{(1/2)}))/c^4 - 1/3*x^3*(a + b*\operatorname{arctanh}(c*x^{(1/2)}))/c^2 - (a + b*\operatorname{arctanh}(c*x^{(1/2)}))^2/b/c^8 + 2*(a + b*\operatorname{arctanh}(c*x^{(1/2)}))*\ln(2/(1 - c*x^{(1/2)}))/c^8 + b*\operatorname{polylog}(2, 1 - 2/(1 - c*x^{(1/2)}))/c^8 - 11/6*b*x^{(1/2)}/c^7$

Rubi [A]

time = 0.42, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {45, 6127, 6037, 308, 212, 327, 6131, 6055, 2449, 2352}

$$-\frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{6c^8} + \frac{2 \log\left(\frac{2}{1 - c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x}))}{c^8} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^6} - \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2c^4} - \frac{x^3(a + b \tanh^{-1}(c\sqrt{x}))}{3c^2} + \frac{b \operatorname{Li}_2\left(1 - \frac{2}{1 - c\sqrt{x}}\right)}{c^8} + \frac{11b \tanh^{-1}(c\sqrt{x})}{6c^8} - \frac{11b\sqrt{x}}{6c^7} - \frac{5bx^{3/2}}{18c^5} - \frac{bx^{5/2}}{15c^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^3*(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]]))/(1 - c^2*x), x]$

[Out] $(-11*b*\operatorname{Sqrt}[x])/(6*c^7) - (5*b*x^{(3/2)})/(18*c^5) - (b*x^{(5/2)})/(15*c^3) + (11*b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])/(6*c^8) - (x*(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]]))/c^6 - (x^2*(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]]))/(2*c^4) - (x^3*(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]]))/(3*c^2) - (a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])^2/(b*c^8) + (2*(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])*\operatorname{Log}[2/(1 - c*\operatorname{Sqrt}[x])])/c^8 + (b*\operatorname{PolyLog}[2, 1 - 2/(1 - c*\operatorname{Sqrt}[x])])/c^8$

Rule 45

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n, x_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 212

$\operatorname{Int}[(a + b*x^2)^{-1}, x_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 308

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 327

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2352

Int[Log[(c_)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2449

Int[Log[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] := Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 6037

Int[((a_) + ArcTanh[(c_)*(x_)^(n_)])*(b_)^(p_)*(x_)^(m_), x_Symbol] := Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c^n*(p/(m + 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]

Rule 6055

Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)), x_Symbol] := Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 6127

Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)*((f_)*(x_)^(m_))/((d_) + (e_)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

Rule 6131

Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3(a + b \tanh^{-1}(c\sqrt{x}))}{1 - c^2x} dx &= 2\text{Subst}\left(\int \frac{x^7(a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx, x, \sqrt{x}\right) \\
 &= -\frac{2\text{Subst}\left(\int x^5(a + b \tanh^{-1}(cx)) dx, x, \sqrt{x}\right)}{c^2} + \frac{2\text{Subst}\left(\int \frac{x^5(a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx, x, \sqrt{x}\right)}{c^2} \\
 &= -\frac{x^3(a + b \tanh^{-1}(c\sqrt{x}))}{3c^2} - \frac{2\text{Subst}\left(\int x^3(a + b \tanh^{-1}(cx)) dx, x, \sqrt{x}\right)}{c^4} \\
 &= -\frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2c^4} - \frac{x^3(a + b \tanh^{-1}(c\sqrt{x}))}{3c^2} - \frac{2\text{Subst}\left(\int x(a + b \tanh^{-1}(cx)) dx, x, \sqrt{x}\right)}{c^6} \\
 &= -\frac{b\sqrt{x}}{3c^7} - \frac{bx^{3/2}}{9c^5} - \frac{bx^{5/2}}{15c^3} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^6} - \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2c^4} \\
 &= -\frac{11b\sqrt{x}}{6c^7} - \frac{5bx^{3/2}}{18c^5} - \frac{bx^{5/2}}{15c^3} + \frac{b \tanh^{-1}(c\sqrt{x})}{3c^8} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^6} \\
 &= -\frac{11b\sqrt{x}}{6c^7} - \frac{5bx^{3/2}}{18c^5} - \frac{bx^{5/2}}{15c^3} + \frac{11b \tanh^{-1}(c\sqrt{x})}{6c^8} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^6} \\
 &= -\frac{11b\sqrt{x}}{6c^7} - \frac{5bx^{3/2}}{18c^5} - \frac{bx^{5/2}}{15c^3} + \frac{11b \tanh^{-1}(c\sqrt{x})}{6c^8} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^6}
 \end{aligned}$$

Mathematica [A]

time = 0.48, size = 160, normalized size = 0.82

$$\frac{165bc\sqrt{x} + 90ac^2x + 25b^2c^3x^{3/2} + 45ac^4x^2 + 6b^2c^5x^{5/2} + 30ac^6x^3 - 90b \tanh^{-1}(c\sqrt{x})^2 + 15b \tanh^{-1}(c\sqrt{x}) \left(-11 + 6c^2x + 3c^4x^2 + 2c^6x^3 - 12 \log\left(1 + e^{-2 \tanh^{-1}(c\sqrt{x})}\right) \right) + 90a \log(1 - c^2x) + 90b \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(c\sqrt{x})}\right)}{90c^8}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(a + b*ArcTanh[c*Sqrt[x]]))/(1 - c^2*x), x]

[Out] -1/90*(165*b*c*Sqrt[x] + 90*a*c^2*x + 25*b*c^3*x^(3/2) + 45*a*c^4*x^2 + 6*b*c^5*x^(5/2) + 30*a*c^6*x^3 - 90*b*ArcTanh[c*Sqrt[x]]^2 + 15*b*ArcTanh[c*Sq

```
rt[x]]*(-11 + 6*c^2*x + 3*c^4*x^2 + 2*c^6*x^3 - 12*Log[1 + E^(-2*ArcTanh[c*
Sqrt[x]])]) + 90*a*Log[1 - c^2*x] + 90*b*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x
]])])/c^8
```

Maple [A]

time = 0.46, size = 277, normalized size = 1.42

method	result
derivativedivides	$2 \left(\frac{a c^6 x^3}{6} + \frac{a c^4 x^2}{4} + \frac{a c^2 x}{2} + \frac{a \ln(c \sqrt{x} - 1)}{2} + \frac{a \ln(1 + c \sqrt{x})}{2} + \frac{b \operatorname{arctanh}(c \sqrt{x}) c^6 x^3}{6} + \frac{b \operatorname{arctanh}(c \sqrt{x}) c^4 x^2}{4} + \frac{b \operatorname{arctanh}(c \sqrt{x}) c^2 x}{2} \right)$
default	$2 \left(\frac{a c^6 x^3}{6} + \frac{a c^4 x^2}{4} + \frac{a c^2 x}{2} + \frac{a \ln(c \sqrt{x} - 1)}{2} + \frac{a \ln(1 + c \sqrt{x})}{2} + \frac{b \operatorname{arctanh}(c \sqrt{x}) c^6 x^3}{6} + \frac{b \operatorname{arctanh}(c \sqrt{x}) c^4 x^2}{4} + \frac{b \operatorname{arctanh}(c \sqrt{x}) c^2 x}{2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^8*(1/6*a*c^6*x^3+1/4*a*c^4*x^2+1/2*a*c^2*x+1/2*a*ln(c*x^(1/2)-1)+1/2*a
*ln(1+c*x^(1/2))+1/6*b*arctanh(c*x^(1/2))*c^6*x^3+1/4*b*arctanh(c*x^(1/2))*
c^4*x^2+1/2*b*arctanh(c*x^(1/2))*c^2*x+1/2*b*arctanh(c*x^(1/2))*ln(c*x^(1/2
)-1)+1/2*b*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/2*b*dilog(1/2*c*x^(1/2)+1/2
)-1/4*b*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)+1/8*b*ln(c*x^(1/2)-1)^2+1/4*b
*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))-1/4*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2
*c*x^(1/2)+1/2)-1/8*b*ln(1+c*x^(1/2))^2+1/30*b*c^5*x^(5/2)+5/36*b*c^3*x^(3/
2)+11/12*b*c*x^(1/2)+11/24*b*ln(c*x^(1/2)-1)-11/24*b*ln(1+c*x^(1/2)))
```

Maxima [A]

time = 0.41, size = 246, normalized size = 1.26

$$\frac{1}{80} \left(\frac{2c^6x^3 + 3c^4x^2 + 6cx}{c^8} - \frac{6 \log(c^2x - 1)}{c^8} - \frac{\log(c\sqrt{x} + 1) \log(-1/2c\sqrt{x} + 1/2) + \operatorname{dilog}(1/2c\sqrt{x} + 1/2)}{c^8} + \frac{11 \log(c\sqrt{x} + 1)}{12c^8} - \frac{11 \log(c\sqrt{x} - 1)}{12c^8} - \frac{12bc^5x^{5/2} + 50bc^3x^{3/2} + 45b \log(c\sqrt{x} + 1)^2 - 45b \log(-c\sqrt{x} + 1)^2 + 330bc\sqrt{x} + 15(2b^2c^6x^3 + 3b^2c^4x^2 + 6b^2c^2x + 6b^2)}{180c^8} \log(c\sqrt{x} + 1) - 15(2b^2c^6x^3 + 3b^2c^4x^2 + 6b^2c^2x + 6b^2)}{180c^8} \log(-c\sqrt{x} + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="maxima")
```

```
[Out] -1/6*a*((2*c^4*x^3 + 3*c^2*x^2 + 6*x)/c^6 + 6*log(c^2*x - 1)/c^8) - (log(c*
sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2))*b/c^8
+ 11/12*b*log(c*sqrt(x) + 1)/c^8 - 11/12*b*log(c*sqrt(x) - 1)/c^8 - 1/180*(
12*b*c^5*x^(5/2) + 50*b*c^3*x^(3/2) + 45*b*log(c*sqrt(x) + 1)^2 - 45*b*log(
-c*sqrt(x) + 1)^2 + 330*b*c*sqrt(x) + 15*(2*b*c^6*x^3 + 3*b*c^4*x^2 + 6*b*c
^2*x)*log(c*sqrt(x) + 1) - 15*(2*b*c^6*x^3 + 3*b*c^4*x^2 + 6*b*c^2*x + 6*b*
log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1))/c^8
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^3*arctanh(c*sqrt(x)) + a*x^3)/(c^2*x - 1), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax^3}{c^2x-1} dx - \int \frac{bx^3 \operatorname{atanh}(c\sqrt{x})}{c^2x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(1/2)))/(-c**2*x+1),x)
```

```
[Out] -Integral(a*x**3/(c**2*x - 1), x) - Integral(b*x**3*atanh(c*sqrt(x))/(c**2*x - 1), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)*x^3/(c^2*x - 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3 (a + b \operatorname{atanh}(c \sqrt{x}))}{c^2 x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a + b*atanh(c*x^(1/2))))/(c^2*x - 1),x)
```

```
[Out] int(-(x^3*(a + b*atanh(c*x^(1/2))))/(c^2*x - 1), x)
```

$$3.37 \quad \int \frac{x^2 \left(a + b \tanh^{-1} \left(c \sqrt{x} \right) \right)}{1 - c^2 x} dx$$

Optimal. Leaf size=160

$$-\frac{3b\sqrt{x}}{2c^5} - \frac{bx^{3/2}}{6c^3} + \frac{3b \tanh^{-1}(c\sqrt{x})}{2c^6} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^4} - \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2c^2} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))}{bc^6}$$

[Out] $-1/6*b*x^{(3/2)}/c^3+3/2*b*\operatorname{arctanh}(c*x^{(1/2)})/c^6-x*(a+b*\operatorname{arctanh}(c*x^{(1/2)}))/c^4-1/2*x^2*(a+b*\operatorname{arctanh}(c*x^{(1/2)}))/c^2-(a+b*\operatorname{arctanh}(c*x^{(1/2)}))^2/b/c^6+2*(a+b*\operatorname{arctanh}(c*x^{(1/2)}))*\ln(2/(1-c*x^{(1/2)}))/c^6+b*\operatorname{polylog}(2,1-2/(1-c*x^{(1/2)}))/c^6-3/2*b*x^{(1/2)}/c^5$

Rubi [A]

time = 0.30, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {45, 6127, 6037, 308, 212, 327, 6131, 6055, 2449, 2352}

$$-\frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{bc^6} + \frac{2 \log\left(\frac{2}{1-c\sqrt{x}}\right)(a + b \tanh^{-1}(c\sqrt{x}))}{c^6} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^4} - \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2c^2} + \frac{b \operatorname{Li}_2\left(1 - \frac{2}{1-c\sqrt{x}}\right)}{c^6} + \frac{3b \tanh^{-1}(c\sqrt{x})}{2c^6} - \frac{3b\sqrt{x}}{2c^5} - \frac{bx^{3/2}}{6c^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^2*(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]]))/(1 - c^2*x), x]$

[Out] $(-3*b*\operatorname{Sqrt}[x])/(2*c^5) - (b*x^{(3/2)})/(6*c^3) + (3*b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])/(2*c^6) - (x*(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]]))/c^4 - (x^2*(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]]))/(2*c^2) - (a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])^2/(b*c^6) + (2*(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])*\operatorname{Log}[2/(1 - c*\operatorname{Sqrt}[x])])/c^6 + (b*\operatorname{PolyLog}[2, 1 - 2/(1 - c*\operatorname{Sqrt}[x])])/c^6$

Rule 45

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 212

$\operatorname{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 308

$\operatorname{Int}(x^m/((a + b*x)^n), x_Symbol) \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[x^m, a + b*x^n, x], x] /;$ FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt

$Q[m, 2*n - 1]$

Rule 327

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[c^{(n-1)}(c*x)^{(m-n+1)}((a + b*x^n)^{(p+1)}/(b*(m+n*p+1))), x] - \text{Dist}[a*c^n*((m-n+1)/(b*(m+n*p+1))), \text{Int}[(c*x)^{(m-n)}(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m+n*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2352

$\text{Int}[\text{Log}[(c_*)(x_*)]/((d_*) + (e_*)(x_*)), x_Symbol] \rightarrow \text{Simp}[(-e^{(-1)})*PolyLog[2, 1 - c*x], x] /; \text{FreeQ}\{c, d, e\}, x \ \&\& \ \text{EqQ}[e + c*d, 0]$

Rule 2449

$\text{Int}[\text{Log}[(c_*)/((d_*) + (e_*)(x_*))]/((f_*) + (g_*)(x_*)^2), x_Symbol] \rightarrow \text{Dist}[-e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x \ \&\& \ \text{EqQ}[c, 2*d] \ \&\& \ \text{EqQ}[e^2*f + d^2*g, 0]$

Rule 6037

$\text{Int}[(a_*) + \text{ArcTanh}[(c_*)(x_*)^{(n_*)}](b_*)^{(p_*)}(x_*)^{(m_*)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}((a + b*\text{ArcTanh}[c*x^n])^p/(m+1)), x] - \text{Dist}[b*c*n*(p/(m+1)), \text{Int}[x^{(m+n)}((a + b*\text{ArcTanh}[c*x^n])^{(p-1)})/(1 - c^2*x^{(2*n)}), x], x] /; \text{FreeQ}\{a, b, c, m, n\}, x \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{EqQ}[p, 1] \ || \ (\text{EqQ}[n, 1] \ \&\& \ \text{IntegerQ}[m])) \ \&\& \ \text{NeQ}[m, -1]$

Rule 6055

$\text{Int}[(a_*) + \text{ArcTanh}[(c_*)(x_*)](b_*)^{(p_*)}/((d_*) + (e_*)(x_*)), x_Symbol] \rightarrow \text{Simp}[(-a + b*\text{ArcTanh}[c*x])^p*(\text{Log}[2/(1 + e*(x/d))]/e), x] + \text{Dist}[b*c*(p/e), \text{Int}[(a + b*\text{ArcTanh}[c*x])^{(p-1)}*(\text{Log}[2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{EqQ}[c^2*d^2 - e^2, 0]$

Rule 6127

$\text{Int}[(a_*) + \text{ArcTanh}[(c_*)(x_*)](b_*)^{(p_*)}((f_*)(x_*)^{(m_*)})/((d_*) + (e_*)(x_*)^2), x_Symbol] \rightarrow \text{Dist}[f^2/e, \text{Int}[(f*x)^{(m-2)}(a + b*\text{ArcTanh}[c*x])^p, x], x] - \text{Dist}[d*(f^2/e), \text{Int}[(f*x)^{(m-2)}((a + b*\text{ArcTanh}[c*x])^p/(d + e*x^2)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{GtQ}[m, 1]$

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.))/((d_.) + (e_.)*(x_.)^2),
  x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{1 - c^2x} dx &= 2\text{Subst}\left(\int \frac{x^5(a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx, x, \sqrt{x}\right) \\
&= -\frac{2\text{Subst}\left(\int x^3(a + b \tanh^{-1}(cx)) dx, x, \sqrt{x}\right)}{c^2} + \frac{2\text{Subst}\left(\int \frac{x^3(a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx, x, \sqrt{x}\right)}{c^2} \\
&= -\frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2c^2} - \frac{2\text{Subst}\left(\int x(a + b \tanh^{-1}(cx)) dx, x, \sqrt{x}\right)}{c^4} \\
&= -\frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^4} - \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2c^2} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))}{bc^6} \\
&= -\frac{3b\sqrt{x}}{2c^5} - \frac{bx^{3/2}}{6c^3} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^4} - \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2c^2} \\
&= -\frac{3b\sqrt{x}}{2c^5} - \frac{bx^{3/2}}{6c^3} + \frac{3b \tanh^{-1}(c\sqrt{x})}{2c^6} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^4} - \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2c^2} \\
&= -\frac{3b\sqrt{x}}{2c^5} - \frac{bx^{3/2}}{6c^3} + \frac{3b \tanh^{-1}(c\sqrt{x})}{2c^6} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^4} - \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2c^2}
\end{aligned}$$

Mathematica [A]

time = 0.29, size = 130, normalized size = 0.81

$$\frac{9bc\sqrt{x} + 6ac^2x + bc^3x^{3/2} + 3ac^4x^2 - 6b \tanh^{-1}(c\sqrt{x})^2 + 3b \tanh^{-1}(c\sqrt{x}) \left(-3 + 2c^2x + c^4x^2 - 4 \log\left(1 + e^{-2 \tanh^{-1}(c\sqrt{x})}\right)\right) + 6a \log(1 - c^2x) + 6b \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(c\sqrt{x})}\right)}{6c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*Sqrt[x]]))/(1 - c^2*x), x]
```

```
[Out] -1/6*(9*b*c*Sqrt[x] + 6*a*c^2*x + b*c^3*x^(3/2) + 3*a*c^4*x^2 - 6*b*ArcTanh
[c*Sqrt[x]]^2 + 3*b*ArcTanh[c*Sqrt[x]]*(-3 + 2*c^2*x + c^4*x^2 - 4*Log[1 +
E^(-2*ArcTanh[c*Sqrt[x]])]) + 6*a*Log[1 - c^2*x] + 6*b*PolyLog[2, -E^(-2*Ar
cTanh[c*Sqrt[x]])])/c^6
```

Maple [A]

time = 0.42, size = 244, normalized size = 1.52

method	result
derivativedivides	$2 \left(\frac{a c^4 x^2}{4} + \frac{a c^2 x}{2} + \frac{a \ln(c\sqrt{x} - 1)}{2} + \frac{a \ln(1 + c\sqrt{x})}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) c^4 x^2}{4} + \frac{b \operatorname{arctanh}(c\sqrt{x}) c^2 x}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x})}{2} \right)$
default	$2 \left(\frac{a c^4 x^2}{4} + \frac{a c^2 x}{2} + \frac{a \ln(c\sqrt{x} - 1)}{2} + \frac{a \ln(1 + c\sqrt{x})}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) c^4 x^2}{4} + \frac{b \operatorname{arctanh}(c\sqrt{x}) c^2 x}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x})}{2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x,method=_RETURNVERBOSE)`

[Out] $-2/c^6*(1/4*a*c^4*x^2+1/2*a*c^2*x+1/2*a*\ln(c*x^{(1/2)}-1)+1/2*a*\ln(1+c*x^{(1/2)}))+1/4*b*\operatorname{arctanh}(c*x^{(1/2)})*c^4*x^2+1/2*b*\operatorname{arctanh}(c*x^{(1/2)})*c^2*x+1/2*b*\operatorname{arctanh}(c*x^{(1/2)})*\ln(c*x^{(1/2)}-1)+1/2*b*\operatorname{arctanh}(c*x^{(1/2)})*\ln(1+c*x^{(1/2)})-1/2*b*\operatorname{dilog}(1/2*c*x^{(1/2)}+1/2)-1/4*b*\ln(c*x^{(1/2)}-1)*\ln(1/2*c*x^{(1/2)}+1/2)+1/8*b*\ln(c*x^{(1/2)}-1)^2+1/4*b*\ln(-1/2*c*x^{(1/2)}+1/2)*\ln(1+c*x^{(1/2)})-1/4*b*\ln(-1/2*c*x^{(1/2)}+1/2)*\ln(1/2*c*x^{(1/2)}+1/2)-1/8*b*\ln(1+c*x^{(1/2)})^2+1/12*b*c^3*x^{(3/2)}+3/4*b*c*x^{(1/2)}+3/8*b*\ln(c*x^{(1/2)}-1)-3/8*b*\ln(1+c*x^{(1/2)})$

Maxima [A]

time = 0.39, size = 208, normalized size = 1.30

$$\frac{1}{2} a \left(\frac{c^2 x^2 + 2x}{c^4} + \frac{2 \log(c^2 x - 1)}{c^6} \right) - \frac{(\log(c\sqrt{x} + 1) \log(-\frac{1}{2}c\sqrt{x} + \frac{1}{2}) + \operatorname{Li}(\frac{1}{2}c\sqrt{x} + \frac{1}{2})) b}{4c^6} + \frac{3b \log(c\sqrt{x} + 1)}{4c^6} - \frac{3b \log(c\sqrt{x} - 1)}{4c^6} - \frac{2bc^2 x^2 + 3b \log(c\sqrt{x} + 1)^2 - 3b \log(-c\sqrt{x} + 1)^2 + 18bc\sqrt{x} + 3(bc^2 x^2 + 2bc^2 x) \log(c\sqrt{x} + 1) - 3(bc^2 x^2 + 2bc^2 x + 2b \log(c\sqrt{x} + 1)) \log(-c\sqrt{x} + 1)}{12c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="maxima")`

[Out] $-1/2*a*((c^2*x^2 + 2*x)/c^4 + 2*\log(c^2*x - 1)/c^6) - (\log(c*\operatorname{sqrt}(x) + 1)*\log(-1/2*c*\operatorname{sqrt}(x) + 1/2) + \operatorname{dilog}(1/2*c*\operatorname{sqrt}(x) + 1/2))*b/c^6 + 3/4*b*\log(c*\operatorname{sqrt}(x) + 1)/c^6 - 3/4*b*\log(c*\operatorname{sqrt}(x) - 1)/c^6 - 1/12*(2*b*c^3*x^{(3/2)} + 3*b*\log(c*\operatorname{sqrt}(x) + 1)^2 - 3*b*\log(-c*\operatorname{sqrt}(x) + 1)^2 + 18*b*c*\operatorname{sqrt}(x) + 3*(b*c^4*x^2 + 2*b*c^2*x)*\log(c*\operatorname{sqrt}(x) + 1) - 3*(b*c^4*x^2 + 2*b*c^2*x + 2*b*\log(c*\operatorname{sqrt}(x) + 1))*\log(-c*\operatorname{sqrt}(x) + 1))/c^6$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="fricas")`

[Out] integral(-(b*x^2*arctanh(c*sqrt(x)) + a*x^2)/(c^2*x - 1), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax^2}{c^2x-1} dx - \int \frac{bx^2 \operatorname{atanh}(c\sqrt{x})}{c^2x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*atanh(c*x**(1/2)))/(-c**2*x+1), x)

[Out] -Integral(a*x**2/(c**2*x - 1), x) - Integral(b*x**2*atanh(c*sqrt(x))/(c**2*x - 1), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1), x, algorithm="giac")

[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)*x^2/(c^2*x - 1), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2 (a + b \operatorname{atanh}(c \sqrt{x}))}{c^2 x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2*(a + b*atanh(c*x^(1/2)))/(-c^2*x - 1), x)

[Out] int(-(x^2*(a + b*atanh(c*x^(1/2)))/(-c^2*x - 1), x)

$$3.38 \quad \int \frac{x \left(a + b \tanh^{-1} \left(c \sqrt{x} \right) \right)}{1 - c^2 x} dx$$

Optimal. Leaf size=120

$$-\frac{b\sqrt{x}}{c^3} + \frac{b \tanh^{-1}(c\sqrt{x})}{c^4} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^2} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{bc^4} + \frac{2(a + b \tanh^{-1}(c\sqrt{x})) \log(2/(1 - c\sqrt{x}))}{c^4}$$

[Out] b*arctanh(c*x^(1/2))/c^4-x*(a+b*arctanh(c*x^(1/2)))/c^2-(a+b*arctanh(c*x^(1/2)))^2/b/c^4+2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^4+b*polylog(2,1-2/(1-c*x^(1/2)))/c^4-b*x^(1/2)/c^3

Rubi [A]

time = 0.19, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {45, 6127, 6037, 327, 212, 6131, 6055, 2449, 2352}

$$-\frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{bc^4} + \frac{2 \log\left(\frac{2}{1 - c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x}))}{c^4} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^2} + \frac{b \text{Li}_2\left(1 - \frac{2}{1 - c\sqrt{x}}\right)}{c^4} + \frac{b \tanh^{-1}(c\sqrt{x})}{c^4} - \frac{b\sqrt{x}}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*ArcTanh[c*sqrt[x]]))/(1 - c^2*x), x]

[Out] -((b*sqrt[x])/c^3) + (b*ArcTanh[c*sqrt[x]]/c^4 - (x*(a + b*ArcTanh[c*sqrt[x]]))/c^2 - (a + b*ArcTanh[c*sqrt[x]])^2/(b*c^4) + (2*(a + b*ArcTanh[c*sqrt[x]])*Log[2/(1 - c*sqrt[x])])/c^4 + (b*PolyLog[2, 1 - 2/(1 - c*sqrt[x])])/c^4

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 212

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 327

Int[((c_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],

$x] /; \text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n - 1] \&\& \text{NeQ}[m + n*p + 1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2352

$\text{Int}[\text{Log}[(c_)*(x_)]/((d_)+(e_)*(x_)), x_Symbol] \rightarrow \text{Simp}[(-e^{(-1)})*PolyLog[2, 1 - c*x], x] /; \text{FreeQ}\{c, d, e\}, x\} \&\& \text{EqQ}[e + c*d, 0]$

Rule 2449

$\text{Int}[\text{Log}[(c_)/((d_)+(e_)*(x_))]/((f_)+(g_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[-e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x\} \&\& \text{EqQ}[c, 2*d] \&\& \text{EqQ}[e^2*f + d^2*g, 0]$

Rule 6037

$\text{Int}[(a_ + \text{ArcTanh}[c_*(x_)^{(n_)}]*(b_))^{(p_)}*(x_)^{(m_)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}*((a + b*\text{ArcTanh}[c*x^n])^p/(m+1)), x] - \text{Dist}[b*c*n*(p/(m+1)), \text{Int}[x^{(m+n)}*((a + b*\text{ArcTanh}[c*x^n])^{(p-1)})/(1 - c^2*x^{(2*n)})), x], x] /; \text{FreeQ}\{a, b, c, m, n\}, x\} \&\& \text{IGtQ}[p, 0] \&\& (\text{EqQ}[p, 1] \|\| (\text{EqQ}[n, 1] \&\& \text{IntegerQ}[m])) \&\& \text{NeQ}[m, -1]$

Rule 6055

$\text{Int}[(a_ + \text{ArcTanh}[c_*(x_)]*(b_))^{(p_)}/((d_)+(e_)*(x_)), x_Symbol] \rightarrow \text{Simp}[(-a + b*\text{ArcTanh}[c*x])^p*(\text{Log}[2/(1 + e*(x/d))]/e), x] + \text{Dist}[b*c*(p/e), \text{Int}[(a + b*\text{ArcTanh}[c*x])^{(p-1)}*(\text{Log}[2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[c^2*d^2 - e^2, 0]$

Rule 6127

$\text{Int}[(((a_ + \text{ArcTanh}[c_*(x_)]*(b_))^{(p_)}*((f_)*(x_)^{(m_)}))/((d_)+(e_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[f^2/e, \text{Int}[(f*x)^{(m-2)}*(a + b*\text{ArcTanh}[c*x])^p, x], x] - \text{Dist}[d*(f^2/e), \text{Int}[(f*x)^{(m-2)}*((a + b*\text{ArcTanh}[c*x])^p/(d + e*x^2)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{GtQ}[p, 0] \&\& \text{GtQ}[m, 1]$

Rule 6131

$\text{Int}[(((a_ + \text{ArcTanh}[c_*(x_)]*(b_))^{(p_)}*(x_))/((d_)+(e_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(a + b*\text{ArcTanh}[c*x])^{(p+1)}/(b*e*(p+1)), x] + \text{Dist}[1/(c*d), \text{Int}[(a + b*\text{ArcTanh}[c*x])^p/(1 - c*x), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{1 - c^2x} dx &= 2\text{Subst}\left(\int \frac{x^3(a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx, x, \sqrt{x}\right) \\
&= -\frac{2\text{Subst}\left(\int x(a + b \tanh^{-1}(cx)) dx, x, \sqrt{x}\right)}{c^2} + \frac{2\text{Subst}\left(\int \frac{x(a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx, x, \sqrt{x}\right)}{c^2} \\
&= -\frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^2} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{bc^4} + \frac{2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{1 - c^2x^2} dx, x, \sqrt{x}\right)}{c^2} \\
&= -\frac{b\sqrt{x}}{c^3} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^2} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{bc^4} + \frac{2(a + b \tanh^{-1}(c\sqrt{x}))}{bc^4} \\
&= -\frac{b\sqrt{x}}{c^3} + \frac{b \tanh^{-1}(c\sqrt{x})}{c^4} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^2} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{bc^4} \\
&= -\frac{b\sqrt{x}}{c^3} + \frac{b \tanh^{-1}(c\sqrt{x})}{c^4} - \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{c^2} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{bc^4}
\end{aligned}$$

Mathematica [A]

time = 0.14, size = 96, normalized size = 0.80

$$\frac{bc\sqrt{x} + ac^2x - b \tanh^{-1}(c\sqrt{x})^2 + b \tanh^{-1}(c\sqrt{x}) \left(-1 + c^2x - 2 \log\left(1 + e^{-2 \tanh^{-1}(c\sqrt{x})}\right)\right) + a \log(1 - c^2x) + b \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(c\sqrt{x})}\right)}{c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*ArcTanh[c*Sqrt[x]]))/(1 - c^2*x), x]

[Out] -((b*c*Sqrt[x] + a*c^2*x - b*ArcTanh[c*Sqrt[x]]^2 + b*ArcTanh[c*Sqrt[x]]*(-1 + c^2*x - 2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])])) + a*Log[1 - c^2*x] + b*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/c^4)

Maple [A]

time = 0.36, size = 211, normalized size = 1.76

method	result
derivativedivides	$ -\frac{2 \left(\frac{ac^2x}{2} + \frac{a \ln(c\sqrt{x} - 1)}{2} + \frac{a \ln(1 + c\sqrt{x})}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x})c^2x}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) \ln(c\sqrt{x} - 1)}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x})^2}{2} \right)}{c^4} $

default	$-\frac{2}{c^2} \left(\frac{a c^2 x}{2} + \frac{a \ln(c\sqrt{x}-1)}{2} + \frac{a \ln(1+c\sqrt{x})}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) c^2 x}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) \ln(c\sqrt{x}-1)}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) \ln(1+c\sqrt{x})}{2} \right)$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x,method=_RETURNVERBOSE)`

[Out] $-2/c^4*(1/2*a*c^2*x+1/2*a*\ln(c*x^(1/2)-1)+1/2*a*\ln(1+c*x^(1/2))+1/2*b*\operatorname{arctanh}(c*x^(1/2))*c^2*x+1/2*b*\operatorname{arctanh}(c*x^(1/2))*\ln(c*x^(1/2)-1)+1/2*b*\operatorname{arctanh}(c*x^(1/2))*\ln(1+c*x^(1/2))+1/2*b*c*x^(1/2)+1/4*b*\ln(c*x^(1/2)-1)-1/4*b*\ln(1+c*x^(1/2))+1/8*b*\ln(c*x^(1/2)-1)^2-1/2*b*\operatorname{dilog}(1/2*c*x^(1/2)+1/2)-1/4*b*\ln(c*x^(1/2)-1)*\ln(1/2*c*x^(1/2)+1/2)-1/8*b*\ln(1+c*x^(1/2))^2-1/4*b*\ln(-1/2*c*x^(1/2)+1/2)*\ln(1/2*c*x^(1/2)+1/2)+1/4*b*\ln(-1/2*c*x^(1/2)+1/2)*\ln(1+c*x^(1/2))$

Maxima [A]

time = 0.40, size = 166, normalized size = 1.38

$$-a\left(\frac{x}{c^2} + \frac{\log(c^2x-1)}{c^4}\right) - \frac{(\log(c\sqrt{x}+1)\log(-\frac{1}{2}c\sqrt{x}+\frac{1}{2}) + \operatorname{Li}_2(\frac{1}{2}c\sqrt{x}+\frac{1}{2}))b}{c^4} + \frac{b\log(c\sqrt{x}+1)}{2c^4} - \frac{b\log(c\sqrt{x}-1)}{2c^4} - \frac{2bc^2x\log(c\sqrt{x}+1) + b\log(c\sqrt{x}+1)^2 - b\log(-c\sqrt{x}+1)^2 + 4bc\sqrt{x} - 2(bc^2x + b\log(c\sqrt{x}+1))\log(-c\sqrt{x}+1)}{4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="maxima")`

[Out] $-a*(x/c^2 + \log(c^2*x - 1)/c^4) - (\log(c*\operatorname{sqrt}(x) + 1)*\log(-1/2*c*\operatorname{sqrt}(x) + 1/2) + \operatorname{dilog}(1/2*c*\operatorname{sqrt}(x) + 1/2))*b/c^4 + 1/2*b*\log(c*\operatorname{sqrt}(x) + 1)/c^4 - 1/2*b*\log(c*\operatorname{sqrt}(x) - 1)/c^4 - 1/4*(2*b*c^2*x*\log(c*\operatorname{sqrt}(x) + 1) + b*\log(c*\operatorname{sqrt}(x) + 1)^2 - b*\log(-c*\operatorname{sqrt}(x) + 1)^2 + 4*b*c*\operatorname{sqrt}(x) - 2*(b*c^2*x + b*\log(c*\operatorname{sqrt}(x) + 1))*\log(-c*\operatorname{sqrt}(x) + 1))/c^4$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="fricas")`

[Out] `integral(-(b*x*arctanh(c*sqrt(x)) + a*x)/(c^2*x - 1), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{c^2x-1} dx - \int \frac{bx \operatorname{atanh}(c\sqrt{x})}{c^2x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*atanh(c*x**(1/2)))/(-c**2*x+1),x)

[Out] -Integral(a*x/(c**2*x - 1), x) - Integral(b*x*atanh(c*sqrt(x))/(c**2*x - 1), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="giac")

[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)*x/(c^2*x - 1), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x(a + b \operatorname{atanh}(c\sqrt{x}))}{c^2 x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x*(a + b*atanh(c*x^(1/2)))/(-c^2*x - 1),x)

[Out] int(-(x*(a + b*atanh(c*x^(1/2)))/(-c^2*x - 1), x)

$$3.39 \quad \int \frac{a+b \tanh^{-1}(c\sqrt{x})}{1-c^2x} dx$$

Optimal. Leaf size=78

$$-\frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{bc^2} + \frac{2(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2}{1-c\sqrt{x}}\right)}{c^2} + \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-c\sqrt{x}}\right)}{c^2}$$

[Out] $-(a+b*\operatorname{arctanh}(c*x^{(1/2)}))^2/b/c^2+2*(a+b*\operatorname{arctanh}(c*x^{(1/2)}))*\ln(2/(1-c*x^{(1/2)}))/c^2+b*\operatorname{polylog}(2,1-2/(1-c*x^{(1/2)}))/c^2$

Rubi [A]

time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6069, 6131, 6055, 2449, 2352}

$$-\frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{bc^2} + \frac{2 \log\left(\frac{2}{1-c\sqrt{x}}\right) (a+b \tanh^{-1}(c\sqrt{x}))}{c^2} + \frac{b \operatorname{Li}_2\left(1 - \frac{2}{1-c\sqrt{x}}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])/(1 - c^2*x), x]$

[Out] $-(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])^2/(b*c^2) + (2*(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])*\operatorname{Log}[2/(1 - c*\operatorname{Sqrt}[x])])/c^2 + (b*\operatorname{PolyLog}[2, 1 - 2/(1 - c*\operatorname{Sqrt}[x])])/c^2$

Rule 2352

$\operatorname{Int}[\operatorname{Log}[(c_*)*(x_)]/((d_*) + (e_*)*(x_)), x_Symbol] \rightarrow \operatorname{Simp}[(-e^{(-1)})*\operatorname{PolyLog}[2, 1 - c*x], x] /; \operatorname{FreeQ}\{c, d, e\}, x] \&\& \operatorname{EqQ}[e + c*d, 0]$

Rule 2449

$\operatorname{Int}[\operatorname{Log}[(c_*)/((d_*) + (e_*)*(x_))]/((f_*) + (g_*)*(x_)^2), x_Symbol] \rightarrow \operatorname{Dist}[-e/g, \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \operatorname{FreeQ}\{c, d, e, f, g\}, x] \&\& \operatorname{EqQ}[c, 2*d] \&\& \operatorname{EqQ}[e^2*f + d^2*g, 0]$

Rule 6055

$\operatorname{Int}[(a_*) + \operatorname{ArcTanh}[(c_*)*(x_)]*(b_*)^{(p_*)}/((d_*) + (e_*)*(x_)), x_Symbol] \rightarrow \operatorname{Simp}[(-a + b*\operatorname{ArcTanh}[c*x])^p*(\operatorname{Log}[2/(1 + e*(x/d))]/e), x] + \operatorname{Dist}[b*c*(p/e), \operatorname{Int}[(a + b*\operatorname{ArcTanh}[c*x])^{(p-1)}*(\operatorname{Log}[2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x] \&\& \operatorname{IGtQ}[p, 0] \&\& \operatorname{EqQ}[c^2*d^2 - e^2, 0]$

Rule 6069


```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)/((d_) + (e_.)*(x_)), x_Symbol]
  :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*((a + b*ArcTanh[
c*x^(k*n)])/(d + e*x^k)), x], x, x^(1/k)], x] /; FreeQ[{a, b, c, d, e}, x]
  && FractionQ[n]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
  x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{1 - c^2x} dx &= 2\text{Subst}\left(\int \frac{x(a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx, x, \sqrt{x}\right) \\
&= -\frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{bc^2} + \frac{2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{1 - cx} dx, x, \sqrt{x}\right)}{c} \\
&= -\frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{bc^2} + \frac{2(a + b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2}{1 - c\sqrt{x}}\right)}{c^2} - \frac{b \text{Li}_2\left(\frac{2}{1 - c\sqrt{x}}\right)}{c^2} \quad (2b)S \\
&= -\frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{bc^2} + \frac{2(a + b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2}{1 - c\sqrt{x}}\right)}{c^2} + \frac{b \text{Li}_2\left(\frac{2}{1 - c\sqrt{x}}\right)}{c^2} \quad (2b)S \\
&= -\frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{bc^2} + \frac{2(a + b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2}{1 - c\sqrt{x}}\right)}{c^2} + \frac{b \text{Li}_2\left(\frac{2}{1 - c\sqrt{x}}\right)}{c^2}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 75, normalized size = 0.96

$$\frac{a \log(1 - c^2x)}{c^2} - \frac{b(-\tanh^{-1}(c\sqrt{x}) (\tanh^{-1}(c\sqrt{x}) + 2 \log(1 + e^{-2 \tanh^{-1}(c\sqrt{x})})) + \text{PolyLog}(2, -e^{-2 \tanh^{-1}(c\sqrt{x})}))}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(1 - c^2*x), x]
```

```
[Out] -((a*Log[1 - c^2*x])/c^2) - (b*(-(ArcTanh[c*Sqrt[x]]*(ArcTanh[c*Sqrt[x]] +
2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])])) + PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]
]])))/c^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(70) = 140$.
time = 0.22, size = 162, normalized size = 2.08

method	result
derivativedivides	$2 \left(\frac{a \ln(c\sqrt{x}-1)}{2} + \frac{a \ln(1+c\sqrt{x})}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) \ln(c\sqrt{x}-1)}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) \ln(1+c\sqrt{x})}{2} + \frac{b \ln(c\sqrt{x})}{8} \right)$
default	$2 \left(\frac{a \ln(c\sqrt{x}-1)}{2} + \frac{a \ln(1+c\sqrt{x})}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) \ln(c\sqrt{x}-1)}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) \ln(1+c\sqrt{x})}{2} + \frac{b \ln(c\sqrt{x})}{8} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x,method=_RETURNVERBOSE)`

[Out] $-2/c^2*(1/2*a*\ln(c*x^(1/2)-1)+1/2*a*\ln(1+c*x^(1/2))+1/2*b*\operatorname{arctanh}(c*x^(1/2))*\ln(c*x^(1/2)-1)+1/2*b*\operatorname{arctanh}(c*x^(1/2))*\ln(1+c*x^(1/2))+1/8*b*\ln(c*x^(1/2)-1)^2-1/2*b*\operatorname{dilog}(1/2*c*x^(1/2)+1/2)-1/4*b*\ln(c*x^(1/2)-1)*\ln(1/2*c*x^(1/2)+1/2)-1/8*b*\ln(1+c*x^(1/2))^2-1/4*b*\ln(-1/2*c*x^(1/2)+1/2)*\ln(1/2*c*x^(1/2)+1/2)+1/4*b*\ln(-1/2*c*x^(1/2)+1/2)*\ln(1+c*x^(1/2)))$

Maxima [A]

time = 0.57, size = 101, normalized size = 1.29

$$\frac{(\log(c\sqrt{x}+1)\log(-\frac{1}{2}c\sqrt{x}+\frac{1}{2})+\operatorname{Li}_2(\frac{1}{2}c\sqrt{x}+\frac{1}{2}))b}{c^2} - \frac{a\log(c^2x-1)}{c^2} - \frac{b\log(c\sqrt{x}+1)^2-2b\log(c\sqrt{x}+1)\log(-c\sqrt{x}+1)-b\log(-c\sqrt{x}+1)^2}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="maxima")`

[Out] $-(\log(c*\operatorname{sqrt}(x)+1)*\log(-1/2*c*\operatorname{sqrt}(x)+1/2)+\operatorname{dilog}(1/2*c*\operatorname{sqrt}(x)+1/2))*b/c^2-a*\log(c^2*x-1)/c^2-1/4*(b*\log(c*\operatorname{sqrt}(x)+1)^2-2*b*\log(c*\operatorname{sqrt}(x)+1)*\log(-c*\operatorname{sqrt}(x)+1)-b*\log(-c*\operatorname{sqrt}(x)+1)^2)/c^2$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="fricas")`

[Out] `integral(-(b*arctanh(c*sqrt(x))+a)/(c^2*x-1),x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2x - 1} dx - \int \frac{b \operatorname{atanh}(c\sqrt{x})}{c^2x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**(1/2)))/(-c**2*x+1), x)**[Out]** -Integral(a/(c**2*x - 1), x) - Integral(b*atanh(c*sqrt(x))/(c**2*x - 1), x)**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/(-c^2*x+1), x, algorithm="giac")**[Out]** integrate(-(b*arctanh(c*sqrt(x)) + a)/(c^2*x - 1), x)**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a + b \operatorname{atanh}(c\sqrt{x})}{c^2x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b*atanh(c*x^(1/2)))/(c^2*x - 1), x)**[Out]** int(-(a + b*atanh(c*x^(1/2)))/(c^2*x - 1), x)

$$3.40 \quad \int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x(1-c^2x)} dx$$

Optimal. Leaf size=69

$$\frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{b} + 2(a+b \tanh^{-1}(c\sqrt{x})) \log\left(2 - \frac{2}{1+c\sqrt{x}}\right) - b \text{PolyLog}\left(2, -1 + \frac{2}{1+c\sqrt{x}}\right)$$

[Out] (a+b*arctanh(c*x^(1/2)))^2/b+2*(a+b*arctanh(c*x^(1/2)))*ln(2-2/(1+c*x^(1/2)))-b*polylog(2,-1+2/(1+c*x^(1/2)))

Rubi [A]

time = 0.18, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {36, 29, 31, 1607, 6135, 6079, 2497}

$$\frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{b} + 2 \log\left(2 - \frac{2}{c\sqrt{x} + 1}\right) (a+b \tanh^{-1}(c\sqrt{x})) - b \text{Li}_2\left(\frac{2}{\sqrt{x}c+1} - 1\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x*(1 - c^2*x)), x]

[Out] (a + b*ArcTanh[c*Sqrt[x]])^2/b + 2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2 - 2/(1 + c*Sqrt[x])] - b*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])]

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 1607

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}], Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 6079

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)/((x_)*((d_) + (e_)*(x_))), x
_Symbol] := Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)/((x_)*((d_) + (e_)*(x_)^2)),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{x(1 - c^2x)} dx &= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x - c^2x^3} dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x(1 - c^2x^2)} dx, x, \sqrt{x}\right) \\
&= \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{b} + 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x(1 + cx)} dx, x, \sqrt{x}\right) \\
&= \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{b} + 2(a + b \tanh^{-1}(c\sqrt{x})) \log\left(2 - \frac{2}{1 + c\sqrt{x}}\right) - 2bL \\
&= \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{b} + 2(a + b \tanh^{-1}(c\sqrt{x})) \log\left(2 - \frac{2}{1 + c\sqrt{x}}\right) - bL
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 72, normalized size = 1.04

$$b \tanh^{-1}(c\sqrt{x}) \left(\tanh^{-1}(c\sqrt{x}) + 2 \log\left(1 - e^{-2 \tanh^{-1}(c\sqrt{x})}\right) \right) + a \log(x) - a \log(1 - c^2x) - b \text{PolyLog}\left(2, e^{-2 \tanh^{-1}(c\sqrt{x})}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x*(1 - c^2*x)),x]

[Out] b*ArcTanh[c*Sqrt[x]]*(ArcTanh[c*Sqrt[x]] + 2*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) + a*Log[x] - a*Log[1 - c^2*x] - b*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(61) = 122$.

time = 0.25, size = 217, normalized size = 3.14

method	result
derivativedivides	$-a \ln(1 + c\sqrt{x}) - a \ln(c\sqrt{x} - 1) + 2a \ln(c\sqrt{x}) - b \operatorname{arctanh}(c\sqrt{x}) \ln(1 + c\sqrt{x}) -$
default	$-a \ln(1 + c\sqrt{x}) - a \ln(c\sqrt{x} - 1) + 2a \ln(c\sqrt{x}) - b \operatorname{arctanh}(c\sqrt{x}) \ln(1 + c\sqrt{x}) -$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x^(1/2)))/x/(-c^2*x+1),x,method=_RETURNVERBOSE)

[Out] $-a*\ln(1+c*x^{(1/2)})-a*\ln(c*x^{(1/2)}-1)+2*a*\ln(c*x^{(1/2)})-b*\operatorname{arctanh}(c*x^{(1/2)})*\ln(1+c*x^{(1/2)})-b*\operatorname{arctanh}(c*x^{(1/2)})*\ln(c*x^{(1/2)}-1)+2*b*\operatorname{arctanh}(c*x^{(1/2)})*\ln(c*x^{(1/2)})-b*\operatorname{dilog}(1+c*x^{(1/2)})-b*\ln(c*x^{(1/2)})*\ln(1+c*x^{(1/2)})-b*\operatorname{dilog}(c*x^{(1/2)})-1/4*b*\ln(c*x^{(1/2)}-1)^2+b*\operatorname{dilog}(1/2*c*x^{(1/2)}+1/2)+1/2*b*\ln(c*x^{(1/2)}-1)*\ln(1/2*c*x^{(1/2)}+1/2)+1/4*b*\ln(1+c*x^{(1/2)})^2+1/2*b*\ln(-1/2*c*x^{(1/2)}+1/2)*\ln(1/2*c*x^{(1/2)}+1/2)-1/2*b*\ln(-1/2*c*x^{(1/2)}+1/2)*\ln(1+c*x^{(1/2)})$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(60) = 120$.

time = 0.37, size = 159, normalized size = 2.30

$$-\frac{1}{4}b \log(c\sqrt{x} + 1)^2 + \frac{1}{2}b \log(c\sqrt{x} + 1) \log(-c\sqrt{x} + 1) + \frac{1}{4}b \log(-c\sqrt{x} + 1)^2 - \left(\log(c\sqrt{x} + 1) \log\left(-\frac{1}{2}c\sqrt{x} + \frac{1}{2}\right) + \operatorname{Li}_2\left(\frac{1}{2}c\sqrt{x} + \frac{1}{2}\right) \right) b - (\log(c\sqrt{x}) \log(-c\sqrt{x} + 1) + \operatorname{Li}_2(-c\sqrt{x} + 1)) b + (\log(c\sqrt{x} + 1) \log(-c\sqrt{x}) + \operatorname{Li}_2(c\sqrt{x} + 1)) b - a(\log(c\sqrt{x} + 1) + \log(c\sqrt{x} - 1) - \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x/(-c^2*x+1),x, algorithm="maxima")

[Out] $-1/4*b*\log(c*\operatorname{sqrt}(x) + 1)^2 + 1/2*b*\log(c*\operatorname{sqrt}(x) + 1)*\log(-c*\operatorname{sqrt}(x) + 1) + 1/4*b*\log(-c*\operatorname{sqrt}(x) + 1)^2 - (\log(c*\operatorname{sqrt}(x) + 1)*\log(-1/2*c*\operatorname{sqrt}(x) + 1/2) + \operatorname{dilog}(1/2*c*\operatorname{sqrt}(x) + 1/2))*b - (\log(c*\operatorname{sqrt}(x))*\log(-c*\operatorname{sqrt}(x) + 1) + \operatorname{dilog}(-c*\operatorname{sqrt}(x) + 1))*b + (\log(c*\operatorname{sqrt}(x) + 1)*\log(-c*\operatorname{sqrt}(x)) + \operatorname{dilog}(c*\operatorname{sqrt}(x) + 1))*b - a*(\log(c*\operatorname{sqrt}(x) + 1) + \log(c*\operatorname{sqrt}(x) - 1) - \log(x))$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x/(-c^2*x+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*arctanh(c*sqrt(x)) + a)/(c^2*x^2 - x), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2 x^2 - x} dx - \int \frac{b \operatorname{atanh}(c\sqrt{x})}{c^2 x^2 - x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x/(-c**2*x+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - x), x) - Integral(b*atanh(c*sqrt(x))/(c**2*x**2 - x), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x/(-c^2*x+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)/((c^2*x - 1)*x), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a + b \operatorname{atanh}(c\sqrt{x})}{x(c^2 x - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh(c*x^(1/2)))/(x*(c^2*x - 1)),x)
```

```
[Out] -int((a + b*atanh(c*x^(1/2)))/(x*(c^2*x - 1)), x)
```

$$3.41 \quad \int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x^2(1-c^2x)} dx$$

Optimal. Leaf size=117

$$-\frac{bc}{\sqrt{x}} + bc^2 \tanh^{-1}(c\sqrt{x}) - \frac{a+b \tanh^{-1}(c\sqrt{x})}{x} + \frac{c^2(a+b \tanh^{-1}(c\sqrt{x}))^2}{b} + 2c^2(a+b \tanh^{-1}(c\sqrt{x})) \log$$

[Out] b*c^2*arctanh(c*x^(1/2))+(-a-b*arctanh(c*x^(1/2)))/x+c^2*(a+b*arctanh(c*x^(1/2)))^2/b+2*c^2*(a+b*arctanh(c*x^(1/2)))*ln(2-2/(1+c*x^(1/2)))-b*c^2*polyl
og(2,-1+2/(1+c*x^(1/2)))-b*c/x^(1/2)

Rubi [A]

time = 0.27, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {46, 1607, 6129, 6037, 331, 212, 6135, 6079, 2497}

$$\frac{c^2(a+b \tanh^{-1}(c\sqrt{x}))^2}{b} + 2c^2 \log\left(2 - \frac{2}{c\sqrt{x}+1}\right)(a+b \tanh^{-1}(c\sqrt{x})) - \frac{a+b \tanh^{-1}(c\sqrt{x})}{x} - bc^2 \text{Li}_2\left(\frac{2}{\sqrt{x}c+1} - 1\right) + bc^2 \tanh^{-1}(c\sqrt{x}) - \frac{bc}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x^2*(1 - c^2*x)),x]

[Out] -((b*c)/Sqrt[x]) + b*c^2*ArcTanh[c*Sqrt[x]] - (a + b*ArcTanh[c*Sqrt[x]])/x + (c^2*(a + b*ArcTanh[c*Sqrt[x]])^2)/b + 2*c^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2 - 2/(1 + c*Sqrt[x])] - b*c^2*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])]

Rule 46

Int[((a_) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 331

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m+1)*((a + b*x^n)^(p+1)/(a*c*(m+1))), x] - Dist[b*((m+n*(p+1)+1)/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,

x]

Rule 1607

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 2497

Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 6037

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m + 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]

Rule 6079

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 6129

Int((((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x], x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

Rule 6135

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{x^2(1 - c^2x)} dx &= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^3 - c^2x^5} dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^3(1 - c^2x^2)} dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^3} dx, x, \sqrt{x}\right) + (2c^2) \text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x(1 - c^2x^2)} dx, x, \sqrt{x}\right) \\
&= -\frac{a + b \tanh^{-1}(c\sqrt{x})}{x} + \frac{c^2(a + b \tanh^{-1}(c\sqrt{x}))^2}{b} + (bc)\text{Subst}\left(\int \frac{1}{x^2(1 - c^2x^2)} dx, x, \sqrt{x}\right) \\
&= -\frac{bc}{\sqrt{x}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{x} + \frac{c^2(a + b \tanh^{-1}(c\sqrt{x}))^2}{b} + 2c^2(a + b \tanh^{-1}(c\sqrt{x})) \\
&= -\frac{bc}{\sqrt{x}} + bc^2 \tanh^{-1}(c\sqrt{x}) - \frac{a + b \tanh^{-1}(c\sqrt{x})}{x} + \frac{c^2(a + b \tanh^{-1}(c\sqrt{x}))^2}{b}
\end{aligned}$$

Mathematica [A]

time = 0.22, size = 118, normalized size = 1.01

$$-\frac{a}{x} + 2ac^2 \log(\sqrt{x}) - ac^2 \log(1 - c^2x) - bc^2 \left(\frac{1}{c\sqrt{x}} - \tanh^{-1}(c\sqrt{x}) \left(-\frac{1 - c^2x}{c^2x} + \tanh^{-1}(c\sqrt{x}) + 2 \log(1 - e^{-2 \tanh^{-1}(c\sqrt{x})}) \right) + \text{PolyLog}(2, e^{-2 \tanh^{-1}(c\sqrt{x})}) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x^2*(1 - c^2*x)), x]`

```
[Out] -(a/x) + 2*a*c^2*Log[Sqrt[x]] - a*c^2*Log[1 - c^2*x] - b*c^2*(1/(c*Sqrt[x])
- ArcTanh[c*Sqrt[x]]*(-((1 - c^2*x)/(c^2*x)) + ArcTanh[c*Sqrt[x]] + 2*Log[
1 - E^(-2*ArcTanh[c*Sqrt[x]])]) + PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(105) = 210.

time = 0.26, size = 278, normalized size = 2.38

method	result
derivativedivides	$-2c^2 \left(\frac{a}{2c^2x} - a \ln(c\sqrt{x}) + \frac{a \ln(1+c\sqrt{x})}{2} + \frac{a \ln(c\sqrt{x}-1)}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x})}{2c^2x} - b \operatorname{arctanh}(c\sqrt{x}) \right)$
default	$-2c^2 \left(\frac{a}{2c^2x} - a \ln(c\sqrt{x}) + \frac{a \ln(1+c\sqrt{x})}{2} + \frac{a \ln(c\sqrt{x}-1)}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x})}{2c^2x} - b \operatorname{arctanh}(c\sqrt{x}) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^(1/2)))/x^2/(-c^2*x+1),x,method=_RETURNVERBOSE)`

[Out] $-2*c^2*(1/2*a/c^2/x-a*\ln(c*x^{(1/2)})+1/2*a*\ln(1+c*x^{(1/2)})+1/2*a*\ln(c*x^{(1/2)}-1)+1/2*b*arctanh(c*x^{(1/2)})/c^2/x-b*arctanh(c*x^{(1/2)})*\ln(c*x^{(1/2)})+1/2*b*arctanh(c*x^{(1/2)})*\ln(1+c*x^{(1/2)})+1/2*b*arctanh(c*x^{(1/2)})*\ln(c*x^{(1/2)}-1)+1/2*b/c/x^{(1/2)}-1/4*b*\ln(1+c*x^{(1/2)})+1/4*b*\ln(c*x^{(1/2)}-1)+1/2*b*dilog(1+c*x^{(1/2)})+1/2*b*\ln(c*x^{(1/2)})*\ln(1+c*x^{(1/2)})+1/2*b*dilog(c*x^{(1/2)})+1/8*b*\ln(c*x^{(1/2)}-1)^2-1/2*b*dilog(1/2*c*x^{(1/2)}+1/2)-1/4*b*\ln(c*x^{(1/2)}-1)*\ln(1/2*c*x^{(1/2)}+1/2)-1/8*b*\ln(1+c*x^{(1/2)})^2-1/4*b*\ln(-1/2*c*x^{(1/2)}+1/2)*\ln(1/2*c*x^{(1/2)}+1/2)+1/4*b*\ln(-1/2*c*x^{(1/2)}+1/2)*\ln(1+c*x^{(1/2)})$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(102) = 204$.

time = 0.41, size = 248, normalized size = 2.12

$-(\log(\sqrt{c^2x+1})\log(-\frac{1}{2}\sqrt{c^2x+1})+\frac{1}{2}\log(\frac{1}{2}\sqrt{c^2x+1}))b^2 - (\log(\sqrt{c^2x+1})\log(-\sqrt{c^2x+1})+\frac{1}{2}\log(-\sqrt{c^2x+1}))b^2 + (\log(\sqrt{c^2x+1})\log(-\sqrt{c^2x+1})+\frac{1}{2}\log(\sqrt{c^2x+1}))b^2 + \frac{1}{2}b^2\log(\sqrt{c^2x+1}) - \frac{1}{2}b^2\log(\sqrt{c^2x-1}) - (c^2\log(\sqrt{c^2x+1})+c^2\log(\sqrt{c^2x-1})-c^2\log(c)+\frac{1}{2})a - \frac{b^2\log(\sqrt{c^2x+1})^2 - b^2\log(\sqrt{c^2x+1})^2 + 4b\sqrt{c^2x+1} + 2b\log(\sqrt{c^2x+1}) - 2(b^2\log(\sqrt{c^2x+1})+b)\log(\sqrt{c^2x+1})}{4x}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/x^2/(-c^2*x+1),x, algorithm="maxima")`

[Out] $-(\log(c*\sqrt{x} + 1)*\log(-1/2*c*\sqrt{x} + 1/2) + \operatorname{dilog}(1/2*c*\sqrt{x} + 1/2)) * b * c^2 - (\log(c*\sqrt{x}) * \log(-c*\sqrt{x} + 1) + \operatorname{dilog}(-c*\sqrt{x} + 1)) * b * c^2 + (\log(c*\sqrt{x} + 1) * \log(-c*\sqrt{x}) + \operatorname{dilog}(c*\sqrt{x} + 1)) * b * c^2 + 1/2 * b * c^2 * \log(c*\sqrt{x} + 1) - 1/2 * b * c^2 * \log(c*\sqrt{x} - 1) - (c^2 * \log(c*\sqrt{x} + 1) + c^2 * \log(c*\sqrt{x} - 1) - c^2 * \log(x) + 1/x) * a - 1/4 * (b * c^2 * x * \log(c * \sqrt{x} + 1)^2 - b * c^2 * x * \log(-c * \sqrt{x} + 1)^2 + 4 * b * c * \sqrt{x} + 2 * b * \log(c * \sqrt{x} + 1) - 2 * (b * c^2 * x * \log(c * \sqrt{x} + 1) + b) * \log(-c * \sqrt{x} + 1)) / x$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/x^2/(-c^2*x+1),x, algorithm="fricas")`

[Out] `integral(-(b*arctanh(c*sqrt(x)) + a)/(c^2*x^3 - x^2), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2x^3 - x^2} dx - \int \frac{b \operatorname{atanh}(c\sqrt{x})}{c^2x^3 - x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**(1/2)))/x**2/(-c**2*x+1),x)

[Out] -Integral(a/(c**2*x**3 - x**2), x) - Integral(b*atanh(c*sqrt(x))/(c**2*x**3 - x**2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2/(-c^2*x+1),x, algorithm="giac")

[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)/((c^2*x - 1)*x^2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$- \int \frac{a + b \operatorname{atanh}(c \sqrt{x})}{x^2 (c^2 x - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b*atanh(c*x^(1/2)))/(x^2*(c^2*x - 1)),x)

[Out] -int((a + b*atanh(c*x^(1/2)))/(x^2*(c^2*x - 1)), x)

$$3.42 \quad \int \frac{a+b \tanh^{-1}\left(c\sqrt{x}\right)}{x^3(1-c^2x)} dx$$

Optimal. Leaf size=157

$$-\frac{bc}{6x^{3/2}} - \frac{3bc^3}{2\sqrt{x}} + \frac{3}{2}bc^4 \tanh^{-1}(c\sqrt{x}) - \frac{a+b \tanh^{-1}(c\sqrt{x})}{2x^2} - \frac{c^2(a+b \tanh^{-1}(c\sqrt{x}))}{x} + \frac{c^4(a+b \tanh^{-1}(c\sqrt{x}))}{b}$$

[Out] $-1/6*b*c/x^{(3/2)}+3/2*b*c^4*\arctanh(c*x^{(1/2)})+1/2*(-a-b*\arctanh(c*x^{(1/2)}))/x^2-c^2*(a+b*\arctanh(c*x^{(1/2)}))/x+c^4*(a+b*\arctanh(c*x^{(1/2)}))^2/b+2*c^4*(a+b*\arctanh(c*x^{(1/2)}))*\ln(2-2/(1+c*x^{(1/2)}))-b*c^4*\text{polylog}(2,-1+2/(1+c*x^{(1/2)}))-3/2*b*c^3/x^{(1/2)}$

Rubi [A]

time = 0.34, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {46, 1607, 6129, 6037, 331, 212, 6135, 6079, 2497}

$$\frac{c^4(a+b \tanh^{-1}(c\sqrt{x}))^2}{b} + 2c^4 \log\left(2 - \frac{2}{c\sqrt{x}+1}\right)(a+b \tanh^{-1}(c\sqrt{x})) - \frac{c^2(a+b \tanh^{-1}(c\sqrt{x}))}{x} - \frac{a+b \tanh^{-1}(c\sqrt{x})}{2x^2} - bc^4 \text{Li}_2\left(\frac{2}{\sqrt{x}c+1}-1\right) + \frac{3}{2}bc^4 \tanh^{-1}(c\sqrt{x}) - \frac{3bc^3}{2\sqrt{x}} - \frac{bc}{6x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x^3*(1 - c^2*x)), x]

[Out] $-1/6*(b*c)/x^{(3/2)} - (3*b*c^3)/(2*\text{Sqrt}[x]) + (3*b*c^4*\text{ArcTanh}[c*\text{Sqrt}[x]])/2 - (a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])/(2*x^2) - (c^2*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]]))/x + (c^4*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])^2)/b + 2*c^4*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])*\text{Log}[2 - 2/(1 + c*\text{Sqrt}[x])] - b*c^4*\text{PolyLog}[2, -1 + 2/(1 + c*\text{Sqrt}[x])]$

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 331

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m+1)*((a + b*x^n)^(p+1)/(a*c*(m+1))), x] - Dist[b*((m+n*(p+1)+1)/(a*c^n*(m+1))], Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,

b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1607

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 2497

Int[Log[u]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 6037

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m + 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]

Rule 6079

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_.) + (e_.)*(x_))), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 6129

Int[(((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_)^(m_.)))/((d_.) + (e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x], x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

Rule 6135

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_.) + (e_.)*(x_)^2)), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{x^3(1 - c^2x)} dx &= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^5 - c^2x^7} dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^5(1 - c^2x^2)} dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^5} dx, x, \sqrt{x}\right) + (2c^2) \text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^3(1 - c^2x^2)} dx, x, \sqrt{x}\right) \\
&= -\frac{a + b \tanh^{-1}(c\sqrt{x})}{2x^2} + \frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{x^4(1 - c^2x^2)} dx, x, \sqrt{x}\right) + (2c^2) \text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^3(1 - c^2x^2)} dx, x, \sqrt{x}\right) \\
&= -\frac{bc}{6x^{3/2}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{2x^2} - \frac{c^2(a + b \tanh^{-1}(c\sqrt{x}))}{x} + \frac{c^4(a + b \tanh^{-1}(c\sqrt{x}))}{b} \\
&= -\frac{bc}{6x^{3/2}} - \frac{3bc^3}{2\sqrt{x}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{2x^2} - \frac{c^2(a + b \tanh^{-1}(c\sqrt{x}))}{x} + \frac{c^4(a + b \tanh^{-1}(c\sqrt{x}))}{b} \\
&= -\frac{bc}{6x^{3/2}} - \frac{3bc^3}{2\sqrt{x}} + \frac{3}{2}bc^4 \tanh^{-1}(c\sqrt{x}) - \frac{a + b \tanh^{-1}(c\sqrt{x})}{2x^2} - \frac{c^2(a + b \tanh^{-1}(c\sqrt{x}))}{x}
\end{aligned}$$

Mathematica [A]

time = 0.39, size = 158, normalized size = 1.01

$$-\frac{3a + bc\sqrt{x} + 6a^2x + 9bc^2x^{3/2} - 6bc^4x^2 \tanh^{-1}(c\sqrt{x})^2 - 3b \tanh^{-1}(c\sqrt{x}) (-1 - 2c^2x + 3c^4x^2 + 4c^4x^2 \log(1 - e^{-2 \tanh^{-1}(c\sqrt{x})})) - 6ac^4x^2 \log(x) + 6ac^4x^2 \log(1 - c^2x)}{6x^2} - bc^4 \text{PolyLog}(2, e^{-2 \tanh^{-1}(c\sqrt{x})})$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x^3*(1 - c^2*x)),x]

[Out] -1/6*(3*a + b*c*Sqrt[x] + 6*a*c^2*x + 9*b*c^3*x^(3/2) - 6*b*c^4*x^2*ArcTanh[c*Sqrt[x]]^2 - 3*b*ArcTanh[c*Sqrt[x]]*(-1 - 2*c^2*x + 3*c^4*x^2 + 4*c^4*x^2*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) - 6*a*c^4*x^2*Log[x] + 6*a*c^4*x^2*Log[1 - c^2*x])/x^2 - b*c^4*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(134) = 268.

time = 0.27, size = 311, normalized size = 1.98

method	result
derivativedivides	$ -2c^4 \left(\frac{a \ln(1+c\sqrt{x})}{2} + \frac{a}{4c^4x^2} + \frac{a}{2c^2x} - a \ln(c\sqrt{x}) + \frac{a \ln(c\sqrt{x}-1)}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) \ln(1+c\sqrt{x})}{2} \right) $

default	$-2c^4 \left(\frac{a \ln(1+c\sqrt{x})}{2} + \frac{a}{4c^4x^2} + \frac{a}{2c^2x} - a \ln(c\sqrt{x}) + \frac{a \ln(c\sqrt{x}-1)}{2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) \ln(1+c\sqrt{x})}{2} \right)$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^(1/2)))/x^3/(-c^2*x+1),x,method=_RETURNVERBOSE)`

[Out] $-2*c^4*(1/2*a*\ln(1+c*x^(1/2))+1/4*a/c^4/x^2+1/2*a/c^2/x-a*\ln(c*x^(1/2))+1/2*a*\ln(c*x^(1/2)-1)+1/2*b*\operatorname{arctanh}(c*x^(1/2))*\ln(1+c*x^(1/2))+1/4*b*\operatorname{arctanh}(c*x^(1/2))/c^4/x^2+1/2*b*\operatorname{arctanh}(c*x^(1/2))/c^2/x-b*\operatorname{arctanh}(c*x^(1/2))*\ln(c*x^(1/2))+1/2*b*\operatorname{arctanh}(c*x^(1/2))*\ln(c*x^(1/2)-1)+1/2*b*\operatorname{dilog}(1+c*x^(1/2))+1/2*b*\ln(c*x^(1/2))*\ln(1+c*x^(1/2))+1/2*b*\operatorname{dilog}(c*x^(1/2))-1/2*b*\operatorname{dilog}(1/2*c*x^(1/2)+1/2)-1/4*b*\ln(c*x^(1/2)-1)*\ln(1/2*c*x^(1/2)+1/2)+1/8*b*\ln(c*x^(1/2)-1)^2-1/4*b*\ln(-1/2*c*x^(1/2)+1/2)*\ln(1/2*c*x^(1/2)+1/2)+1/4*b*\ln(-1/2*c*x^(1/2)+1/2)*\ln(1+c*x^(1/2))-1/8*b*\ln(1+c*x^(1/2))^2-3/8*b*\ln(1+c*x^(1/2))+1/12*b/c^3/x^(3/2)+3/4*b/c/x^(1/2)+3/8*b*\ln(c*x^(1/2)-1))$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(130) = 260$.

time = 0.41, size = 291, normalized size = 1.85

$$-\left(\ln(\sqrt{c}+1)\ln\left(\frac{1}{2}\sqrt{c}-\frac{1}{2}\right)+4\left(\frac{1}{2}\sqrt{c}+\frac{1}{2}\right)\right)^{b^2}-\left(\ln(\sqrt{c})\ln(-\sqrt{c}+1)+4\ln(-\sqrt{c}+1)\right)^{b^2}+\left(\ln(\sqrt{c}+1)\ln(-\sqrt{c})+4\ln(\sqrt{c}+1)\right)^{b^2}+\frac{1}{2}b^2\ln(\sqrt{c}+1)-\frac{1}{2}b^2\ln(\sqrt{c}-1)-\frac{1}{2}\left(b^2\ln(\sqrt{c}+1)+2^b\ln(\sqrt{c}-1)-2^b\ln(c)+\frac{2b^2+1}{2}\right)-\frac{3b^2\ln(\sqrt{c}+1)^2-3b^2\ln(-\sqrt{c})^2+1^2+16b^2+24b\sqrt{c}+3(16b^2+8)\ln(\sqrt{c}+1)-3(16b^2+8)\ln(\sqrt{c}-1)}{224}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/x^3/(-c^2*x+1),x, algorithm="maxima")`

[Out] $-(\log(c*\sqrt{x})+1)*\log(-1/2*c*\sqrt{x}+1/2)+\operatorname{dilog}(1/2*c*\sqrt{x}+1/2)*b*c^4-(\log(c*\sqrt{x}))*\log(-c*\sqrt{x}+1)+\operatorname{dilog}(-c*\sqrt{x}+1)*b*c^4+(\log(c*\sqrt{x})+1)*\log(-c*\sqrt{x})+\operatorname{dilog}(c*\sqrt{x}+1)*b*c^4+3/4*b*c^4*\log(c*\sqrt{x}+1)-3/4*b*c^4*\log(c*\sqrt{x}-1)-1/2*(2*c^4*\log(c*\sqrt{x}+1)+2*c^4*\log(c*\sqrt{x}-1)-2*c^4*\log(x)+(2*c^2*x+1)/x^2)*a-1/12*(3*b*c^4*x^2*\log(c*\sqrt{x}+1)^2-3*b*c^4*x^2*\log(-c*\sqrt{x}+1)^2+18*b*c^3*x^(3/2)+2*b*c*\sqrt{x}+3*(2*b*c^2*x+b)*\log(c*\sqrt{x}+1)-3*(2*b*c^4*x^2*\log(c*\sqrt{x}+1)+2*b*c^2*x+b)*\log(-c*\sqrt{x}+1))/x^2$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/x^3/(-c^2*x+1),x, algorithm="fricas")`

[Out] `integral(-(b*arctanh(c*sqrt(x)) + a)/(c^2*x^4 - x^3), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2 x^4 - x^3} dx - \int \frac{b \operatorname{atanh}(c\sqrt{x})}{c^2 x^4 - x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**(1/2)))/x**3/(-c**2*x+1), x)`

[Out] `-Integral(a/(c**2*x**4 - x**3), x) - Integral(b*atanh(c*sqrt(x))/(c**2*x**4 - x**3), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/x^3/(-c^2*x+1), x, algorithm="giac")`

[Out] `integrate(-(b*arctanh(c*sqrt(x)) + a)/((c^2*x - 1)*x^3), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a + b \operatorname{atanh}(c\sqrt{x})}{x^3 (c^2 x - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(a + b*atanh(c*x^(1/2)))/(x^3*(c^2*x - 1)), x)`

[Out] `-int((a + b*atanh(c*x^(1/2)))/(x^3*(c^2*x - 1)), x)`

$$3.43 \quad \int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x^4(1-c^2x)} dx$$

Optimal. Leaf size=192

$$-\frac{bc}{15x^{5/2}} - \frac{5bc^3}{18x^{3/2}} - \frac{11bc^5}{6\sqrt{x}} + \frac{11}{6}bc^6 \tanh^{-1}(c\sqrt{x}) - \frac{a+b \tanh^{-1}(c\sqrt{x})}{3x^3} - \frac{c^2(a+b \tanh^{-1}(c\sqrt{x}))}{2x^2} - \frac{c^4(a+b \tanh^{-1}(c\sqrt{x}))}{x}$$

[Out] $-1/15*b*c/x^{(5/2)}-5/18*b*c^3/x^{(3/2)}+11/6*b*c^6*\arctanh(c*x^{(1/2)})+1/3*(-a-b*\arctanh(c*x^{(1/2)}))/x^3-1/2*c^2*(a+b*\arctanh(c*x^{(1/2)}))/x^2-c^4*(a+b*\arctanh(c*x^{(1/2)}))/x+c^6*(a+b*\arctanh(c*x^{(1/2)}))^2/b+2*c^6*(a+b*\arctanh(c*x^{(1/2)}))*\ln(2-2/(1+c*x^{(1/2)}))-b*c^6*\text{polylog}(2,-1+2/(1+c*x^{(1/2)}))-11/6*b*c^6/x^{(1/2)}$

Rubi [A]

time = 0.40, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {46, 1607, 6129, 6037, 331, 212, 6135, 6079, 2497}

$$\frac{c^6(a+b \tanh^{-1}(c\sqrt{x}))^2}{b} + 2c^6 \log\left(2 - \frac{2}{c\sqrt{x}+1}\right)(a+b \tanh^{-1}(c\sqrt{x})) - \frac{c^4(a+b \tanh^{-1}(c\sqrt{x}))}{x} - \frac{c^2(a+b \tanh^{-1}(c\sqrt{x}))}{2x^2} - \frac{a+b \tanh^{-1}(c\sqrt{x})}{3x^3} - bc^6 \text{Li}_2\left(\frac{2}{\sqrt{x}c+1}-1\right) + \frac{11}{6}bc^6 \tanh^{-1}(c\sqrt{x}) - \frac{11bc^5}{6\sqrt{x}} - \frac{5bc^3}{18x^{3/2}} - \frac{bc}{15x^{5/2}}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcTanh[c*Sqrt[x]])/(x^4*(1 - c^2*x)), x]`

[Out] $-1/15*(b*c)/x^{(5/2)} - (5*b*c^3)/(18*x^{(3/2)}) - (11*b*c^5)/(6*\text{Sqrt}[x]) + (11*b*c^6*\text{ArcTanh}[c*\text{Sqrt}[x]])/6 - (a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])/(3*x^3) - (c^2*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]]))/(2*x^2) - (c^4*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]]))/x + (c^6*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])^2)/b + 2*c^6*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])*\text{Log}[2 - 2/(1 + c*\text{Sqrt}[x])] - b*c^6*\text{PolyLog}[2, -1 + 2/(1 + c*\text{Sqrt}[x])]$

Rule 46

`Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])]`

Rule 212

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 331

`Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m+1)*((a + b*x^n)^(p+1)/(a*c*(m+1))), x] - Dist[b*((m+n*(p+1))`

+ 1)/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1607

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 2497

Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 6037

Int[((a_) + ArcTanh[(c_)*(x_)^(n_)])*(b_)^(p_)*(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m + 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]

Rule 6079

Int[((a_) + ArcTanh[(c_)*(x_)])*(b_)^(p_)/((x_)*((d_) + (e_)*(x_))), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 6129

Int[((a_) + ArcTanh[(c_)*(x_)])*(b_)^(p_)*((f_)*(x_)^(m_))/((d_) + (e_)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x], x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

Rule 6135

Int[((a_) + ArcTanh[(c_)*(x_)])*(b_)^(p_)/((x_)*((d_) + (e_)*(x_)^2)), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{x^4(1 - c^2x)} dx &= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^7 - c^2x^9} dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^7(1 - c^2x^2)} dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^7} dx, x, \sqrt{x}\right) + (2c^2) \text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^5(1 - c^2x^2)} dx, x, \sqrt{x}\right) \\
&= -\frac{a + b \tanh^{-1}(c\sqrt{x})}{3x^3} + \frac{1}{3}(bc)\text{Subst}\left(\int \frac{1}{x^6(1 - c^2x^2)} dx, x, \sqrt{x}\right) + (2c^2) \text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^5(1 - c^2x^2)} dx, x, \sqrt{x}\right) \\
&= -\frac{bc}{15x^{5/2}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{3x^3} - \frac{c^2(a + b \tanh^{-1}(c\sqrt{x}))}{2x^2} + \frac{1}{3}(bc^3) \text{Subst}\left(\int \frac{1}{x^6(1 - c^2x^2)} dx, x, \sqrt{x}\right) \\
&= -\frac{bc}{15x^{5/2}} - \frac{5bc^3}{18x^{3/2}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{3x^3} - \frac{c^2(a + b \tanh^{-1}(c\sqrt{x}))}{2x^2} - \frac{c^4(a + b \tanh^{-1}(c\sqrt{x}))}{6\sqrt{x}} \\
&= -\frac{bc}{15x^{5/2}} - \frac{5bc^3}{18x^{3/2}} - \frac{11bc^5}{6\sqrt{x}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{3x^3} - \frac{c^2(a + b \tanh^{-1}(c\sqrt{x}))}{2x^2} \\
&= -\frac{bc}{15x^{5/2}} - \frac{5bc^3}{18x^{3/2}} - \frac{11bc^5}{6\sqrt{x}} + \frac{11}{6}bc^6 \tanh^{-1}(c\sqrt{x}) - \frac{a + b \tanh^{-1}(c\sqrt{x})}{3x^3} - \frac{c^2(a + b \tanh^{-1}(c\sqrt{x}))}{2x^2}
\end{aligned}$$

Mathematica [A]

time = 0.56, size = 187, normalized size = 0.97

$$\frac{30a + 6bc\sqrt{x} + 45ac^2x + 25bc^2x^{3/2} + 90ac^4x^2 + 165bc^5x^{5/2} - 90bc^6x^3 \tanh^{-1}(c\sqrt{x})^2 - 15b \tanh^{-1}(c\sqrt{x}) (-2 - 3c^2x - 6c^4x^2 + 11c^6x^3 + 12c^6x^3 \log(1 - e^{-2 \tanh^{-1}(c\sqrt{x})})) - 90ac^6x^3 \log(x) + 90ac^6x^3 \log(1 - c^2x)}{90x^3} - bc^6 \text{PolyLog}(2, e^{-2 \tanh^{-1}(c\sqrt{x})})$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x^4*(1 - c^2*x)), x]

[Out] -1/90*(30*a + 6*b*c*Sqrt[x] + 45*a*c^2*x + 25*b*c^3*x^(3/2) + 90*a*c^4*x^2 + 165*b*c^5*x^(5/2) - 90*b*c^6*x^3*ArcTanh[c*Sqrt[x]]^2 - 15*b*ArcTanh[c*Sqrt[x]]*(-2 - 3*c^2*x - 6*c^4*x^2 + 11*c^6*x^3 + 12*c^6*x^3*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) - 90*a*c^6*x^3*Log[x] + 90*a*c^6*x^3*Log[1 - c^2*x])/x^3 - b*c^6*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(161) = 322.

time = 0.28, size = 344, normalized size = 1.79

method	result
--------	--------

derivativedivides	$-2c^6 \left(\frac{a}{2c^2x} + \frac{a}{4c^4x^2} + \frac{a}{6c^6x^3} - \frac{11b \ln(1+c\sqrt{x})}{24} + \frac{11b}{12c\sqrt{x}} - a \ln(c\sqrt{x}) - \frac{b \ln(c\sqrt{x}-1) \ln\left(\frac{c}{c}\right)}{4} \right)$
default	$-2c^6 \left(\frac{a}{2c^2x} + \frac{a}{4c^4x^2} + \frac{a}{6c^6x^3} - \frac{11b \ln(1+c\sqrt{x})}{24} + \frac{11b}{12c\sqrt{x}} - a \ln(c\sqrt{x}) - \frac{b \ln(c\sqrt{x}-1) \ln\left(\frac{c}{c}\right)}{4} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^4/(-c^2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c^6*(1/2*a/c^2/x+1/4*a/c^4/x^2+1/6*a/c^6/x^3-11/24*b*ln(1+c*x^(1/2))+1/4
*b*arctanh(c*x^(1/2))/c^4/x^2+1/2*b*arctanh(c*x^(1/2))/c^2/x+11/12*b/c/x^(1
/2)-a*ln(c*x^(1/2))+1/2*b*dilog(1+c*x^(1/2))+1/2*b*dilog(c*x^(1/2))+1/2*b*a
rctanh(c*x^(1/2))*ln(c*x^(1/2)-1)+1/2*b*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-
1/4*b*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)+1/4*b*ln(-1/2*c*x^(1/2)+1/2)*ln
(1+c*x^(1/2))-1/4*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2*c*x^(1/2)+1/2)+1/30*b/c^5
/x^(5/2)+5/36*b/c^3/x^(3/2)+1/2*a*ln(c*x^(1/2)-1)+1/2*a*ln(1+c*x^(1/2))-1/2
*b*dilog(1/2*c*x^(1/2)+1/2)+1/8*b*ln(c*x^(1/2)-1)^2-1/8*b*ln(1+c*x^(1/2))^2
+11/24*b*ln(c*x^(1/2)-1)+1/6*b*arctanh(c*x^(1/2))/c^6/x^3-b*arctanh(c*x^(1/
2))*ln(c*x^(1/2))+1/2*b*ln(c*x^(1/2))*ln(1+c*x^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(157) = 314.

time = 0.41, size = 330, normalized size = 1.72

-(4*log(1+c*sqrt(x))-4*log(1-c*sqrt(x)))/c^2+11*b/c^6*(1/2*c*sqrt(x)+1/2)-11*b/c^6*(1/2*c*sqrt(x)-1/2)+5*b/c^5*x^(5/2)+5*b/c^3*x^(3/2)+11/24*b*ln(1+c*sqrt(x))-11/24*b*ln(1-c*sqrt(x))+1/4*b*arctanh(c*sqrt(x))*ln(1+c*sqrt(x))-1/4*b*arctanh(c*sqrt(x))*ln(c*sqrt(x)-1)+1/2*b*dilog(1+c*sqrt(x))+1/2*b*dilog(c*sqrt(x))-1/2*b*ln(c*sqrt(x)-1)*ln(1+c*sqrt(x))+1/2*b*ln(1+c*sqrt(x))*ln(c*sqrt(x)-1)+1/2*b*ln(c*sqrt(x))*ln(1+c*sqrt(x))-1/4*b*ln(1+c*sqrt(x)-1)*ln(1/2*c*sqrt(x)+1/2)+1/4*b*ln(-1/2*c*sqrt(x)+1/2)*ln(1+c*sqrt(x))-1/4*b*ln(-1/2*c*sqrt(x)+1/2)*ln(1/2*c*sqrt(x)+1/2)+1/30*b/c^5/x^(5/2)+5/36*b/c^3/x^(3/2)+1/2*a*ln(1+c*sqrt(x))-1/2*a*ln(c*sqrt(x)-1)+1/2*b*ln(1+c*sqrt(x))-1/2*b*ln(c*sqrt(x)-1)^2-1/8*b*ln(1+c*sqrt(x))^2+11/24*b*ln(c*sqrt(x)-1)+1/6*b*arctanh(c*sqrt(x))/c^6/x^3-b*arctanh(c*sqrt(x))*ln(c*sqrt(x))+1/2*b*ln(c*sqrt(x))*ln(1+c*sqrt(x)))

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4/(-c^2*x+1),x, algorithm="maxima")
```

```
[Out] -(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2)
)*b*c^6 - (log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1))*b*c^
6 + (log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b*c^6 + 11/
12*b*c^6*log(c*sqrt(x) + 1) - 11/12*b*c^6*log(c*sqrt(x) - 1) - 1/6*(6*c^6*l
og(c*sqrt(x) + 1) + 6*c^6*log(c*sqrt(x) - 1) - 6*c^6*log(x) + (6*c^4*x^2 +
3*c^2*x + 2)/x^3)*a - 1/180*(45*b*c^6*x^3*log(c*sqrt(x) + 1)^2 - 45*b*c^6*x
^3*log(-c*sqrt(x) + 1)^2 + 330*b*c^5*x^(5/2) + 50*b*c^3*x^(3/2) + 12*b*c*sq
rt(x) + 15*(6*b*c^4*x^2 + 3*b*c^2*x + 2*b)*log(c*sqrt(x) + 1) - 15*(6*b*c^6
*x^3*log(c*sqrt(x) + 1) + 6*b*c^4*x^2 + 3*b*c^2*x + 2*b)*log(-c*sqrt(x) + 1
))/x^3
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4/(-c^2*x+1),x, algorithm="fricas")

[Out] integral(-(b*arctanh(c*sqrt(x)) + a)/(c^2*x^5 - x^4), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2 x^5 - x^4} dx - \int \frac{b \operatorname{atanh}(c\sqrt{x})}{c^2 x^5 - x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**(1/2)))/x**4/(-c**2*x+1),x)

[Out] -Integral(a/(c**2*x**5 - x**4), x) - Integral(b*atanh(c*sqrt(x))/(c**2*x**5 - x**4), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4/(-c^2*x+1),x, algorithm="giac")

[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)/((c^2*x - 1)*x^4), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a + b \operatorname{atanh}(c\sqrt{x})}{x^4 (c^2 x - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b*atanh(c*x^(1/2)))/(x^4*(c^2*x - 1)),x)

[Out] -int((a + b*atanh(c*x^(1/2)))/(x^4*(c^2*x - 1)), x)

$$3.44 \quad \int \frac{x^2 \left(a + b \tanh^{-1} \left(c \sqrt{x} \right) \right)}{d + ex} dx$$

Optimal. Leaf size=460

$$-\frac{bd\sqrt{x}}{ce^2} + \frac{b\sqrt{x}}{2c^3e} + \frac{bx^{3/2}}{6ce} + \frac{bd \tanh^{-1}(c\sqrt{x})}{c^2e^2} - \frac{b \tanh^{-1}(c\sqrt{x})}{2c^4e} - \frac{dx(a + b \tanh^{-1}(c\sqrt{x}))}{e^2} + \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2e}$$

[Out] $1/6*b*x^{(3/2)}/c/e+b*d*\arctanh(c*x^{(1/2)})/c^2/e^2-1/2*b*\arctanh(c*x^{(1/2)})/c^4/e-d*x*(a+b*\arctanh(c*x^{(1/2)}))/e^2+1/2*x^2*(a+b*\arctanh(c*x^{(1/2)}))/e-2*d^2*(a+b*\arctanh(c*x^{(1/2)}))*\ln(2/(1+c*x^{(1/2)}))/e^3+d^2*(a+b*\arctanh(c*x^{(1/2)}))*\ln(2*c*((-d)^{(1/2)}-e^{(1/2)})*x^{(1/2)})/(c*(-d)^{(1/2)}-e^{(1/2)})/(1+c*x^{(1/2)})/e^3+d^2*(a+b*\arctanh(c*x^{(1/2)}))*\ln(2*c*((-d)^{(1/2)}+e^{(1/2)})*x^{(1/2)})/(c*(-d)^{(1/2)}+e^{(1/2)})/(1+c*x^{(1/2)})/e^3+b*d^2*\text{polylog}(2,1-2/(1+c*x^{(1/2)}))/e^3-1/2*b*d^2*\text{polylog}(2,1-2*c*((-d)^{(1/2)}-e^{(1/2)})*x^{(1/2)})/(c*(-d)^{(1/2)}-e^{(1/2)})/(1+c*x^{(1/2)})/e^3-1/2*b*d^2*\text{polylog}(2,1-2*c*((-d)^{(1/2)}+e^{(1/2)})*x^{(1/2)})/(c*(-d)^{(1/2)}+e^{(1/2)})/(1+c*x^{(1/2)})/e^3-b*d*x^{(1/2)}/c/e^2+1/2*b*x^{(1/2)}/c^3/e$

Rubi [A]

time = 0.58, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {45, 6127, 6037, 308, 212, 327, 6191, 6057, 2449, 2352, 2497}

$$\frac{2d^2 \log\left(\frac{d+bx}{\sqrt{d+ex}}\right)}{c^2} + \frac{d^2(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{d+\sqrt{d+ex}-\sqrt{d+ex}}{\sqrt{d+ex}(\sqrt{d+ex}-\sqrt{d+ex})}\right)}{c^2} + \frac{d^2(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{d+\sqrt{d+ex}+\sqrt{d+ex}}{\sqrt{d+ex}(\sqrt{d+ex}+\sqrt{d+ex})}\right)}{c^2} + \frac{d^2(a+b \tanh^{-1}(c\sqrt{x}))}{c^2} + \frac{b \tanh^{-1}(c\sqrt{x})}{2c^4} + \frac{bd\sqrt{x}}{ce^2} + \frac{b \tanh^{-1}(c\sqrt{x})}{2c^4e} + \frac{M^2 \text{Li}\left(1-\frac{d}{\sqrt{d+ex}}\right)}{c^2} + \frac{M^2 \text{Li}\left(1-\frac{d+\sqrt{d+ex}-\sqrt{d+ex}}{\sqrt{d+ex}(\sqrt{d+ex}-\sqrt{d+ex})}\right)}{c^2} + \frac{M^2 \text{Li}\left(1-\frac{d+\sqrt{d+ex}+\sqrt{d+ex}}{\sqrt{d+ex}(\sqrt{d+ex}+\sqrt{d+ex})}\right)}{c^2} + \frac{bd\sqrt{x}}{ce^2} + \frac{bd \tanh^{-1}(c\sqrt{x})}{c^2e^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^2*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]))]/(d + e*x), x]$

[Out] $-((b*d*\text{Sqrt}[x])/(c*e^2)) + (b*\text{Sqrt}[x])/(2*c^3*e) + (b*x^{(3/2)})/(6*c*e) + (b*d*\text{ArcTanh}[c*\text{Sqrt}[x])/(c^2*e^2) - (b*\text{ArcTanh}[c*\text{Sqrt}[x])/(2*c^4*e) - (d*x*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]))/e^2 + (x^2*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]))/(2*e) - (2*d^2*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]))*\text{Log}[2/(1 + c*\text{Sqrt}[x])]/e^3 + (d^2*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]))*\text{Log}[(2*c*(\text{Sqrt}[-d] - \text{Sqrt}[e]*\text{Sqrt}[x])]/((c*\text{Sqrt}[-d] - \text{Sqrt}[e])*(1 + c*\text{Sqrt}[x]))]/e^3 + (d^2*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]))*\text{Log}[(2*c*(\text{Sqrt}[-d] + \text{Sqrt}[e]*\text{Sqrt}[x])]/((c*\text{Sqrt}[-d] + \text{Sqrt}[e])*(1 + c*\text{Sqrt}[x]))]/e^3 + (b*d^2*\text{PolyLog}[2, 1 - 2/(1 + c*\text{Sqrt}[x])]/e^3 - (b*d^2*\text{PolyLog}[2, 1 - (2*c*(\text{Sqrt}[-d] - \text{Sqrt}[e]*\text{Sqrt}[x])]/((c*\text{Sqrt}[-d] - \text{Sqrt}[e])*(1 + c*\text{Sqrt}[x]))]/(2*e^3) - (b*d^2*\text{PolyLog}[2, 1 - (2*c*(\text{Sqrt}[-d] + \text{Sqrt}[e]*\text{Sqrt}[x])]/((c*\text{Sqrt}[-d] + \text{Sqrt}[e])*(1 + c*\text{Sqrt}[x]))]/(2*e^3)$

Rule 45

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}$

$x]$ && $\text{NeQ}[b*c - a*d, 0]$ && $\text{IGtQ}[m, 0]$ && $(\text{!IntegerQ}[n] \text{ || } (\text{EqQ}[c, 0] \text{ \&\& } \text{LeQ}[7*m + 4*n + 4, 0]) \text{ || } \text{LtQ}[9*m + 5*(n + 1), 0] \text{ || } \text{GtQ}[m + n + 2, 0])$

Rule 212

$\text{Int}[\text{((a_)} + \text{(b_)}*(x_)^2)^{-1}, x_Symbol] \text{ :> } \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] \text{ /; } \text{FreeQ}[\{a, b\}, x] \text{ \&\& } \text{NegQ}[a/b] \text{ \&\& } (\text{GtQ}[a, 0] \text{ || } \text{LtQ}[b, 0])$

Rule 308

$\text{Int}[(x_)^{(m_)} / ((a_) + (b_)*(x_)^{(n_)}), x_Symbol] \text{ :> } \text{Int}[\text{PolynomialDivide}[x^m, a + b*x^n, x], x] \text{ /; } \text{FreeQ}[\{a, b\}, x] \text{ \&\& } \text{IGtQ}[m, 0] \text{ \&\& } \text{IGtQ}[n, 0] \text{ \&\& } \text{GtQ}[m, 2*n - 1]$

Rule 327

$\text{Int}[\text{((c_)}*(x_))^{(m_)} * ((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \text{ :> } \text{Simp}[c^{(n - 1)}*(c*x)^{(m - n + 1)}*((a + b*x^n)^{(p + 1)} / (b*(m + n*p + 1))), x] - \text{Dist}[a*c^n*((m - n + 1) / (b*(m + n*p + 1))), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] \text{ /; } \text{FreeQ}[\{a, b, c, p\}, x] \text{ \&\& } \text{IGtQ}[n, 0] \text{ \&\& } \text{GtQ}[m, n - 1] \text{ \&\& } \text{NeQ}[m + n*p + 1, 0] \text{ \&\& } \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2352

$\text{Int}[\text{Log}[(c_)*(x_)] / ((d_) + (e_)*(x_)), x_Symbol] \text{ :> } \text{Simp}[(-e^{-1}) * \text{PolyLog}[2, 1 - c*x], x] \text{ /; } \text{FreeQ}[\{c, d, e\}, x] \text{ \&\& } \text{EqQ}[e + c*d, 0]$

Rule 2449

$\text{Int}[\text{Log}[(c_)/((d_) + (e_)*(x_))] / ((f_) + (g_)*(x_)^2), x_Symbol] \text{ :> } \text{Dist}[-e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x] / (1 - 2*d*x), x], x, 1/(d + e*x)], x] \text{ /; } \text{FreeQ}[\{c, d, e, f, g\}, x] \text{ \&\& } \text{EqQ}[c, 2*d] \text{ \&\& } \text{EqQ}[e^2*f + d^2*g, 0]$

Rule 2497

$\text{Int}[\text{Log}[u_]*(Pq_)^{(m_)}, x_Symbol] \text{ :> } \text{With}[\{C = \text{FullSimplify}[Pq^m * ((1 - u) / D[u, x])]\}, \text{Simp}[C * \text{PolyLog}[2, 1 - u], x] \text{ /; } \text{FreeQ}[C, x] \text{ /; } \text{IntegerQ}[m] \text{ \&\& } \text{PolyQ}[Pq, x] \text{ \&\& } \text{RationalFunctionQ}[u, x] \text{ \&\& } \text{LeQ}[\text{RationalFunctionExponents}[u, x][[2]], \text{Expon}[Pq, x]]$

Rule 6037

$\text{Int}[\text{((a_)} + \text{ArcTanh}[(c_)*(x_)^{(n_)}]) * (b_)^{(p_)} * (x_)^{(m_)}, x_Symbol] \text{ :> } \text{Simp}[x^{(m + 1)} * ((a + b * \text{ArcTanh}[c*x^n])^p / (m + 1)), x] - \text{Dist}[b*c*n*(p / (m + 1)), \text{Int}[x^{(m + n)} * ((a + b * \text{ArcTanh}[c*x^n])^{(p - 1)} / (1 - c^2*x^{(2*n)})), x]$

, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]

Rule 6057

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-a + b*ArcTanh[c*x])*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])*(Log[2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 6127

Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^p_)*((f_.)*(x_))^(m_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

Rule 6191

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^q_, x_Symbol] := With[{u = ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{d + ex} dx &= 2\text{Subst}\left(\int \frac{x^5(a + b \tanh^{-1}(cx))}{d + ex^2} dx, x, \sqrt{x}\right) \\
&= \frac{2\text{Subst}\left(\int x^3(a + b \tanh^{-1}(cx)) dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\text{Subst}\left(\int \frac{x^3(a + b \tanh^{-1}(cx))}{d + ex^2} dx, x, \sqrt{x}\right)}{e} \\
&= \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2e} - \frac{(2d)\text{Subst}\left(\int x(a + b \tanh^{-1}(cx)) dx, x, \sqrt{x}\right)}{e^2} \\
&= -\frac{dx(a + b \tanh^{-1}(c\sqrt{x}))}{e^2} + \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2e} + \frac{(bcd)\text{Subst}\left(\int \frac{1}{\sqrt{x}} dx, x, \sqrt{x}\right)}{e^2} \\
&= -\frac{bd\sqrt{x}}{ce^2} + \frac{b\sqrt{x}}{2c^3e} + \frac{bx^{3/2}}{6ce} - \frac{dx(a + b \tanh^{-1}(c\sqrt{x}))}{e^2} + \frac{x^2(a + b \tanh^{-1}(c\sqrt{x}))}{2e} \\
&= -\frac{bd\sqrt{x}}{ce^2} + \frac{b\sqrt{x}}{2c^3e} + \frac{bx^{3/2}}{6ce} + \frac{bd \tanh^{-1}(c\sqrt{x})}{c^2e^2} - \frac{b \tanh^{-1}(c\sqrt{x})}{2c^4e} - \frac{dx(a + b \tanh^{-1}(c\sqrt{x}))}{e^2} \\
&= -\frac{bd\sqrt{x}}{ce^2} + \frac{b\sqrt{x}}{2c^3e} + \frac{bx^{3/2}}{6ce} + \frac{bd \tanh^{-1}(c\sqrt{x})}{c^2e^2} - \frac{b \tanh^{-1}(c\sqrt{x})}{2c^4e} - \frac{dx(a + b \tanh^{-1}(c\sqrt{x}))}{e^2} \\
&= -\frac{bd\sqrt{x}}{ce^2} + \frac{b\sqrt{x}}{2c^3e} + \frac{bx^{3/2}}{6ce} + \frac{bd \tanh^{-1}(c\sqrt{x})}{c^2e^2} - \frac{b \tanh^{-1}(c\sqrt{x})}{2c^4e} - \frac{dx(a + b \tanh^{-1}(c\sqrt{x}))}{e^2}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 2.66, size = 558, normalized size = 1.21

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*ArcTanh[c*Sqrt[x]]))/(d + e*x), x]

[Out] (-6*a*d*e*x + 3*a*e^2*x^2 + 6*a*d^2*Log[d + e*x] + (b*(2*c*e*(-3*c^2*d + 2*e)*Sqrt[x] + c*e^2*Sqrt[x]*(-1 + c^2*x) - 6*(c^2*d - e)*e*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]] + 3*e^2*(-1 + c^2*x)^2*ArcTanh[c*Sqrt[x]] - 6*c^4*d^2*(ArcTanh[c*Sqrt[x]]*(ArcTanh[c*Sqrt[x]] + 2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]]))]) -

$$\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c*\text{Sqrt}[x]])}] + 3*c^4*d^2*(2*\text{ArcTanh}[c*\text{Sqrt}[x]]^2 - (4*I)*\text{ArcSin}[\text{Sqrt}[(c^2*d)/(c^2*d + e)]]*\text{ArcTanh}[(c*e*\text{Sqrt}[x])/ \text{Sqrt}[-(c^2*d*e)]] + 2*((-I)*\text{ArcSin}[\text{Sqrt}[(c^2*d)/(c^2*d + e)]] + \text{ArcTanh}[c*\text{Sqrt}[x]])*\text{Log}[(-2*\text{Sqrt}[-(c^2*d*e)] + e*(-1 + E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])})) + c^2*d*(1 + E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])}))]/((c^2*d + e)*E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])}))] + 2*(I*\text{ArcSin}[\text{Sqrt}[(c^2*d)/(c^2*d + e)]] + \text{ArcTanh}[c*\text{Sqrt}[x]])*\text{Log}[(2*\text{Sqrt}[-(c^2*d*e)] + e*(-1 + E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])})) + c^2*d*(1 + E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])}))]/((c^2*d + e)*E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])}))] - \text{PolyLog}[2, (-(c^2*d) + e - 2*\text{Sqrt}[-(c^2*d*e)])/((c^2*d + e)*E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])}))] - \text{PolyLog}[2, (-(c^2*d) + e + 2*\text{Sqrt}[-(c^2*d*e)])/((c^2*d + e)*E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])}))])]/c^4/(6*e^3)$$

Maple [A]

time = 0.65, size = 706, normalized size = 1.53

method	result
derivativedivides	$\frac{-\frac{a c^6 dx}{e^2} + \frac{a c^6 x^2}{2e} + \frac{a c^6 d^2 \ln(c^2 e x + c^2 d)}{e^3} - \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) dx}{e^2} + \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) x^2}{2e} + \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) d^2 \ln(c^2 e x + c^2 d)}{e^3}}{-\frac{a c^6 dx}{e^2} + \frac{a c^6 x^2}{2e} + \frac{a c^6 d^2 \ln(c^2 e x + c^2 d)}{e^3} - \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) dx}{e^2} + \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) x^2}{2e} + \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) d^2 \ln(c^2 e x + c^2 d)}{e^3}}$
default	$\frac{-\frac{a c^6 dx}{e^2} + \frac{a c^6 x^2}{2e} + \frac{a c^6 d^2 \ln(c^2 e x + c^2 d)}{e^3} - \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) dx}{e^2} + \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) x^2}{2e} + \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) d^2 \ln(c^2 e x + c^2 d)}{e^3}}{-\frac{a c^6 dx}{e^2} + \frac{a c^6 x^2}{2e} + \frac{a c^6 d^2 \ln(c^2 e x + c^2 d)}{e^3} - \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) dx}{e^2} + \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) x^2}{2e} + \frac{b c^6 \operatorname{arctanh}(c \sqrt{x}) d^2 \ln(c^2 e x + c^2 d)}{e^3}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arctanh(c*x^(1/2)))/(e*x+d), x, method=_RETURNVERBOSE)`

[Out]
$$\frac{2}{c^6} * (-\frac{1}{2} * a * c^6 / e^2 * d * x + \frac{1}{4} * a * c^6 / e * x^2 + \frac{1}{2} * a * c^6 * d^2 / e^3 * \ln(c^2 * e * x + c^2 * d) - \frac{1}{2} * b * c^6 * \operatorname{arctanh}(c * x^{(1/2)}) / e^2 * d * x + \frac{1}{4} * b * c^6 * \operatorname{arctanh}(c * x^{(1/2)}) / e * x^2 + \frac{1}{2} * b * c^6 * \operatorname{arctanh}(c * x^{(1/2)}) * d^2 / e^3 * \ln(c^2 * e * x + c^2 * d) - \frac{1}{2} * b * c^5 / e^2 * d * x^{(1/2)} + \frac{1}{12} * b * c^5 / e * x^{(3/2)} + \frac{1}{4} * b * c^3 / e * x^{(1/2)} - \frac{1}{4} * b * c^4 / e^2 * \ln(c * x^{(1/2)} - 1) * d + \frac{1}{8} * b * c^2 * \ln(c * x^{(1/2)} - 1) / e + \frac{1}{4} * b * c^4 / e^2 * \ln(1 + c * x^{(1/2)}) * d - \frac{1}{8} * b * c^2 * \ln(1 + c * x^{(1/2)}) / e - \frac{1}{4} * b * c^6 * d^2 / e^3 * \ln(1 + c * x^{(1/2)}) * \ln(c^2 * e * x + c^2 * d) + \frac{1}{4} * b * c^6 * d^2 / e^3 * \ln(1 + c * x^{(1/2)}) * \ln((c * (-d * e)^{(1/2)} - e * (1 + c * x^{(1/2)}) + e) / (c * (-d * e)^{(1/2)} + e)) + \frac{1}{4} * b * c^6 * d^2 / e^3 * \ln(1 + c * x^{(1/2)}) * \ln((c * (-d * e)^{(1/2)} + e * (1 + c * x^{(1/2)}) - e) / (c * (-d * e)^{(1/2)} - e)) + \frac{1}{4} * b * c^6 * d^2 / e^3 * \operatorname{dilog}((c * (-d * e)^{(1/2)} - e * (1 + c * x^{(1/2)}) + e) / (c * (-d * e)^{(1/2)} + e)) + \frac{1}{4} * b * c^6 * d^2 / e^3 * \operatorname{dilog}((c * (-d * e)^{(1/2)} + e * (1 + c * x^{(1/2)}) - e) / (c * (-d * e)^{(1/2)} - e)) + \frac{1}{4} * b * c^6 * d^2 / e^3 * \ln(c * x^{(1/2)} - 1) * \ln(c^2 * e * x + c^2 * d) - \frac{1}{4} * b * c^6 * d^2 / e^3 * \ln(c * x^{(1/2)} - 1) * \ln((c * (-d * e)^{(1/2)} - e * (c * x^{(1/2)} - 1) - e) / (c * (-d * e)^{(1/2)} - e)) - \frac{1}{4} * b * c^6 * d^2 / e^3 * \ln(c * x^{(1/2)} - 1) * \ln((c * (-d * e)^{(1/2)} + e * (c * x^{(1/2)} - 1) + e) / (c * (-d * e)^{(1/2)} + e)) - \frac{1}{4} * b * c^6 * d^2 / e^3 * \operatorname{dilog}((c * (-d * e)^{(1/2)} - e * (c * x^{(1/2)} - 1) - e) / (c * (-d * e)^{(1/2)} - e)) - \frac{1}{4} * b * c^6 * d^2 / e^3 * \operatorname{dilog}((c * (-d * e)^{(1/2)} + e * (c * x^{(1/2)} - 1) + e) / (c * (-d * e)^{(1/2)} + e))$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*d^2*e^(-3)*log(x*e + d) + (x^2*e - 2*d*x)*e^(-2))*a + b*integrate(1/2*x^2*log(c*sqrt(x) + 1)/(x*e + d), x) - b*integrate(1/2*x^2*log(-c*sqrt(x) + 1)/(x*e + d), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*sqrt(x)) + a*x^2)/(x*e + d), x)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(1/2)))/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)*x^2/(e*x + d), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (a + b \operatorname{atanh}(c \sqrt{x}))}{d + e x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x^(1/2))))/(d + e*x),x)
```

```
[Out] int((x^2*(a + b*atanh(c*x^(1/2))))/(d + e*x), x)
```

$$3.45 \quad \int \frac{x \left(a + b \tanh^{-1} \left(c \sqrt{x} \right) \right)}{d + ex} dx$$

Optimal. Leaf size=374

$$\frac{b\sqrt{x}}{ce} - \frac{b \tanh^{-1}(c\sqrt{x})}{c^2e} + \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{e} + \frac{2d(a + b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2}{1+c\sqrt{x}}\right)}{e^2} - \frac{d(a + b \tanh^{-1}(c\sqrt{x}))}{e}$$

[Out] $-b \operatorname{arctanh}(c \sqrt{x}) / c^2 / e + x (a + b \operatorname{arctanh}(c \sqrt{x})) / e + 2 d (a + b \operatorname{arctanh}(c \sqrt{x})) \ln(2 / (1 + c \sqrt{x})) / e^2 - d (a + b \operatorname{arctanh}(c \sqrt{x})) \ln(2 c ((-d)^{1/2} - e^{1/2}) \sqrt{x}) / (c (-d)^{1/2} - e^{1/2}) / (1 + c \sqrt{x}) / e^2 - d (a + b \operatorname{arctanh}(c \sqrt{x})) \ln(2 c ((-d)^{1/2} + e^{1/2}) \sqrt{x}) / (c (-d)^{1/2} + e^{1/2}) / (1 + c \sqrt{x}) / e^2 - b d \operatorname{polylog}(2, 1 - 2 / (1 + c \sqrt{x})) / e^2 + 1/2 b d \operatorname{polylog}(2, 1 - 2 c ((-d)^{1/2} - e^{1/2}) \sqrt{x}) / (c (-d)^{1/2} - e^{1/2}) / (1 + c \sqrt{x}) / e^2 + 1/2 b d \operatorname{polylog}(2, 1 - 2 c ((-d)^{1/2} + e^{1/2}) \sqrt{x}) / (c (-d)^{1/2} + e^{1/2}) / (1 + c \sqrt{x}) / e^2 + b x^{1/2} / c / e$

Rubi [A]

time = 0.37, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {45, 6127, 6037, 327, 212, 6191, 6057, 2449, 2352, 2497}

$$\frac{2d \log\left(\frac{1}{\sqrt{x+1}}\right) (a + b \operatorname{tanh}^{-1}(c\sqrt{x}))}{e^2} - \frac{d(a + b \operatorname{tanh}^{-1}(c\sqrt{x})) \log\left(\frac{a(\sqrt{-d} - \sqrt{c}\sqrt{x})}{(c\sqrt{x+1})(\sqrt{-d} - \sqrt{c}\sqrt{x})}\right)}{e^2} - \frac{d(a + b \operatorname{tanh}^{-1}(c\sqrt{x})) \log\left(\frac{a(\sqrt{-d} + \sqrt{c}\sqrt{x})}{(c\sqrt{x+1})(\sqrt{-d} + \sqrt{c}\sqrt{x})}\right)}{e^2} + \frac{x(a + b \operatorname{tanh}^{-1}(c\sqrt{x}))}{e} - \frac{b \operatorname{tanh}^{-1}(c\sqrt{x})}{c^2e} - \frac{b d \ln\left(1 - \frac{1}{\sqrt{x+1}}\right)}{e^2} + \frac{b d \ln\left(1 - \frac{a(\sqrt{-d} - \sqrt{c}\sqrt{x})}{(c\sqrt{x+1})(\sqrt{-d} - \sqrt{c}\sqrt{x})}\right)}{2e^2} + \frac{b d \ln\left(1 - \frac{a(\sqrt{-d} + \sqrt{c}\sqrt{x})}{(c\sqrt{x+1})(\sqrt{-d} + \sqrt{c}\sqrt{x})}\right)}{2e^2} + \frac{b\sqrt{x}}{ce}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x(a + b \operatorname{ArcTanh}[c \operatorname{Sqrt}[x]])) / (d + e x), x]$

[Out] $(b \operatorname{Sqrt}[x]) / (c e) - (b \operatorname{ArcTanh}[c \operatorname{Sqrt}[x]]) / (c^2 e) + (x(a + b \operatorname{ArcTanh}[c \operatorname{Sqrt}[x]])) / e + (2 d (a + b \operatorname{ArcTanh}[c \operatorname{Sqrt}[x]]) \operatorname{Log}[2 / (1 + c \operatorname{Sqrt}[x])] / e^2 - (d (a + b \operatorname{ArcTanh}[c \operatorname{Sqrt}[x]]) \operatorname{Log}[(2 c (\operatorname{Sqrt}[-d] - \operatorname{Sqrt}[e] \operatorname{Sqrt}[x])) / ((c \operatorname{Sqrt}[-d] - \operatorname{Sqrt}[e]) (1 + c \operatorname{Sqrt}[x]))]) / e^2 - (d (a + b \operatorname{ArcTanh}[c \operatorname{Sqrt}[x]]) \operatorname{Log}[(2 c (\operatorname{Sqrt}[-d] + \operatorname{Sqrt}[e] \operatorname{Sqrt}[x])) / ((c \operatorname{Sqrt}[-d] + \operatorname{Sqrt}[e]) (1 + c \operatorname{Sqrt}[x]))]) / e^2 - (b d \operatorname{PolyLog}[2, 1 - 2 / (1 + c \operatorname{Sqrt}[x])] / e^2 + (b d \operatorname{PolyLog}[2, 1 - (2 c (\operatorname{Sqrt}[-d] - \operatorname{Sqrt}[e] \operatorname{Sqrt}[x])) / ((c \operatorname{Sqrt}[-d] - \operatorname{Sqrt}[e]) (1 + c \operatorname{Sqrt}[x]))]) / (2 e^2) + (b d \operatorname{PolyLog}[2, 1 - (2 c (\operatorname{Sqrt}[-d] + \operatorname{Sqrt}[e] \operatorname{Sqrt}[x])) / ((c \operatorname{Sqrt}[-d] + \operatorname{Sqrt}[e]) (1 + c \operatorname{Sqrt}[x]))]) / (2 e^2)$

Rule 45

$\operatorname{Int}[(a_. + (b_.)(x_)^m) * ((c_.) + (d_.)(x_)^n), x_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b x)^m (c + d x)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x \ \&\& \operatorname{NeQ}[b c - a d, 0] \ \&\& \operatorname{IGtQ}[m, 0] \ \&\& (!\operatorname{IntegerQ}[n] || (\operatorname{EqQ}[c, 0] \ \&\& \operatorname{LeQ}[7 m + 4 n + 4, 0]) || \operatorname{LtQ}[9 m + 5 (n + 1), 0]) || \operatorname{GtQ}[m + n + 2, 0])$

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n
- 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
&& IntegerQ[m])) && NeQ[m, -1]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x)
)/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x))]/e), x]) /; FreeQ[{a, b, c, d,
```

e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)*((f_.)*(x_.))^(m_.))/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6191

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)*((f_.)*(x_.))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] := With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{d + ex} dx &= 2\text{Subst}\left(\int \frac{x^3(a + b \tanh^{-1}(cx))}{d + ex^2} dx, x, \sqrt{x}\right) \\
&= \frac{2\text{Subst}\left(\int x(a + b \tanh^{-1}(cx)) dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\text{Subst}\left(\int \frac{x(a + b \tanh^{-1}(cx))}{d + ex^2} dx, x, \sqrt{x}\right)}{e} \\
&= \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{e} - \frac{(bc)\text{Subst}\left(\int \frac{x^2}{1 - c^2x^2} dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\text{Subst}\left(\int \frac{x}{d + ex^2} dx, x, \sqrt{x}\right)}{e} \\
&= \frac{b\sqrt{x}}{ce} + \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{e} + \frac{d\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{\sqrt{-d} - \sqrt{e}x} dx, x, \sqrt{x}\right)}{e^{3/2}} - \frac{2d(a + b \tanh^{-1}(c\sqrt{x}))}{e} \\
&= \frac{b\sqrt{x}}{ce} - \frac{b \tanh^{-1}(c\sqrt{x})}{c^2e} + \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{e} + \frac{2d(a + b \tanh^{-1}(c\sqrt{x}))}{e} \\
&= \frac{b\sqrt{x}}{ce} - \frac{b \tanh^{-1}(c\sqrt{x})}{c^2e} + \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{e} + \frac{2d(a + b \tanh^{-1}(c\sqrt{x}))}{e} \\
&= \frac{b\sqrt{x}}{ce} - \frac{b \tanh^{-1}(c\sqrt{x})}{c^2e} + \frac{x(a + b \tanh^{-1}(c\sqrt{x}))}{e} + \frac{2d(a + b \tanh^{-1}(c\sqrt{x}))}{e}
\end{aligned}$$

Mathematica [A]

time = 1.03, size = 337, normalized size = 0.90

$$\frac{2acx - 2ad \log(d + ex) + \frac{2\left(a\sqrt{x} + b \tanh^{-1}(c\sqrt{x})\right)\left(\sqrt{x}\right)^2 \tanh^{-1}(c\sqrt{x}) - (a + b \tanh^{-1}(c\sqrt{x}))\left(\sqrt{x}\right)^2 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(c\sqrt{x})}\right)\right)}{2c^2} - \text{li}\left(2 \tanh^{-1}(c\sqrt{x})\left(-\tanh^{-1}(c\sqrt{x}) + \log\left(1 + \frac{(d + e)x^2 \tanh^{-1}(c\sqrt{x})}{d - 2cx - d\sqrt{e-x}}\right) + \log\left(1 + \frac{(d + e)x^2 \tanh^{-1}(c\sqrt{x})}{d + 2cx + d\sqrt{e-x}}\right)\right) + \text{PolyLog}\left(2, \frac{(d + e)x^2 \tanh^{-1}(c\sqrt{x})}{d - 2cx - d\sqrt{e-x}}\right) + \text{PolyLog}\left(2, \frac{(d + e)x^2 \tanh^{-1}(c\sqrt{x})}{d + 2cx + d\sqrt{e-x}}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*ArcTanh[c*Sqrt[x]]))/(d + e*x), x]

[Out] (2*a*e*x - 2*a*d*Log[d + e*x] + (2*b*(c*e*Sqrt[x] + c^2*d*ArcTanh[c*Sqrt[x]]^2 + ArcTanh[c*Sqrt[x]]*(-e + c^2*e*x + 2*c^2*d*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])])) - c^2*d*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])]))/c^2 - b*d*(2*ArcTanh[c*Sqrt[x]]*(-ArcTanh[c*Sqrt[x]] + Log[1 + ((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]])]))/(c^2*d - 2*c*Sqrt[-d]*Sqrt[e] - e) + Log[1 + ((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]])]))/(c^2*d + 2*c*Sqrt[-d]*Sqrt[e] - e) + PolyLog[2, -((

$$c^2*d + e)*E^{(2*ArcTanh[c*sqrt[x]])}/(c^2*d - 2*c*sqrt[-d]*sqrt[e] - e))] +$$

$$\text{PolyLog}[2, -(((c^2*d + e)*E^{(2*ArcTanh[c*sqrt[x]])})/(c^2*d + 2*c*sqrt[-d]*$$

$$\text{sqrt}[e] - e)))]/(2*e^2)$$

Maple [A]

time = 0.64, size = 589, normalized size = 1.57

method	result
derivativedivides	$\frac{\frac{a c^4 x}{e} - \frac{a c^4 d \ln(c^2 e x + c^2 d)}{e^2} + \frac{b c^4 \operatorname{arctanh}(c \sqrt{x}) x}{e} - \frac{b c^4 \operatorname{arctanh}(c \sqrt{x}) d \ln(c^2 e x + c^2 d)}{e^2} + \frac{b c^4 d \ln(1 + c \sqrt{x}) \ln(c^2 e x + c^2 d)}{2 e^2}}$
default	$\frac{\frac{a c^4 x}{e} - \frac{a c^4 d \ln(c^2 e x + c^2 d)}{e^2} + \frac{b c^4 \operatorname{arctanh}(c \sqrt{x}) x}{e} - \frac{b c^4 \operatorname{arctanh}(c \sqrt{x}) d \ln(c^2 e x + c^2 d)}{e^2} + \frac{b c^4 d \ln(1 + c \sqrt{x}) \ln(c^2 e x + c^2 d)}{2 e^2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x,method=_RETURNVERBOSE)`

[Out] $2/c^4*(1/2*a*c^4/e*x-1/2*a*c^4*d/e^2*\ln(c^2*e*x+c^2*d)+1/2*b*c^4*arctanh(c*x^{(1/2)})/e*x-1/2*b*c^4*arctanh(c*x^{(1/2)})*d/e^2*\ln(c^2*e*x+c^2*d)+1/4*b*c^4*d/e^2*\ln(1+c*x^{(1/2)})*\ln(c^2*e*x+c^2*d)-1/4*b*c^4*d/e^2*\ln(1+c*x^{(1/2)})*\ln((c*(-d*e)^{(1/2)}-e*(1+c*x^{(1/2)}))+e)/(c*(-d*e)^{(1/2)}+e))-1/4*b*c^4*d/e^2*\ln(1+c*x^{(1/2)})*\ln((c*(-d*e)^{(1/2)}+e*(1+c*x^{(1/2)}))-e)/(c*(-d*e)^{(1/2)}-e))-1/4*b*c^4*d/e^2*dilog((c*(-d*e)^{(1/2)}-e*(1+c*x^{(1/2)}))+e)/(c*(-d*e)^{(1/2)}+e))-1/4*b*c^4*d/e^2*dilog((c*(-d*e)^{(1/2)}+e*(1+c*x^{(1/2)}))-e)/(c*(-d*e)^{(1/2)}-e))-1/4*b*c^4*d/e^2*\ln(c*x^{(1/2)}-1)*\ln(c^2*e*x+c^2*d)+1/4*b*c^4*d/e^2*\ln(c*x^{(1/2)}-1)*\ln((c*(-d*e)^{(1/2)}-e*(c*x^{(1/2)}-1))-e)/(c*(-d*e)^{(1/2)}-e))+1/4*b*c^4*d/e^2*\ln(c*x^{(1/2)}-1)*\ln((c*(-d*e)^{(1/2)}+e*(c*x^{(1/2)}-1)+e)/(c*(-d*e)^{(1/2)}+e))+1/4*b*c^4*d/e^2*dilog((c*(-d*e)^{(1/2)}-e*(c*x^{(1/2)}-1))-e)/(c*(-d*e)^{(1/2)}-e))+1/4*b*c^4*d/e^2*dilog((c*(-d*e)^{(1/2)}+e*(c*x^{(1/2)}-1)+e)/(c*(-d*e)^{(1/2)}+e))+1/2*b*c^3/e*x^{(1/2)}+1/4*b*c^2*\ln(c*x^{(1/2)}-1)/e-1/4*b*c^2*\ln(1+c*x^{(1/2)})/e)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="maxima")`

[Out] $-(d*e^{(-2)}*\log(x*e + d) - x*e^{(-1)})*a + b*\text{integrate}(1/2*x*\log(c*\text{sqrt}(x) + 1)/(x*e + d), x) - b*\text{integrate}(1/2*x*\log(-c*\text{sqrt}(x) + 1)/(x*e + d), x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="fricas")``[Out] integral((b*x*arctanh(c*sqrt(x)) + a*x)/(x*e + d), x)`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{atanh}(c\sqrt{x}))}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*(a+b*atanh(c*x**(1/2)))/(e*x+d),x)``[Out] Integral(x*(a + b*atanh(c*sqrt(x)))/(d + e*x), x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="giac")``[Out] integrate((b*arctanh(c*sqrt(x)) + a)*x/(e*x + d), x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x(a + b \operatorname{atanh}(c\sqrt{x}))}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((x*(a + b*atanh(c*x^(1/2))))/(d + e*x),x)``[Out] int((x*(a + b*atanh(c*x^(1/2))))/(d + e*x), x)`

$$3.46 \quad \int \frac{a+b \tanh^{-1}\left(c\sqrt{x}\right)}{d+ex} dx$$

Optimal. Leaf size=318

$$\frac{2(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2}{1+c\sqrt{x}}\right)}{e} + \frac{(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2c(\sqrt{-d}-\sqrt{e}\sqrt{x})}{(c\sqrt{-d}-\sqrt{e})(1+c\sqrt{x})}\right)}{e} + \dots$$

[Out] $-2*(a+b*\operatorname{arctanh}(c*x^{(1/2)}))*\ln(2/(1+c*x^{(1/2)}))/e+(a+b*\operatorname{arctanh}(c*x^{(1/2)}))*\ln(2*c*((-d)^{(1/2)}-e^{(1/2)}*x^{(1/2)})/(c*(-d)^{(1/2)}-e^{(1/2)})/(1+c*x^{(1/2)}))/e+(a+b*\operatorname{arctanh}(c*x^{(1/2)}))*\ln(2*c*((-d)^{(1/2)}+e^{(1/2)}*x^{(1/2)})/(c*(-d)^{(1/2)}+e^{(1/2)})/(1+c*x^{(1/2)}))/e+b*\operatorname{polylog}(2,1-2/(1+c*x^{(1/2)}))/e-1/2*b*\operatorname{polylog}(2,1-2*c*((-d)^{(1/2)}-e^{(1/2)}*x^{(1/2)})/(c*(-d)^{(1/2)}-e^{(1/2)})/(1+c*x^{(1/2)}))/e-1/2*b*\operatorname{polylog}(2,1-2*c*((-d)^{(1/2)}+e^{(1/2)}*x^{(1/2)})/(c*(-d)^{(1/2)}+e^{(1/2)})/(1+c*x^{(1/2)}))/e$

Rubi [A]

time = 0.23, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6069, 6191, 6057, 2449, 2352, 2497}

$$\frac{(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2(\sqrt{-d}-\sqrt{e}\sqrt{x})}{(c\sqrt{x}+1)(\sqrt{-d}-\sqrt{e})}\right)}{e} + \frac{(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2(\sqrt{-d}+\sqrt{e}\sqrt{x})}{(c\sqrt{x}+1)(\sqrt{-d}+\sqrt{e})}\right)}{e} - 2 \log\left(\frac{2}{1+c\sqrt{x}}\right) \frac{(a+b \tanh^{-1}(c\sqrt{x}))}{e} - \frac{\operatorname{Li}_2\left(1-\frac{2(\sqrt{-d}-\sqrt{e}\sqrt{x})}{(c\sqrt{-d}-\sqrt{e})(\sqrt{x}+1)}\right)}{2e} - \frac{\operatorname{Li}_2\left(1-\frac{2(\sqrt{-d}+\sqrt{e}\sqrt{x})}{(\sqrt{-d}+e\sqrt{e})(\sqrt{x}+1)}\right)}{2e} + \frac{\operatorname{Li}_2\left(1-\frac{2}{\sqrt{x}+1}\right)}{e}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])/(d + e*x), x]$

[Out] $(-2*(a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])*\operatorname{Log}[2/(1 + c*\operatorname{Sqrt}[x])])/e + ((a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])*\operatorname{Log}[(2*c*(\operatorname{Sqrt}[-d] - \operatorname{Sqrt}[e]*\operatorname{Sqrt}[x]))/((c*\operatorname{Sqrt}[-d] - \operatorname{Sqrt}[e])*(1 + c*\operatorname{Sqrt}[x]))])/e + ((a + b*\operatorname{ArcTanh}[c*\operatorname{Sqrt}[x]])*\operatorname{Log}[(2*c*(\operatorname{Sqrt}[-d] + \operatorname{Sqrt}[e]*\operatorname{Sqrt}[x]))/((c*\operatorname{Sqrt}[-d] + \operatorname{Sqrt}[e])*(1 + c*\operatorname{Sqrt}[x]))])/e + (b*\operatorname{PolyLog}[2, 1 - 2/(1 + c*\operatorname{Sqrt}[x])])/e - (b*\operatorname{PolyLog}[2, 1 - (2*c*(\operatorname{Sqrt}[-d] - \operatorname{Sqrt}[e]*\operatorname{Sqrt}[x]))/((c*\operatorname{Sqrt}[-d] - \operatorname{Sqrt}[e])*(1 + c*\operatorname{Sqrt}[x]))])/e - (b*\operatorname{PolyLog}[2, 1 - (2*c*(\operatorname{Sqrt}[-d] + \operatorname{Sqrt}[e]*\operatorname{Sqrt}[x]))/((c*\operatorname{Sqrt}[-d] + \operatorname{Sqrt}[e])*(1 + c*\operatorname{Sqrt}[x]))])/e$

Rule 2352

$\operatorname{Int}[\operatorname{Log}[(c_*)*(x_)]/((d_*) + (e_*)*(x_)), x_Symbol] \rightarrow \operatorname{Simp}[(-e^{-1})*\operatorname{PolyLog}[2, 1 - c*x], x] /; \operatorname{FreeQ}\{c, d, e, x\} \ \&\& \operatorname{EqQ}[e + c*d, 0]$

Rule 2449

$\operatorname{Int}[\operatorname{Log}[(c_*)/((d_*) + (e_*)*(x_))]/((f_*) + (g_*)*(x_)^2), x_Symbol] \rightarrow \operatorname{Dist}[-e/g, \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \operatorname{FreeQ}\{$

c, d, e, f, g, x && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 2497

Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 6057

Int[((a_) + ArcTanh[(c_)*(x_)])*(b_)/((d_) + (e_)*(x_)), x_Symbol] := Simp[(-a + b*ArcTanh[c*x])*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])*(Log[2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/e), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 6069

Int[((a_) + ArcTanh[(c_)*(x_)^(n_)])*(b_)/((d_) + (e_)*(x_)), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*((a + b*ArcTanh[c*x^(k*n)])/(d + e*x^k)), x], x, x^(1/k)], x] /; FreeQ[{a, b, c, d, e}, x] && FractionQ[n]

Rule 6191

Int[((a_) + ArcTanh[(c_)*(x_)])*(b_)^(p_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(q_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{d + ex} dx &= 2 \text{Subst} \left(\int \frac{x(a + b \tanh^{-1}(cx))}{d + ex^2} dx, x, \sqrt{x} \right) \\
&= 2 \text{Subst} \left(\int \left(-\frac{a + b \tanh^{-1}(cx)}{2\sqrt{e}(\sqrt{-d} - \sqrt{e}x)} + \frac{a + b \tanh^{-1}(cx)}{2\sqrt{e}(\sqrt{-d} + \sqrt{e}x)} \right) dx, x, \sqrt{x} \right) \\
&= -\frac{\text{Subst} \left(\int \frac{a + b \tanh^{-1}(cx)}{\sqrt{-d} - \sqrt{e}x} dx, x, \sqrt{x} \right)}{\sqrt{e}} + \frac{\text{Subst} \left(\int \frac{a + b \tanh^{-1}(cx)}{\sqrt{-d} + \sqrt{e}x} dx, x, \sqrt{x} \right)}{\sqrt{e}} \\
&= -\frac{2(a + b \tanh^{-1}(c\sqrt{x})) \log \left(\frac{2}{1 + c\sqrt{x}} \right)}{e} + \frac{(a + b \tanh^{-1}(c\sqrt{x})) \log \left(\frac{2c}{c\sqrt{x}} \right)}{e} \\
&= -\frac{2(a + b \tanh^{-1}(c\sqrt{x})) \log \left(\frac{2}{1 + c\sqrt{x}} \right)}{e} + \frac{(a + b \tanh^{-1}(c\sqrt{x})) \log \left(\frac{2c}{c\sqrt{x}} \right)}{e} \\
&= -\frac{2(a + b \tanh^{-1}(c\sqrt{x})) \log \left(\frac{2}{1 + c\sqrt{x}} \right)}{e} + \frac{(a + b \tanh^{-1}(c\sqrt{x})) \log \left(\frac{2c}{c\sqrt{x}} \right)}{e}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.48, size = 432, normalized size = 1.36

$$\frac{(-2 \operatorname{atanh}^{-1}(c\sqrt{x}) + b \operatorname{atanh}^{-1}(c\sqrt{x})) \log\left(\frac{2}{1+c\sqrt{x}}\right) + 2 \operatorname{atanh}^{-1}(c\sqrt{x}) \log\left(\frac{2c}{c\sqrt{x}}\right) - (-2 \operatorname{atanh}^{-1}(c\sqrt{x}) + b \operatorname{atanh}^{-1}(c\sqrt{x})) \log\left(\frac{2}{1+c\sqrt{x}}\right) + (-2 \operatorname{atanh}^{-1}(c\sqrt{x}) + b \operatorname{atanh}^{-1}(c\sqrt{x})) \log\left(\frac{2c}{c\sqrt{x}}\right)}{e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(d + e*x), x]

[Out] (a*Log[d + e*x])/e - (b*(-2*ArcTanh[c*Sqrt[x]]^2 + (4*I)*ArcSin[Sqrt[(c^2*d)/(c^2*d + e)]]*ArcTanh[(c*e*Sqrt[x])/Sqrt[-(c^2*d*e)]] + 2*ArcTanh[c*Sqrt[x]]*(ArcTanh[c*Sqrt[x]] + 2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])])) - 2*((-I)*ArcSin[Sqrt[(c^2*d)/(c^2*d + e)]] + ArcTanh[c*Sqrt[x]])*Log[(-2*Sqrt[-(c^2*d*e)] + e*(-1 + E^(2*ArcTanh[c*Sqrt[x]])) + c^2*d*(1 + E^(2*ArcTanh[c*Sqrt[x]])))]/((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]])) - 2*(I*ArcSin[Sqrt[(c^2*d)/(c^2*d + e)]] + ArcTanh[c*Sqrt[x]])*Log[(2*Sqrt[-(c^2*d*e)] + e*(-1 + E^(2*ArcTanh[c*Sqrt[x]])) + c^2*d*(1 + E^(2*ArcTanh[c*Sqrt[x]])))]/((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]])) - 2*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])] + PolyLo

$g[2, (-(c^2*d) + e - 2*\sqrt{-(c^2*d*e)})/((c^2*d + e)*E^{(2*ArcTanh[c*\sqrt{x}]]))] + PolyLog[2, (-(c^2*d) + e + 2*\sqrt{-(c^2*d*e)})/((c^2*d + e)*E^{(2*ArcTanh[c*\sqrt{x}]))})]/(2*e)$

Maple [A]

time = 0.56, size = 505, normalized size = 1.59

method	result
derivativedivides	$\frac{\frac{a c^2 \ln(c^2 e x + c^2 d)}{e} + \frac{b c^2 \ln(c^2 e x + c^2 d) \operatorname{arctanh}(c \sqrt{x})}{e} - \frac{b c^2 \ln(1 + c \sqrt{x}) \ln(c^2 e x + c^2 d)}{2e} + \frac{b c^2 \ln(1 + c \sqrt{x}) \ln\left(\frac{c \sqrt{-d e}}{c \sqrt{e}}\right)}{2e}}{\frac{a c^2 \ln(c^2 e x + c^2 d)}{e} + \frac{b c^2 \ln(c^2 e x + c^2 d) \operatorname{arctanh}(c \sqrt{x})}{e} - \frac{b c^2 \ln(1 + c \sqrt{x}) \ln(c^2 e x + c^2 d)}{2e} + \frac{b c^2 \ln(1 + c \sqrt{x}) \ln\left(\frac{c \sqrt{-d e}}{c \sqrt{e}}\right)}{2e}}$
default	$\frac{\frac{a c^2 \ln(c^2 e x + c^2 d)}{e} + \frac{b c^2 \ln(c^2 e x + c^2 d) \operatorname{arctanh}(c \sqrt{x})}{e} - \frac{b c^2 \ln(1 + c \sqrt{x}) \ln(c^2 e x + c^2 d)}{2e} + \frac{b c^2 \ln(1 + c \sqrt{x}) \ln\left(\frac{c \sqrt{-d e}}{c \sqrt{e}}\right)}{2e}}{\frac{a c^2 \ln(c^2 e x + c^2 d)}{e} + \frac{b c^2 \ln(c^2 e x + c^2 d) \operatorname{arctanh}(c \sqrt{x})}{e} - \frac{b c^2 \ln(1 + c \sqrt{x}) \ln(c^2 e x + c^2 d)}{2e} + \frac{b c^2 \ln(1 + c \sqrt{x}) \ln\left(\frac{c \sqrt{-d e}}{c \sqrt{e}}\right)}{2e}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^(1/2)))/(e*x+d),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{2}{c^2} \left(\frac{1}{2} a c^2 / e \ln(c^2 e x + c^2 d) + \frac{1}{2} b c^2 / e \ln(c^2 e x + c^2 d) \operatorname{arctanh}(c x^{1/2}) - \frac{1}{4} b c^2 / e \ln(1 + c x^{1/2}) \ln(c^2 e x + c^2 d) + \frac{1}{4} b c^2 \ln(1 + c x^{1/2}) / e \ln((c(-d e)^{1/2} - e(1 + c x^{1/2})) / (c(-d e)^{1/2} + e)) + \frac{1}{4} b c^2 \ln(1 + c x^{1/2}) / e \ln((c(-d e)^{1/2} + e(1 + c x^{1/2})) / (c(-d e)^{1/2} - e)) + \frac{1}{4} b c^2 / e \operatorname{dilog}((c(-d e)^{1/2} - e(1 + c x^{1/2})) / (c(-d e)^{1/2} + e)) + \frac{1}{4} b c^2 / e \operatorname{dilog}((c(-d e)^{1/2} + e(1 + c x^{1/2})) / (c(-d e)^{1/2} - e)) + \frac{1}{4} b c^2 / e \ln(c x^{1/2} - 1) \ln(c^2 e x + c^2 d) - \frac{1}{4} b c^2 \ln(c x^{1/2} - 1) / e \ln((c(-d e)^{1/2} - e(c x^{1/2} - 1) - e) / (c(-d e)^{1/2} - e)) - \frac{1}{4} b c^2 \ln(c x^{1/2} - 1) / e \ln((c(-d e)^{1/2} + e(c x^{1/2} - 1) + e) / (c(-d e)^{1/2} + e)) - \frac{1}{4} b c^2 / e \operatorname{dilog}((c(-d e)^{1/2} - e(c x^{1/2} - 1) - e) / (c(-d e)^{1/2} - e)) - \frac{1}{4} b c^2 / e \operatorname{dilog}((c(-d e)^{1/2} + e(c x^{1/2} - 1) + e) / (c(-d e)^{1/2} + e)) \right)$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="maxima")`

[Out]
$$a e^{-1} \log(x e + d) + b \operatorname{integrate}(1/2 \log(c \sqrt{x} + 1) / (x e + d), x) - b \operatorname{integrate}(1/2 \log(-c \sqrt{x} + 1) / (x e + d), x)$$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="fricas")`

[Out] `integral((b*arctanh(c*sqrt(x)) + a)/(x*e + d), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}(c\sqrt{x})}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**(1/2)))/(e*x+d),x)`

[Out] `Integral((a + b*atanh(c*sqrt(x)))/(d + e*x), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*sqrt(x)) + a)/(e*x + d), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{atanh}(c\sqrt{x})}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c*x^(1/2)))/(d + e*x),x)`

[Out] `int((a + b*atanh(c*x^(1/2)))/(d + e*x), x)`

$$3.47 \quad \int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x(d+ex)} dx$$

Optimal. Leaf size=358

$$\frac{2(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2}{1+c\sqrt{x}}\right)}{d} - \frac{(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2c(\sqrt{-d}-\sqrt{e}\sqrt{x})}{(c\sqrt{-d}-\sqrt{e})(1+c\sqrt{x})}\right)}{d} - \frac{(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2c(\sqrt{-d}+\sqrt{e}\sqrt{x})}{(c\sqrt{-d}+\sqrt{e})(1+c\sqrt{x})}\right)}{d}$$

[Out] a*ln(x)/d+2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1+c*x^(1/2)))/d-(a+b*arctanh(c*x^(1/2)))*ln(2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2))/(1+c*x^(1/2)))/d-(a+b*arctanh(c*x^(1/2)))*ln(2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(-d)^(1/2)+e^(1/2))/(1+c*x^(1/2)))/d-b*polylog(2,-c*x^(1/2))/d+b*polylog(2,c*x^(1/2))/d-b*polylog(2,1-2/(1+c*x^(1/2)))/d+1/2*b*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2))/(1+c*x^(1/2)))/d+1/2*b*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(-d)^(1/2)+e^(1/2))/(1+c*x^(1/2)))/d

Rubi [A]

time = 0.43, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {36, 29, 31, 1607, 6139, 6031, 6191, 6057, 2449, 2352, 2497}

$$\frac{(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2(\sqrt{-d}-\sqrt{e}\sqrt{x})}{(c\sqrt{x+1})(\sqrt{-d}-\sqrt{e})}\right)}{d} - \frac{(a+b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2(\sqrt{-d}+\sqrt{e}\sqrt{x})}{(c\sqrt{x+1})(\sqrt{-d}+\sqrt{e})}\right)}{d} + \frac{2 \log\left(\frac{2}{\sqrt{x+1}}\right) (a+b \tanh^{-1}(c\sqrt{x}))}{d} + \frac{a \log(x)}{d} + \frac{M_1\left(1-\frac{x(\sqrt{-d}-\sqrt{e}\sqrt{x})}{(c\sqrt{-d}-\sqrt{e})(\sqrt{x+1})}\right)}{2d} + \frac{M_1\left(1-\frac{x(\sqrt{-d}+\sqrt{e}\sqrt{x})}{(c\sqrt{-d}+\sqrt{e})(\sqrt{x+1})}\right)}{2d} - \frac{M_1\left(1-\frac{2}{\sqrt{x+1}}\right)}{d} - \frac{M_1(-c\sqrt{x})}{d} + \frac{M_1(c\sqrt{x})}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x*(d + e*x)), x]

[Out] (2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 + c*Sqrt[x])])/d - ((a + b*ArcTanh[c*Sqrt[x]])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] - Sqrt[e])*(1 + c*Sqrt[x]))])/d - ((a + b*ArcTanh[c*Sqrt[x]])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(1 + c*Sqrt[x]))])/d + (a*Log[x])/d - (b*PolyLog[2, 1 - 2/(1 + c*Sqrt[x])])/d + (b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] - Sqrt[e])*(1 + c*Sqrt[x]))])/d + (b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(1 + c*Sqrt[x]))])/d - (b*PolyLog[2, -(c*Sqrt[x])])/d + (b*PolyLog[2, c*Sqrt[x]])/d

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
  x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
  x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6191

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_)^(m_.))*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] := With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{x(d + ex)} dx &= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{dx + ex^3} dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x(d + ex^2)} dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \left(\frac{a + b \tanh^{-1}(cx)}{dx} - \frac{ex(a + b \tanh^{-1}(cx))}{d(d + ex^2)}\right) dx, x, \sqrt{x}\right) \\
&= \frac{2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x} dx, x, \sqrt{x}\right)}{d} - \frac{(2e)\text{Subst}\left(\int \frac{x(a + b \tanh^{-1}(cx))}{d + ex^2} dx, x, \sqrt{x}\right)}{d} \\
&= \frac{a \log(x)}{d} - \frac{b\text{Li}_2(-c\sqrt{x})}{d} + \frac{b\text{Li}_2(c\sqrt{x})}{d} - \frac{(2e)\text{Subst}\left(\int \left(-\frac{a + b \tanh^{-1}(cx)}{2\sqrt{e}(\sqrt{-d} - \sqrt{e})}\right) dx, x, \sqrt{x}\right)}{d} \\
&= \frac{a \log(x)}{d} - \frac{b\text{Li}_2(-c\sqrt{x})}{d} + \frac{b\text{Li}_2(c\sqrt{x})}{d} + \frac{\sqrt{e} \text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{\sqrt{-d} - \sqrt{e} x} dx, x, \sqrt{x}\right)}{d} \\
&= \frac{2(a + b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2}{1 + c\sqrt{x}}\right)}{d} - \frac{(a + b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2c(\sqrt{-d} - \sqrt{e})}{c\sqrt{-d} - \sqrt{e}}\right)}{d} \\
&= \frac{2(a + b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2}{1 + c\sqrt{x}}\right)}{d} - \frac{(a + b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2c(\sqrt{-d} - \sqrt{e})}{c\sqrt{-d} - \sqrt{e}}\right)}{d} \\
&= \frac{2(a + b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2}{1 + c\sqrt{x}}\right)}{d} - \frac{(a + b \tanh^{-1}(c\sqrt{x})) \log\left(\frac{2c(\sqrt{-d} - \sqrt{e})}{c\sqrt{-d} - \sqrt{e}}\right)}{d}
\end{aligned}$$

Mathematica [A]

time = 0.83, size = 302, normalized size = 0.84

$$\frac{-4b \tanh^{-1}(c\sqrt{x})^2 - 4b \tanh^{-1}(c\sqrt{x}) \log(1 - e^{-2 \tanh^{-1}(c\sqrt{x})}) + 2b \tanh^{-1}(c\sqrt{x}) \log\left(1 + \frac{(c^2 d + e) \tanh^{-1}(c\sqrt{x})}{d + 2c\sqrt{d}\sqrt{e} - e}\right) + 2b \tanh^{-1}(c\sqrt{x}) \log\left(1 + \frac{(c^2 d + e) \tanh^{-1}(c\sqrt{x})}{d + 2c\sqrt{d}\sqrt{e} - e}\right) - 2a \log(x) + 2a \log(d + ex) + 2b \text{PolyLog}(2, e^{-2 \tanh^{-1}(c\sqrt{x})}) + 8 \text{PolyLog}(2, -\frac{(c^2 d + e) \tanh^{-1}(c\sqrt{x})}{d + 2c\sqrt{d}\sqrt{e} - e}) + 8 \text{PolyLog}(2, -\frac{(c^2 d + e) \tanh^{-1}(c\sqrt{x})}{d + 2c\sqrt{d}\sqrt{e} - e})}{2d}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x*(d + e*x)), x]`

```
[Out] -1/2*(-4*b*ArcTanh[c*Sqrt[x]]^2 - 4*b*ArcTanh[c*Sqrt[x]]*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])] + 2*b*ArcTanh[c*Sqrt[x]]*Log[1 + ((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]])])]
```

$$\frac{[c\sqrt{x}]]}{(c^2d - 2c\sqrt{-d}\sqrt{e} - e)} + 2b\operatorname{ArcTanh}[c\sqrt{x}] * \operatorname{Log}[1 + ((c^2d + e)E^{(2\operatorname{ArcTanh}[c\sqrt{x}])})]/(c^2d + 2c\sqrt{-d}\sqrt{e} - e) - 2a\operatorname{Log}[x] + 2a\operatorname{Log}[d + ex] + 2b\operatorname{PolyLog}[2, E^{(-2\operatorname{ArcTanh}[c\sqrt{x}])}] + b\operatorname{PolyLog}[2, -(((c^2d + e)E^{(2\operatorname{ArcTanh}[c\sqrt{x}])})/(c^2d - 2c\sqrt{-d}\sqrt{e} - e))] + b\operatorname{PolyLog}[2, -(((c^2d + e)E^{(2\operatorname{ArcTanh}[c\sqrt{x}])})/(c^2d + 2c\sqrt{-d}\sqrt{e} - e)))]/d$$

Maple [A]

time = 0.64, size = 540, normalized size = 1.51

method	result
derivativedivides	$-\frac{a \ln(c^2ex+c^2d)}{d} + \frac{2a \ln(c\sqrt{x})}{d} - \frac{b \operatorname{arctanh}(c\sqrt{x}) \ln(c^2ex+c^2d)}{d} + \frac{2b \operatorname{arctanh}(c\sqrt{x}) \ln(c\sqrt{x})}{d} + \frac{b \ln(c\sqrt{x})}{d}$
default	$-\frac{a \ln(c^2ex+c^2d)}{d} + \frac{2a \ln(c\sqrt{x})}{d} - \frac{b \operatorname{arctanh}(c\sqrt{x}) \ln(c^2ex+c^2d)}{d} + \frac{2b \operatorname{arctanh}(c\sqrt{x}) \ln(c\sqrt{x})}{d} + \frac{b \ln(c\sqrt{x})}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^(1/2)))/x/(e*x+d),x,method=_RETURNVERBOSE)`

[Out]
$$-a/d*\ln(c^2*e*x+c^2*d)+2*a/d*\ln(c*x^(1/2))-b*\operatorname{arctanh}(c*x^(1/2))/d*\ln(c^2*e*x+c^2*d)+2*b*\operatorname{arctanh}(c*x^(1/2))/d*\ln(c*x^(1/2))+1/2*b/d*\ln(1+c*x^(1/2))*\ln(c^2*e*x+c^2*d)-1/2*b/d*\ln(1+c*x^(1/2))*\ln((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^(1/2)+e))-1/2*b/d*\ln(1+c*x^(1/2))*\ln((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e)/(c*(-d*e)^(1/2)-e))-1/2*b/d*\operatorname{dilog}((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^(1/2)+e))-1/2*b/d*\operatorname{dilog}((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e)/(c*(-d*e)^(1/2)-e))-1/2*b/d*\ln(c*x^(1/2)-1)*\ln(c^2*e*x+c^2*d)+1/2*b/d*\ln(c*x^(1/2)-1)*\ln((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e)/(c*(-d*e)^(1/2)-e))+1/2*b/d*\ln(c*x^(1/2)-1)*\ln((c*(-d*e)^(1/2)+e*(c*x^(1/2)-1)+e)/(c*(-d*e)^(1/2)+e))+1/2*b/d*\operatorname{dilog}((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e)/(c*(-d*e)^(1/2)-e))+1/2*b/d*\operatorname{dilog}((c*(-d*e)^(1/2)+e*(c*x^(1/2)-1)+e)/(c*(-d*e)^(1/2)+e))-b/d*\operatorname{dilog}(1+c*x^(1/2))-b/d*\ln(c*x^(1/2))*\ln(1+c*x^(1/2))-b/d*\operatorname{dilog}(c*x^(1/2))$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/x/(e*x+d),x, algorithm="maxima")`

[Out]
$$-a*(\log(x*e + d)/d - \log(x)/d) + b*\operatorname{integrate}(1/2*\log(c*\sqrt{x} + 1)/((x^(3/2)*e + d*\sqrt{x})*\sqrt{x}), x) - b*\operatorname{integrate}(1/2*\log(-c*\sqrt{x} + 1)/((x^(3/2)*e + d*\sqrt{x})*\sqrt{x}), x)$$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*sqrt(x)) + a)/(x^2*e + d*x), x)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)/((e*x + d)*x), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{atanh}(c \sqrt{x})}{x (d + e x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/(x*(d + e*x)),x)
```

```
[Out] int((a + b*atanh(c*x^(1/2)))/(x*(d + e*x)), x)
```

3.48
$$\int \frac{a+b \tanh^{-1}\left(c \sqrt{x}\right)}{x^2(d+e x)} d x$$

Optimal. Leaf size=413

$$-\frac{b c}{d \sqrt{x}}+\frac{b c^2 \tanh^{-1}\left(c \sqrt{x}\right)}{d}-\frac{a+b \tanh^{-1}\left(c \sqrt{x}\right)}{d x}-\frac{2 e\left(a+b \tanh^{-1}\left(c \sqrt{x}\right)\right) \log \left(\frac{2}{1+c \sqrt{x}}\right)}{d^2}+\frac{e\left(a+b \tanh^{-1}\left(c \sqrt{x}\right)\right)}{d^2}$$

[Out] $b*c^2*arctanh(c*x^(1/2))/d+(-a-b*arctanh(c*x^(1/2)))/d/x-a*e*ln(x)/d^2-2*e*(a+b*arctanh(c*x^(1/2)))*ln(2/(1+c*x^(1/2)))/d^2+e*(a+b*arctanh(c*x^(1/2)))*ln(2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2))/(1+c*x^(1/2)))/d^2+e*(a+b*arctanh(c*x^(1/2)))*ln(2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(-d)^(1/2)+e^(1/2))/(1+c*x^(1/2)))/d^2+b*e*polylog(2,-c*x^(1/2))/d^2-b*e*polylog(2,c*x^(1/2))/d^2+b*e*polylog(2,1-2/(1+c*x^(1/2)))/d^2-1/2*b*e*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2))/(1+c*x^(1/2)))/d^2-1/2*b*e*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(-d)^(1/2)+e^(1/2))/(1+c*x^(1/2)))/d^2-b*c/d/x^(1/2)$

Rubi [A]

time = 0.53, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {46, 1607, 6129, 6037, 331, 212, 6139, 6031, 6191, 6057, 2449, 2352, 2497}

$$\frac{2c \log\left(\frac{2}{1+c \sqrt{x}}\right)+e\left(a+b \tanh^{-1}\left(c \sqrt{x}\right)\right)}{d^2}+\frac{b c^2 \tanh^{-1}\left(c \sqrt{x}\right)}{d}+\frac{e\left(a+b \tanh^{-1}\left(c \sqrt{x}\right)\right) \log\left(\frac{2}{1+c \sqrt{x}}\right)}{d^2}+\frac{e\left(a+b \tanh^{-1}\left(c \sqrt{x}\right)\right)}{d^2}-\frac{a+b \tanh^{-1}\left(c \sqrt{x}\right)}{d x}-\frac{2 e\left(a+b \tanh^{-1}\left(c \sqrt{x}\right)\right) \log\left(\frac{2}{1+c \sqrt{x}}\right)}{d^2}+\frac{e\left(a+b \tanh^{-1}\left(c \sqrt{x}\right)\right)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x^2*(d + e*x)),x]

[Out] $-((b*c)/(d*Sqrt[x]))+(b*c^2*ArcTanh[c*Sqrt[x]])/d-(a+b*ArcTanh[c*Sqrt[x]])/(d*x)-(2*e*(a+b*ArcTanh[c*Sqrt[x]])*Log[2/(1+c*Sqrt[x])])/d^2+(e*(a+b*ArcTanh[c*Sqrt[x]])*Log[(2*c*(Sqrt[-d]-Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d]-Sqrt[e])*(1+c*Sqrt[x]))])/d^2+(e*(a+b*ArcTanh[c*Sqrt[x]])*Log[(2*c*(Sqrt[-d]+Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d]+Sqrt[e])*(1+c*Sqrt[x]))])/d^2-(a*e*Log[x])/d^2+(b*e*PolyLog[2,1-2/(1+c*Sqrt[x])])/d^2-(b*e*PolyLog[2,1-(2*c*(Sqrt[-d]-Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d]-Sqrt[e])*(1+c*Sqrt[x]))])/d^2-(b*e*PolyLog[2,1-(2*c*(Sqrt[-d]+Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d]+Sqrt[e])*(1+c*Sqrt[x]))])/d^2+(b*e*PolyLog[2,-(c*Sqrt[x])])/d^2-(b*e*PolyLog[2,c*Sqrt[x]])/d^2$

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +

$n + 2, 0]$)

Rule 212

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2])) \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot (x/\text{Rt}[a, 2])], x] /;$ $\text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 331

$\text{Int}[(c_ \cdot)(x_)^{(m_)} \cdot ((a_ + (b_ \cdot)(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(c \cdot x)^{(m+1)} \cdot ((a + b \cdot x^n)^{(p+1)} / (a \cdot c \cdot (m+1))), x] - \text{Dist}[b \cdot ((m+n \cdot (p+1) + 1) / (a \cdot c^n \cdot (m+1))), \text{Int}[(c \cdot x)^{(m+n)} \cdot (a + b \cdot x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 1607

$\text{Int}[(u_ \cdot)((a_ \cdot)(x_)^{(p_)} + (b_ \cdot)(x_)^{(q_)})^{(n_)}, x_Symbol] \rightarrow \text{Int}[u \cdot x^{(n \cdot p)} \cdot (a + b \cdot x^{(q-p)})^n, x] /;$ $\text{FreeQ}\{a, b, p, q\}, x] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{PosQ}[q-p]$

Rule 2352

$\text{Int}[\text{Log}[(c_ \cdot)(x_)] / ((d_ + (e_ \cdot)(x_)), x_Symbol] \rightarrow \text{Simp}[(-e^{-1}) \cdot \text{PolyLog}[2, 1 - c \cdot x], x] /;$ $\text{FreeQ}\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c \cdot d, 0]$

Rule 2449

$\text{Int}[\text{Log}[(c_ / ((d_ + (e_ \cdot)(x_)))] / ((f_ + (g_ \cdot)(x_)^2), x_Symbol] \rightarrow \text{Dist}[-e/g, \text{Subst}[\text{Int}[\text{Log}[2 \cdot d \cdot x] / (1 - 2 \cdot d \cdot x), x], x, 1/(d + e \cdot x)], x] /;$ $\text{FreeQ}\{c, d, e, f, g\}, x] \ \&\& \ \text{EqQ}[c, 2 \cdot d] \ \&\& \ \text{EqQ}[e^2 \cdot f + d^2 \cdot g, 0]$

Rule 2497

$\text{Int}[\text{Log}[u_] \cdot (Pq_)^{(m_)}, x_Symbol] \rightarrow \text{With}[\{C = \text{FullSimplify}[Pq^m \cdot ((1-u)/D[u, x])]\}, \text{Simp}[C \cdot \text{PolyLog}[2, 1-u], x] /;$ $\text{FreeQ}[C, x] /;$ $\text{IntegerQ}[m] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{RationalFunctionQ}[u, x] \ \&\& \ \text{LeQ}[\text{RationalFunctionExponents}[u, x][[2]], \text{Expon}[Pq, x]]$

Rule 6031

$\text{Int}[(a_ + \text{ArcTanh}[(c_ \cdot)(x_)] \cdot (b_ \cdot)) / (x_), x_Symbol] \rightarrow \text{Simp}[a \cdot \text{Log}[x], x] + (-\text{Simp}[(b/2) \cdot \text{PolyLog}[2, (-c) \cdot x], x] + \text{Simp}[(b/2) \cdot \text{PolyLog}[2, c \cdot x], x]) /;$ $\text{FreeQ}\{a, b, c\}, x]$

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
&& IntegerQ[m])) && NeQ[m, -1]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)*(b_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x)
)/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)*(b_.)]^(p_.)*((f_.)*(x_)^(m_.)))/((d_.) + (
e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)*(b_.)]*(x_)^(m_.)))/((d_.) + (e_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6191

```
Int[((a_.) + ArcTanh[(c_.)*(x_)*(b_.)]^(p_.)*((f_.)*(x_)^(m_.))*((d_.) + (e
_.)*(x_)^2)^(q_.), x_Symbol] := With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{x^2(d + ex)} dx &= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{dx^3 + ex^5} dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^3(d + ex^2)} dx, x, \sqrt{x}\right) \\
&= \frac{2\text{Subst}\left(\int \frac{a+b \tanh^{-1}(cx)}{x^3} dx, x, \sqrt{x}\right)}{d} - \frac{(2e)\text{Subst}\left(\int \frac{a+b \tanh^{-1}(cx)}{x(d+ex^2)} dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{a + b \tanh^{-1}(c\sqrt{x})}{dx} + \frac{(bc)\text{Subst}\left(\int \frac{1}{x^2(1-c^2x^2)} dx, x, \sqrt{x}\right)}{d} - \frac{(2e)\text{Subst}\left(\int \frac{1}{x(d+ex^2)} dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{bc}{d\sqrt{x}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{dx} + \frac{(bc^3)\text{Subst}\left(\int \frac{1}{1-c^2x^2} dx, x, \sqrt{x}\right)}{d} - \frac{(2e)\text{Subst}\left(\int \frac{1}{x(d+ex^2)} dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{bc}{d\sqrt{x}} + \frac{bc^2 \tanh^{-1}(c\sqrt{x})}{d} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{dx} - \frac{ae \log(x)}{d^2} + \frac{be \text{Li}_2\left(-\frac{c\sqrt{x}}{d}\right)}{d^2} \\
&= -\frac{bc}{d\sqrt{x}} + \frac{bc^2 \tanh^{-1}(c\sqrt{x})}{d} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{dx} - \frac{ae \log(x)}{d^2} + \frac{be \text{Li}_2\left(-\frac{c\sqrt{x}}{d}\right)}{d^2} \\
&= -\frac{bc}{d\sqrt{x}} + \frac{bc^2 \tanh^{-1}(c\sqrt{x})}{d} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{dx} - \frac{2e(a + b \tanh^{-1}(c\sqrt{x}))}{d^2} \\
&= -\frac{bc}{d\sqrt{x}} + \frac{bc^2 \tanh^{-1}(c\sqrt{x})}{d} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{dx} - \frac{2e(a + b \tanh^{-1}(c\sqrt{x}))}{d^2} \\
&= -\frac{bc}{d\sqrt{x}} + \frac{bc^2 \tanh^{-1}(c\sqrt{x})}{d} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{dx} - \frac{2e(a + b \tanh^{-1}(c\sqrt{x}))}{d^2}
\end{aligned}$$

Mathematica [A]

time = 1.13, size = 360, normalized size = 0.87

$$\frac{a}{dx} - \frac{2ae \log(\sqrt{x})}{d^2} + \frac{ae \log(d + ex)}{d^2} + 2bc^2 \left(-\frac{bc}{\sqrt{x}} + \frac{bc^2 \tanh^{-1}(c\sqrt{x})}{d} + \frac{\left(\frac{d-dx^2}{d}\right) + c \tanh^{-1}(c\sqrt{x}) + 2b \log(1 - e^{-2 \tanh^{-1}(c\sqrt{x})})}{2c^2 d} - d \text{PolyLog}(2, e^{-2 \tanh^{-1}(c\sqrt{x})}) + \frac{c \left(2 \tanh^{-1}(c\sqrt{x}) \left(-\tanh^{-1}(c\sqrt{x}) + \log\left(1 + \frac{c^2 dx + c^2 \sqrt{d}\sqrt{x}}{d}\right) + \log\left(1 + \frac{c^2 dx + c^2 \sqrt{d}\sqrt{x}}{d}\right)\right) + \text{PolyLog}(2, \frac{c^2 dx + c^2 \sqrt{d}\sqrt{x}}{d}) + \text{PolyLog}(2, \frac{c^2 dx + c^2 \sqrt{d}\sqrt{x}}{d})}{2c^2 d} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x^2*(d + e*x)),x]
```

```
[Out] -(a/(d*x)) - (2*a*e*Log[Sqrt[x]])/d^2 + (a*e*Log[d + e*x])/d^2 + 2*b*c^4*(-
1/2*((c*d)/Sqrt[x] + ArcTanh[c*Sqrt[x]]*((d*(1 - c^2*x))/x + e*ArcTanh[c*Sq
rt[x]] + 2*e*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) - e*PolyLog[2, E^(-2*ArcTa
nh[c*Sqrt[x]])])/(c^4*d^2) + (e*(2*ArcTanh[c*Sqrt[x]]*(-ArcTanh[c*Sqrt[x]]
+ Log[1 + ((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]])])/(c^2*d - 2*c*Sqrt[-d]*Sqrt
[e] - e)) + Log[1 + ((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]])])/(c^2*d + 2*c*Sqr
t[-d]*Sqrt[e] - e)) + PolyLog[2, -(((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))/
(c^2*d - 2*c*Sqrt[-d]*Sqrt[e] - e))] + PolyLog[2, -(((c^2*d + e)*E^(2*ArcTa
nh[c*Sqrt[x]])/(c^2*d + 2*c*Sqrt[-d]*Sqrt[e] - e)))]/(4*c^4*d^2))
```

Maple [A]

time = 0.67, size = 683, normalized size = 1.65

method	result
derivativedivides	$2c^2 \left(\frac{ae \ln(c^2 ex + c^2 d)}{2c^2 d^2} - \frac{a}{2d c^2 x} - \frac{ae \ln(c\sqrt{x})}{c^2 d^2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) e \ln(c^2 ex + c^2 d)}{2c^2 d^2} - \frac{b \operatorname{arctanh}(c\sqrt{x})}{2d c^2 x} \right)$
default	$2c^2 \left(\frac{ae \ln(c^2 ex + c^2 d)}{2c^2 d^2} - \frac{a}{2d c^2 x} - \frac{ae \ln(c\sqrt{x})}{c^2 d^2} + \frac{b \operatorname{arctanh}(c\sqrt{x}) e \ln(c^2 ex + c^2 d)}{2c^2 d^2} - \frac{b \operatorname{arctanh}(c\sqrt{x})}{2d c^2 x} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^2*(1/2*a/c^2*e/d^2*ln(c^2*e*x+c^2*d)-1/2*a/d/c^2/x-a/c^2/d^2*e*ln(c*x^(
1/2))+1/2*b/c^2*arctanh(c*x^(1/2))*e/d^2*ln(c^2*e*x+c^2*d)-1/2*b*arctanh(c*
x^(1/2))/d/c^2/x-b/c^2*arctanh(c*x^(1/2))/d^2*e*ln(c*x^(1/2))+1/4*b/d*ln(1+
c*x^(1/2))-1/4*b/d*ln(c*x^(1/2)-1)-1/2*b/d/c/x^(1/2)+1/2*b/c^2/d^2*e*dilog(
1+c*x^(1/2))+1/2*b/c^2/d^2*e*ln(c*x^(1/2))*ln(1+c*x^(1/2))+1/2*b/c^2/d^2*e*
dilog(c*x^(1/2))-1/4*b/c^2/d^2*e*ln(1+c*x^(1/2))*ln(c^2*e*x+c^2*d)+1/4*b/c^
2/d^2*e*ln(1+c*x^(1/2))*ln((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^(1/
2)+e))+1/4*b/c^2/d^2*e*ln(1+c*x^(1/2))*ln((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e
)/(c*(-d*e)^(1/2)-e))+1/4*b/c^2/d^2*e*dilog((c*(-d*e)^(1/2)-e*(1+c*x^(1/2)
)+e)/(c*(-d*e)^(1/2)+e))+1/4*b/c^2/d^2*e*dilog((c*(-d*e)^(1/2)+e*(1+c*x^(1/2
))-e)/(c*(-d*e)^(1/2)-e))+1/4*b/c^2/d^2*e*ln(c*x^(1/2)-1)*ln(c^2*e*x+c^2*d)
-1/4*b/c^2/d^2*e*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e)/(c(
-d*e)^(1/2)-e))-1/4*b/c^2/d^2*e*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(1/2)+e*(c*x^(
1/2)-1)+e)/(c*(-d*e)^(1/2)+e))-1/4*b/c^2/d^2*e*dilog((c*(-d*e)^(1/2)-e*(c*x
```

$$\frac{\sqrt{x+1}-e}{(c\sqrt{x+1}-e)} - \frac{1}{4} \frac{b}{c^2 d^2 e} \operatorname{dilog}\left(\frac{c\sqrt{x+1}+e}{c\sqrt{x+1}-e}\right)$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2/(e*x+d),x, algorithm="maxima")

[Out] a*(e*log(x*e + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + b*integrate(1/2*log(c*sqrt(x) + 1)/((x^(5/2)*e + d*x^(3/2))*sqrt(x)), x) - b*integrate(1/2*log(-c*sqrt(x) + 1)/((x^(5/2)*e + d*x^(3/2))*sqrt(x)), x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2/(e*x+d),x, algorithm="fricas")

[Out] integral((b*arctanh(c*sqrt(x)) + a)/(x^3*e + d*x^2), x)

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**(1/2)))/x**2/(e*x+d),x)

[Out] Timed out

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2/(e*x+d),x, algorithm="giac")

[Out] integrate((b*arctanh(c*sqrt(x)) + a)/((e*x + d)*x^2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{atanh}(c \sqrt{x})}{x^2 (d + e x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^(1/2)))/(x^2*(d + e*x)),x)

[Out] int((a + b*atanh(c*x^(1/2)))/(x^2*(d + e*x)), x)


```
Int[((a_) + (b_.)*(x_)^(m_))*((c_) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)/(x_), x_Symbol] := Simp[a*Log[x], x] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
```

; FreeQ[{a, b, c}, x]

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
&& IntegerQ[m])) && NeQ[m, -1]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := S
imp[(- (a + b*ArcTanh[c*x])*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/(c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/(c*d + e)*(1 + c*x))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_)^(m_.))/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6191

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_)^(m_.))*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] := With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c,
d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{x^3(d + ex)} dx &= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{dx^5 + ex^7} dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^5(d + ex^2)} dx, x, \sqrt{x}\right) \\
&= \frac{2\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^5} dx, x, \sqrt{x}\right)}{d} - \frac{(2e)\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^3(d + ex^2)} dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{a + b \tanh^{-1}(c\sqrt{x})}{2dx^2} + \frac{(bc)\text{Subst}\left(\int \frac{1}{x^4(1 - c^2x^2)} dx, x, \sqrt{x}\right)}{2d} - \frac{(2e)\text{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^3} dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{bc}{6dx^{3/2}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{2dx^2} + \frac{e(a + b \tanh^{-1}(c\sqrt{x}))}{d^2x} + \frac{(bc^3)\text{Subst}\left(\int \frac{1}{x^4} dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{2dx^2} + \frac{e(a + b \tanh^{-1}(c\sqrt{x}))}{d^2x} \\
&= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4 \tanh^{-1}(c\sqrt{x})}{2d} - \frac{bc^2e \tanh^{-1}(c\sqrt{x})}{d^2} - \frac{a}{d} \\
&= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4 \tanh^{-1}(c\sqrt{x})}{2d} - \frac{bc^2e \tanh^{-1}(c\sqrt{x})}{d^2} - \frac{a}{d} \\
&= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4 \tanh^{-1}(c\sqrt{x})}{2d} - \frac{bc^2e \tanh^{-1}(c\sqrt{x})}{d^2} - \frac{a}{d} \\
&= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4 \tanh^{-1}(c\sqrt{x})}{2d} - \frac{bc^2e \tanh^{-1}(c\sqrt{x})}{d^2} - \frac{a}{d} \\
&= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4 \tanh^{-1}(c\sqrt{x})}{2d} - \frac{bc^2e \tanh^{-1}(c\sqrt{x})}{d^2} - \frac{a}{d} \\
&= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4 \tanh^{-1}(c\sqrt{x})}{2d} - \frac{bc^2e \tanh^{-1}(c\sqrt{x})}{d^2} - \frac{a}{d}
\end{aligned}$$

Mathematica [A]

time = 1.83, size = 394, normalized size = 0.78

$\frac{3bd^2 - 6bdex - 6bd^2 \log(x) + 6bd^2 \log(d + ex) + 4\left(\frac{d^2 \sqrt{x}}{d + ex} + 2bdx - 6bx\right) - 3 \tanh^{-1}\left(\frac{c\sqrt{x}}{d + ex}\right) \left(d - 1 + c^2x\right) \left(d + c^2dx - 2bx\right) + 2b^2 \tanh^{-1}\left(\frac{c\sqrt{x}}{d + ex}\right) + 4b^2 \log\left(1 - c^2 \tanh^{-1}\left(\frac{c\sqrt{x}}{d + ex}\right)\right) + 6b^2 \log\left(\frac{2x - 2c \tanh^{-1}\left(\frac{c\sqrt{x}}{d + ex}\right)}{d + ex}\right) + 3b^2 \left(2 \tanh^{-1}\left(\frac{c\sqrt{x}}{d + ex}\right) - \tanh^{-1}\left(\frac{c\sqrt{x}}{d + ex}\right) + \log\left(1 + \frac{2d \sqrt{x} \tanh^{-1}\left(\frac{c\sqrt{x}}{d + ex}\right)}{d + ex}\right) + \log\left(1 + \frac{2d \sqrt{x} \tanh^{-1}\left(\frac{c\sqrt{x}}{d + ex}\right)}{d + ex}\right)\right) + 2b \log\left(\frac{2x - 2c \tanh^{-1}\left(\frac{c\sqrt{x}}{d + ex}\right)}{d + ex}\right) + 2b \log\left(\frac{2x - 2c \tanh^{-1}\left(\frac{c\sqrt{x}}{d + ex}\right)}{d + ex}\right)\right)$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x^3*(d + e*x)),x]

[Out]
$$-1/6*(3*a*d^2 - 6*a*d*e*x - 6*a*e^2*x^2*\text{Log}[x] + 6*a*e^2*x^2*\text{Log}[d + e*x] + b*(c*d*\text{Sqrt}[x]*(d + 3*c^2*d*x - 6*e*x) - 3*\text{ArcTanh}[c*\text{Sqrt}[x]]*(d*(-1 + c^2*x)*(d + c^2*d*x - 2*e*x) + 2*e^2*x^2*\text{ArcTanh}[c*\text{Sqrt}[x]] + 4*e^2*x^2*\text{Log}[1 - E^{(-2*\text{ArcTanh}[c*\text{Sqrt}[x]])}])) + 6*e^2*x^2*\text{PolyLog}[2, E^{(-2*\text{ArcTanh}[c*\text{Sqrt}[x])}]] + 3*e^2*x^2*(2*\text{ArcTanh}[c*\text{Sqrt}[x]]*(-\text{ArcTanh}[c*\text{Sqrt}[x]] + \text{Log}[1 + ((c^2*d + e)*E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])})]/(c^2*d - 2*c*\text{Sqrt}[-d]*\text{Sqrt}[e] - e)) + \text{Log}[1 + ((c^2*d + e)*E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])})]/(c^2*d + 2*c*\text{Sqrt}[-d]*\text{Sqrt}[e] - e))) + \text{PolyLog}[2, -(((c^2*d + e)*E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])})]/(c^2*d - 2*c*\text{Sqrt}[-d]*\text{Sqrt}[e] - e))] + \text{PolyLog}[2, -(((c^2*d + e)*E^{(2*\text{ArcTanh}[c*\text{Sqrt}[x])})]/(c^2*d + 2*c*\text{Sqrt}[-d]*\text{Sqrt}[e] - e)))))/(d^3*x^2)$$

Maple [A]

time = 0.65, size = 808, normalized size = 1.60

method	result
derivativedivides	$2c^4 \left(-\frac{b}{12d c^3 x^{\frac{3}{2}}} - \frac{a}{4d c^4 x^2} + \frac{be}{2c^3 d^2 \sqrt{x}} - \frac{b \ln(1+c\sqrt{x})e}{4c^2 d^2} + \frac{b \ln(c\sqrt{x}-1)e}{4c^2 d^2} - \frac{a e^2 \ln(c^2 e x + c^2 d)}{2c^4 d^3} + \dots \right)$
default	$2c^4 \left(-\frac{b}{12d c^3 x^{\frac{3}{2}}} - \frac{a}{4d c^4 x^2} + \frac{be}{2c^3 d^2 \sqrt{x}} - \frac{b \ln(1+c\sqrt{x})e}{4c^2 d^2} + \frac{b \ln(c\sqrt{x}-1)e}{4c^2 d^2} - \frac{a e^2 \ln(c^2 e x + c^2 d)}{2c^4 d^3} + \dots \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x^(1/2)))/x^3/(e*x+d),x,method=_RETURNVERBOSE)

[Out]
$$2*c^4*(1/2*b/c^4*arctanh(c*x^(1/2))/d^2*e/x-1/12*b/d/c^3/x^(3/2)-1/4*a/d/c^4/x^2+1/2*b/c^3/d^2/x^(1/2)*e-1/4*b*arctanh(c*x^(1/2))/d/c^4/x^2-1/4*b/c^2/d^2*\ln(1+c*x^(1/2))*e+1/4*b/c^2/d^2*\ln(c*x^(1/2)-1)*e-1/2*b/c^4/d^3*e^2*dilog(1+c*x^(1/2))-1/2*b/c^4/d^3*e^2*dilog(c*x^(1/2))-1/4*b/c^4/d^3*e^2*dilog((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^(1/2)+e))-1/4*b/c^4/d^3*e^2*dilog((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e)/(c*(-d*e)^(1/2)-e))+1/4*b/c^4/d^3*e^2*dilog((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e)/(c*(-d*e)^(1/2)-e))+1/4*b/c^4/d^3*e^2*dilog((c*(-d*e)^(1/2)+e*(c*x^(1/2)-1)+e)/(c*(-d*e)^(1/2)+e))-1/2*a/c^4*e^2/d^3*\ln(c^2*e*x+c^2*d)+a/c^4/d^3*e^2*\ln(c*x^(1/2))+1/2*a/c^4/d^2*e/x+1/8*b/d*\ln(1+c*x^(1/2))-1/8*b/d*\ln(c*x^(1/2)-1)-1/4*b/d/c/x^(1/2)-1/2*b/c^4*arctanh(c*x^(1/2))*e^2/d^3*\ln(c^2*e*x+c^2*d)+b/c^4*arctanh(c*x^(1/2))/d^3*e$$

$$\begin{aligned} &^2*\ln(c*x^{(1/2)})-1/2*b/c^4/d^3*e^2*\ln(c*x^{(1/2)})*\ln(1+c*x^{(1/2)})+1/4*b/c^4/ \\ &d^3*e^2*\ln(1+c*x^{(1/2)})*\ln(c^2*e*x+c^2*d)-1/4*b/c^4/d^3*e^2*\ln(1+c*x^{(1/2)}) \\ &*\ln((c*(-d*e)^{(1/2)}-e*(1+c*x^{(1/2)}))+e)/(c*(-d*e)^{(1/2)}+e))-1/4*b/c^4/d^3*e^ \\ &2*\ln(1+c*x^{(1/2)})*\ln((c*(-d*e)^{(1/2)}+e*(1+c*x^{(1/2)}))-e)/(c*(-d*e)^{(1/2)}-e) \\ &-1/4*b/c^4/d^3*e^2*\ln(c*x^{(1/2)}-1)*\ln(c^2*e*x+c^2*d)+1/4*b/c^4/d^3*e^2*\ln(c \\ &*x^{(1/2)}-1)*\ln((c*(-d*e)^{(1/2)}-e*(c*x^{(1/2)}-1))-e)/(c*(-d*e)^{(1/2)}-e))+1/4*b \\ &/c^4/d^3*e^2*\ln(c*x^{(1/2)}-1)*\ln((c*(-d*e)^{(1/2)}+e*(c*x^{(1/2)}-1))+e)/(c*(-d*e \\ &)^{(1/2)}+e))) \end{aligned}$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3/(e*x+d),x, algorithm="maxima")

[Out] $-1/2*a*(2*e^2*\log(x*e + d)/d^3 - 2*e^2*\log(x)/d^3 - (2*x*e - d)/(d^2*x^2))$
 $+ b*\integrate(1/2*\log(c*\sqrt{x} + 1)/((x^{(7/2)}*e + d*x^{(5/2)})*\sqrt{x}), x)$
 $- b*\integrate(1/2*\log(-c*\sqrt{x} + 1)/((x^{(7/2)}*e + d*x^{(5/2)})*\sqrt{x}), x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3/(e*x+d),x, algorithm="fricas")

[Out] $\integral((b*\arctanh(c*\sqrt{x}) + a)/(x^4*e + d*x^3), x)$

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**(1/2)))/x**3/(e*x+d),x)

[Out] Timed out

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3/(e*x+d),x, algorithm="giac")

[Out] integrate((b*arctanh(c*sqrt(x)) + a)/((e*x + d)*x^3), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{atanh}(c \sqrt{x})}{x^3 (d + e x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^(1/2)))/(x^3*(d + e*x)),x)

[Out] int((a + b*atanh(c*x^(1/2)))/(x^3*(d + e*x)), x)

Chapter 4

Appendix

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4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*           Small rewrite of logic in main function to make it*)
(*           match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*           is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*           antiderivative*)
(* "A" if result can be considered optimal*)
```

```

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A","none"}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
        ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
      ,(*ELSE*)(*result does not contains complex*)
      If[leafCountResult<=2*leafCountOptimal,
        finalresult={"A","none"}
        ,(*ELSE*)
        finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
      ]
    ]
    ,(*ELSE*)(*expnResult>expnOptimal*)
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      finalresult={"C","Result contains higher order function than in optimal. Order "<
    ,
    finalresult={"F","Contains unresolved integral."}
  ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)

```

(*9 = unknown function*)

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType, expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]], 2]],
      Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3, ExpnType[expn[[1]]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
    If[Head[expn]===RootSum,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
    9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

```

```

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
  }

```



```

ExpIntegralE, ExpIntegralEi, LogIntegral,
SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
Gamma, LogGamma, PolyGamma,
Zeta, PolyLog, ProductLog,
EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]

```

4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B","result has leaf size over 500,000. Avoiding possible recursion issues";
  fi;

  leaf_count_optimal := leafcount(optimal);
  ExpnType_result := ExpnType(result);
  ExpnType_optimal := ExpnType(optimal);

```

```

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#   is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
#   antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of r
                    convert(leaf_count_result,string)," vs. $2 (" ,
                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
            end if
        else #result contains complex but optimal is not
            if debug then
                print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex

```

```

    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well")
    fi;
    if leaf_count_result<=2*leaf_count_optimal then
        if debug then
            print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A","";
    else
        if debug then
            print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of o
                        convert(leaf_count_result,string)," $ vs. $2(",
                        convert(leaf_count_optimal,string),")=",convert(2*leaf_cou

    fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                  convert(ExpnType_result,string)," vs. order ",
                  convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function

```

```

# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+`) or type(expn,'*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,

```

```

    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

```

```

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type(expn,'*')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:

```

```

    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
            else:
                grade = "C"
                grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

```



```
#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#      Albert Rich to use with Sagemath. This is used to
#      grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#      'arctan2', 'floor', 'abs', 'log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#      issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

```

```

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)

```

```

    return 1
  else:
    return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
  else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or instan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_resu

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```